DYNAMICS OF TOPOLOGICAL EXCITATIONS IN LOW DIMENSIONAL MAGNETIC SYSTEMS

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Outline

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  - Topological Excitations
  - Quantization
  - Mobility

- **Applications to Magnetic Systems**
  - Domain walls
  - Quantum Hall Skyrmions

- **Conclusions**

Introduction

- **Why magnetic systems?**
- **Solitons and the dynamical properties**
  - J.A. Krumhansl and J.R. Schrieffer, 'PRB 11', 3535 (1975).
  - H.J. Mikeska (1978-1990)
- **Wall motion is dissipative**
  - Landau and Litshitz, 'Physik A', Soviet Unions, 153 (1935).
- **Quantum dissipation**
  - Caldeira and Leggett, 'Physica 121A', 587 (1983).
- **Walls and Magnons, the same origin!**
  - Castro Neto and Caldeira, 'PRL 67', 1960 (1991).
  - Castro Neto and Caldeira, 'PRE 48', 4037 (1993).

Dynamics of topological excitations

• Topological excitations

Let us consider systems described by

$$\mathcal{L}(x,t) = \frac{1}{2}\left(\dot{\phi}^2 - \phi'^2\right) - U(\phi). \qquad (1)$$

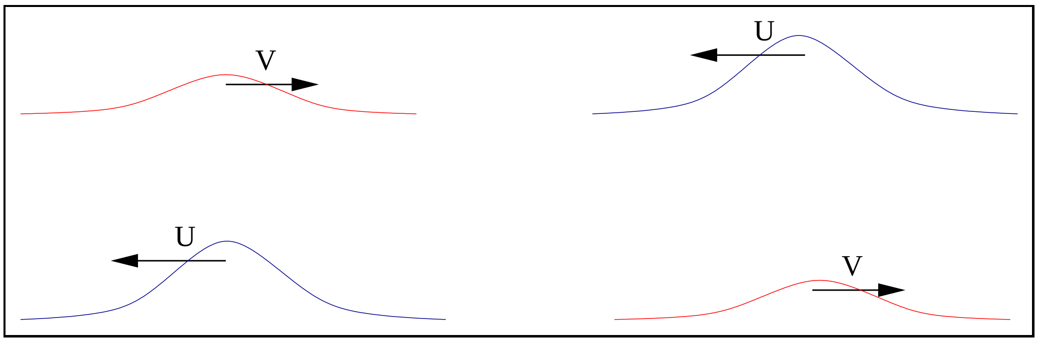
Energy density

$$\varepsilon(x,t) = \frac{1}{2}\left(\dot{\phi}^2 + \phi'^2\right) + U(\phi). \qquad (2)$$

Solitonic solutions

· Localization

· Conservation of the speed and form



Equation of motion

$$\ddot{\phi} - \phi'' = -\frac{\partial U(\phi)}{\partial \phi}. \tag{3}$$

Solution

$$\phi'' = \frac{\partial U(\phi)}{\partial \phi} \quad + \quad \text{Lorentz transformation.} \tag{4}$$

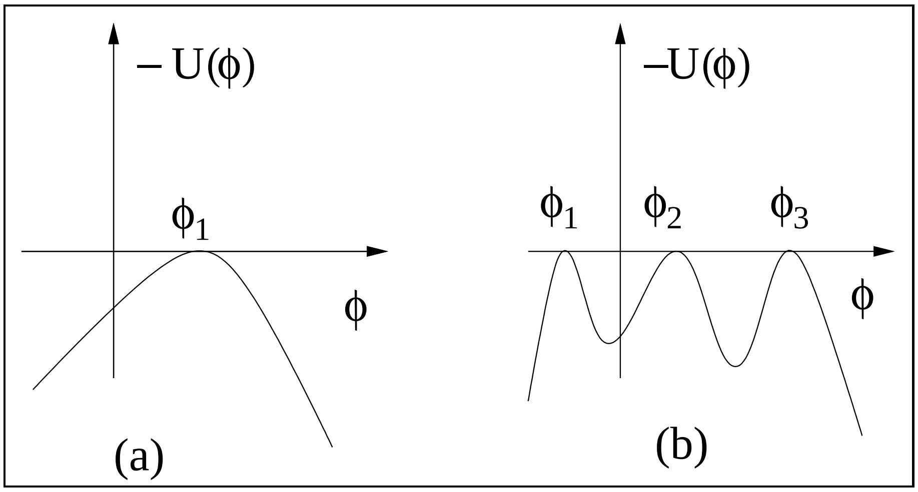
Boundary conditions

$$\lim_{x \to \pm\infty} \phi(x) = \text{Minima of U.} \tag{5}$$

Localized solutions have the form

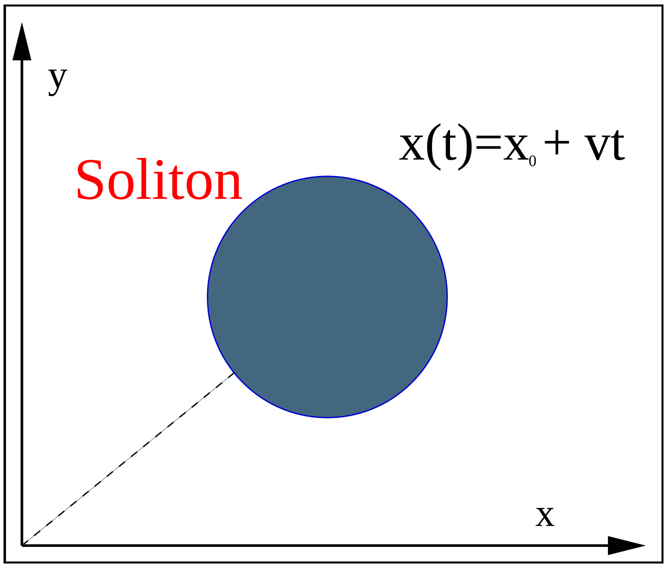
$$x - x_0 = \pm \int_{\phi(x_0)}^{\phi(x)} \frac{d\phi}{\sqrt{2U(\phi)}}. \tag{6}$$

Topological analysis



$$\boxed{\textbf{Topological charge} \propto \varphi(+\infty) - \varphi(-\infty)}$$

- **Quantization**



The solution of this problem is the expansion of the functional $V[\phi]$ up to second order which yields

$$\left[-\frac{d^2}{dx^2}+\frac{d^2U}{d\phi^2}\right]_{\phi_0}\eta_i(x)=\omega_i^2\eta_i(x), \tag{7}$$

We can write

$$\eta_i(x,t)\equiv\sum_i c_i(t)\eta_i(x), \tag{8}$$

and then

$$E_{n_i}=E_0+\hbar\sum_i(n_i+\frac{1}{2})\omega_i. \tag{9}$$

Collective coordinate method

$$\boxed{\boxed{\phi(x,t) = \phi_0(x - X(t)) + \sum_{i=1}^{\infty} q_i(t)\eta_i(x - X(t))}} \tag{10}$$

Using this expansion in 2-D

$$\boxed{\boxed{\widehat{H} = \frac{1}{2M}\left(\widehat{\mathbf{P}} - \widehat{\mathbf{P}}_{mg}\right)^2 + \sum_{mn} \hbar\omega_{mn} b^{\dagger}_{mn} b_{mn}}} \tag{11}$$

where

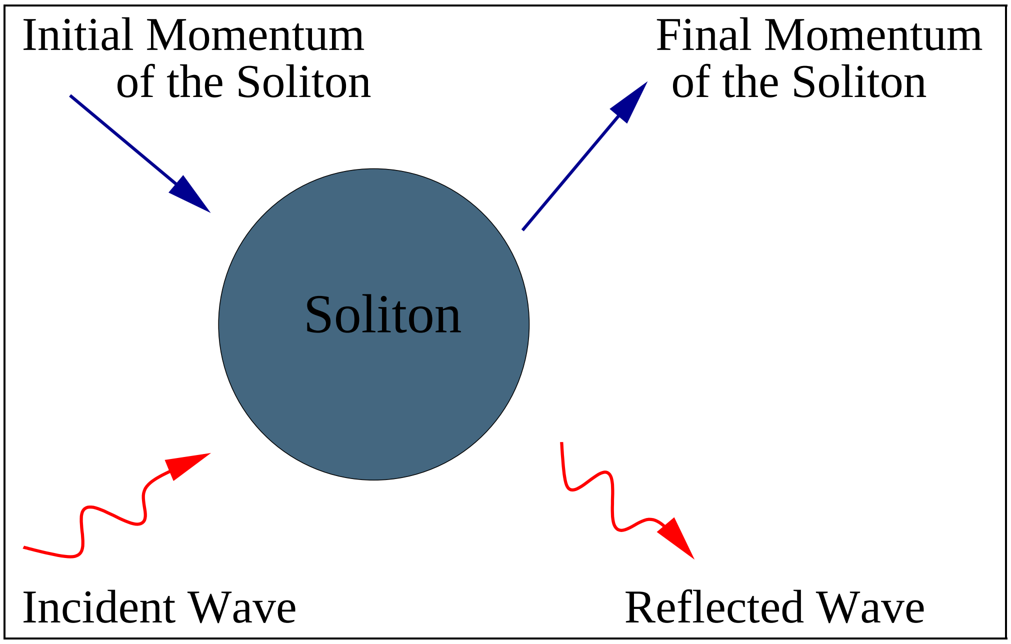
$$\widehat{\mathbf{P}}_{mg} = \sum_{mn,kl} \mathbf{D}_{mn,kl} b^{\dagger}_{mn} b_{kl} + \sum_{mn,kl} \mathbf{C}_{mn,kl}\left(b_{mn}b_{kl} - b^{\dagger}_{mn}b^{\dagger}_{kl}\right), \tag{12}$$

$$\mathbf{D}_{mn,kl} = \frac{1}{2}\left[\sqrt{\left(\frac{\omega_{kl}}{\omega_{mn}}\right)} + \sqrt{\left(\frac{\omega_{mn}}{\omega_{kl}}\right)}\right]\mathbf{G}_{mn,kl}, \tag{13}$$

$$\mathbf{C}_{mn,kl} = \frac{1}{4}\left[\sqrt{\left(\frac{\omega_{kl}}{\omega_{mn}}\right)} - \sqrt{\left(\frac{\omega_{mn}}{\omega_{kl}}\right)}\right]\mathbf{G}_{mn,kl}, \tag{14}$$

$$\mathbf{G}_{mn,kl} = -\frac{i}{\hbar}\int_{\infty}^{\infty} \eta_{mn}(\mathbf{x})(\nabla\eta_{kl}(\mathbf{x}))d\mathbf{x}. \tag{15}$$

$$\boxed{\widehat{H} = \frac{1}{2M}\left(\widehat{\mathbf{P}} - \widehat{\mathbf{P}}_{mg}\right)^2 + \sum_{mn} \hbar\omega_{mn} b^{\dagger}_{mn} b_{mn}} \qquad (16)$$



- **Dynamics of the topological excitations**

The density operator for the composite system

$$\widehat{\rho}(t) = \exp(-\frac{i}{\hbar}\widehat{H}t)\widehat{\rho}(0)\exp(\frac{i}{\hbar}\widehat{H}t) \qquad (17)$$

We assume that initially

$$\widehat{\rho}(\mathbf{x}', \mathbf{R}', \mathbf{y}', \mathbf{Q}', 0) = \widehat{\rho}_s(\mathbf{x}', \mathbf{y}', 0)\widehat{\rho}_r(\mathbf{R}', \mathbf{Q}', 0), \qquad (18)$$

The equation of motion for the center of mass of the soliton

$$\boxed{\boxed{\ddot{R}_i\,(t) + \int_0^t \gamma_{i,j}(t-t')\,\dot{R}_j\,(t')dt' = 0,}} \qquad (19)$$

Brownian particle

The decay function $\gamma_{i,j}$ has the form

$$\begin{aligned} \gamma_{i,j}(t) \quad &= \quad \frac{\hbar}{M}\Theta(t)\int d\omega \int d\omega' S_{i,j}(\omega,\omega') \times \\ &\quad (\overline{N}(\omega) - \overline{N}(\omega'))\frac{(\omega+\omega')^2(\omega'-\omega)}{\omega\omega'} \times \\ &\quad \cos(\omega'-\omega)t, \end{aligned} \qquad (20)$$

and the components of the scattering matrix S are

$$S_{i,j}(\omega,\omega') = \sum_{mn,kl} \mathcal{M}_{i,j}(mn,kl)\delta(\omega-\omega_{mn})\delta(\omega'-\omega_{kl}). \qquad (21)$$

where

$$\mathcal{M}_{i,j} \quad = \quad \begin{pmatrix} |G^{(1)}_{mn,kl}|^2 & 0 \\ 0 & |G^{(2)}_{mn,kl}|^2 \end{pmatrix} \qquad (22)$$

$$\mathbf{G}_{mn,kl} = -\frac{i}{\hbar}\int \eta_{mn}(\nabla\eta_{kl})dr \qquad (23)$$

In the long time approximation

$$\varphi = (\omega' + \omega)/2 \qquad , \qquad \eta = (\omega' - \omega) \tag{24}$$

Then

$$\gamma_{i,j}(t) = \overline{\gamma}_{i,j}(T)\delta(t), \tag{25}$$

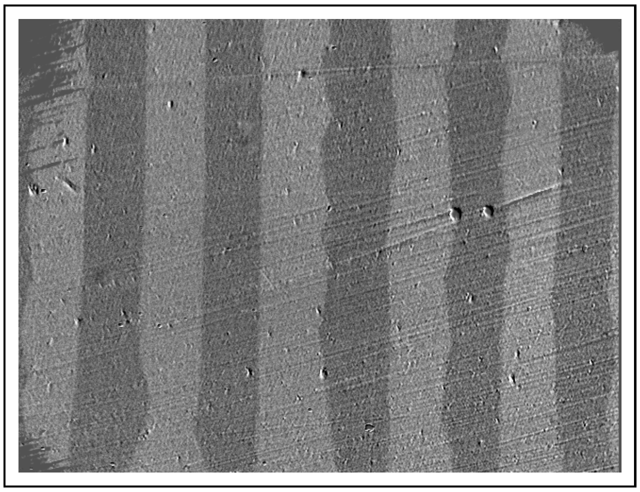
where

2-D case

$$\boxed{\boxed{\overline{\gamma}_{i,j}(T) = \frac{8\hbar}{M} \lim_{\eta\to 0} \int_0^\infty \frac{\partial \overline{N}(\varphi)}{\partial \varphi} S_{i,j}(\varphi,\eta)\eta^2 d\varphi}} \tag{26}$$

Applications to magnetic systems

- **Bloch walls**



Magnetic domains on a CoFeSiB strip*

Hamiltonian

$$
\begin{aligned}
H &= -\sum_{\langle ij\rangle} J_x S_i^{(x)} S_j^{(x)} + J_y S_i^{(y)} S_j^{(y)} + J_z S_i^{(z)} S_j^{(z)} \\
&\quad -\frac{\mu}{\hbar}\mathbf{B}\cdot\sum_i \mathbf{S}_i, \qquad (27)
\end{aligned}
$$

where $J_x > J_y > J_z > 0$.

We shall describe the spins as

$$
\mathbf{S}_i = S(\sin\theta_i\cos\varphi_i, \sin\theta_i\sin\varphi_i, \cos\theta_i) \qquad (28)
$$

*Courtesy of K. Pirota and M. Knobel, LMBT-IFGW, UNICAMP.

We can describe the system starting from

$$\mathcal{L} = \frac{1}{2c^2}\left(\frac{d\varphi}{dt}\right)^2 - \frac{1}{2}\left(\frac{d\varphi}{dz}\right)^2 - U(\varphi), \tag{29}$$

where

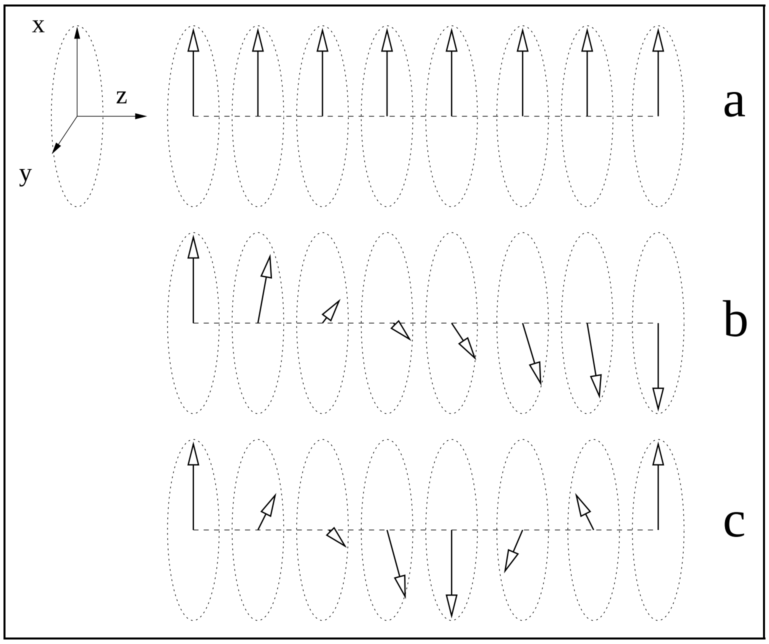
$$U(\varphi) = A_1(1 - \cos\varphi) + \frac{A_2}{2}(1 - \cos 2\varphi). \tag{30}$$

Localized solutions

$$\varphi(z) = 2\tan^{-1}[\exp\sqrt{2A_2}(z - z_0)], \quad A_1 = 0 \tag{31}$$

$$\varphi(z) = 2\tan^{-1}[\frac{\cosh\rho}{\sinh(z/\lambda)}], \tag{32}$$

Bloch walls



Bloch walls

The potential felt by the fluctuations

$$U(z) = \frac{1}{\lambda^2}\Big[1 - 2\mathrm{sech}^2(\frac{z}{\lambda}+\rho) - 2\mathrm{sech}^2(\frac{z}{\lambda}-\rho) + 2\mathrm{sech}(\frac{z}{\lambda}+\rho)\mathrm{sech}(\frac{z}{\lambda}-\rho)\Big]. \tag{33}$$

Mobility of the wall for B=0

In this case

$$U(z) = \eta^2(1 - 2\mathrm{sech}^2\eta z). \tag{34}$$

On the other hand, for 1D

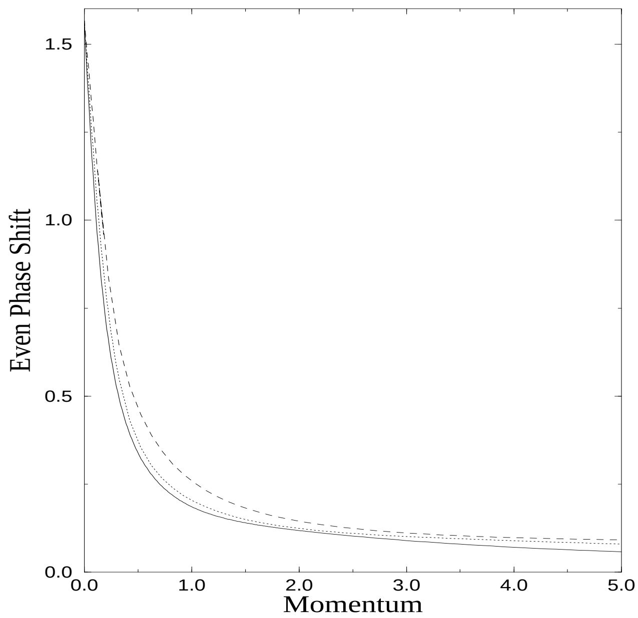
$$\bar{\gamma}(T) = \frac{\hbar}{2\pi M}\int_0^\infty E\mathcal{R}(E)\frac{d\bar{N}}{dE}dE \tag{35}$$

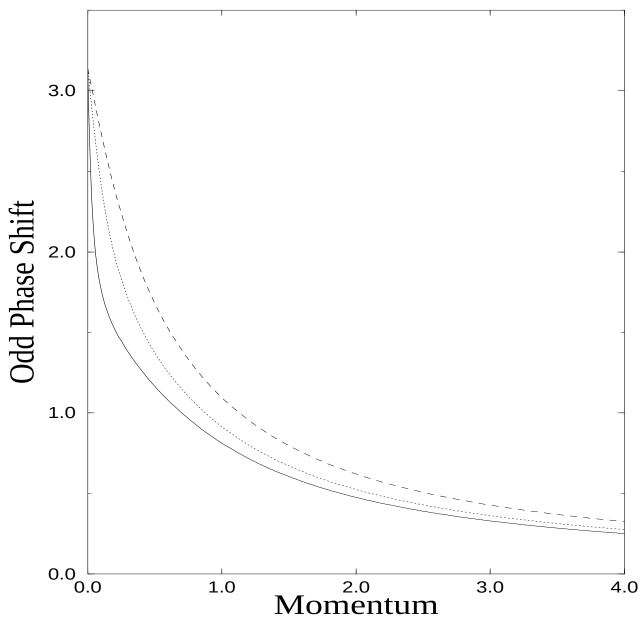
$$\mathcal{R}(k) = \sin^2\left(\delta^e(k) - \delta^o(k)\right), \tag{36}$$

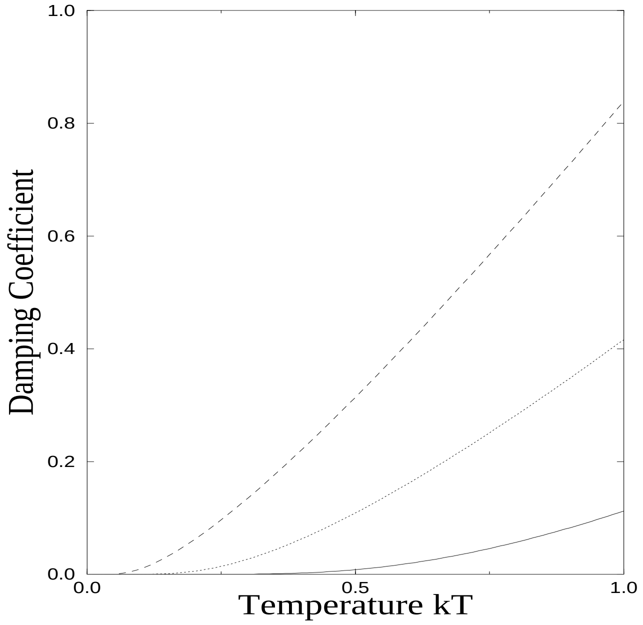
It can be shown that

$$\delta^{e,o}(k) = \arctan(\eta/k) \qquad \to \mathcal{R}(k) = 0. \tag{37}$$

so, for B=0 the mobility is infinite



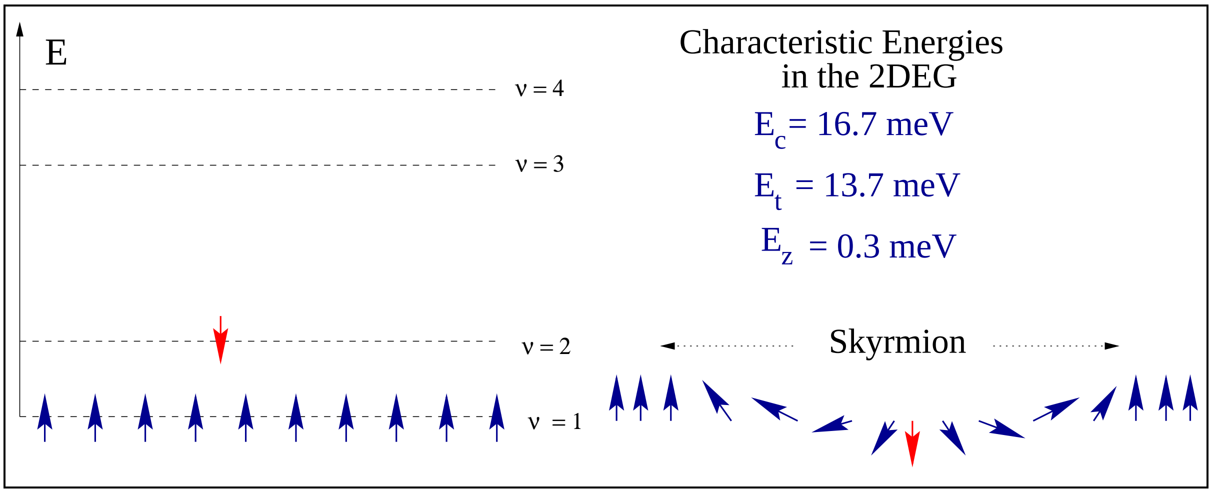




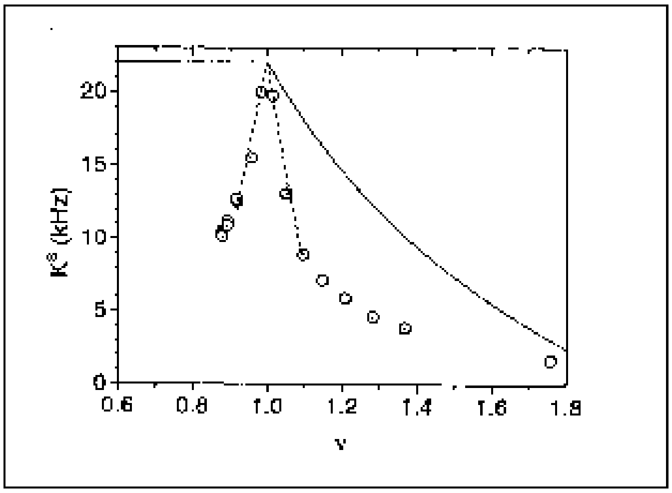
Mobility of the wall for B$\gg$(J$_x - J_y$)

The continuous, dotted, and dashed lines correspond to $\rho = 0.14$, $\rho = 0.31$, and $\rho = 0.50$, respectively

- **Quantum Hall Skyrmions**



Knight shift vs the Landau filling factor, open circles are experimental data*



*S.E. Barrett et al, *Phys. Rev. Lett*,**72** 1368 (1994)

From the theoretical point of view

$$\mathcal{L}_{eff}(\mathbf{r}) = \alpha\mathcal{A}(\mathbf{n}(\mathbf{r}))\cdot\partial_t\mathbf{n}(\mathbf{r}) - \alpha^{'}(\nabla\mathbf{n}(\mathbf{r}))^2 - g\mu_B\mathbf{n}(\mathbf{r})\cdot\mathbf{B} - \frac{1}{2}\int dr'^2 V(\mathbf{r}-\mathbf{r}')q(\mathbf{r})q(\mathbf{r}') \quad (38)$$

where

$$q(\mathbf{r}) = \frac{1}{8\pi}\varepsilon_{\mu\nu}\mathbf{n}\cdot(\partial_\mu\mathbf{n}\times\partial_\nu\mathbf{n}) \quad (39)$$

and

$$\mathcal{A}(\mathbf{n}(\mathbf{r})) = \partial_t\mathbf{n}(t,\tau)\times\partial_\tau\mathbf{n}(t,\tau) \quad (40)$$

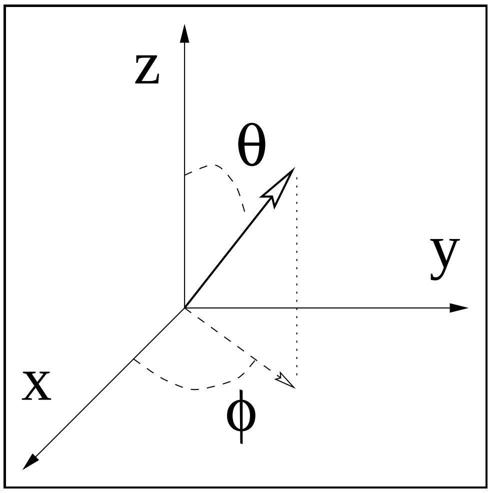
The non-linear sigma model

$$\nabla^2\mathbf{n}(\mathbf{r}) - \left(\mathbf{n}(\mathbf{r})\cdot\nabla^2\mathbf{n}(\mathbf{r})\right)\mathbf{n}(\mathbf{r}) = 0. \quad (41)$$

Solution

$$n_{1(2)} = \frac{4\lambda x(y)}{r^2+4\lambda^2}, \quad n_3 = \frac{r^2-4\lambda^2}{r^2+4\lambda^2} \quad (42)$$

New variables



The equation for the fluctuations about Θ

$$\left\{-\nabla^2 + U(r)\right\} u_{m,\mathbf{k}}(\mathbf{r}) = e_m u_{m,\mathbf{k}}(\mathbf{r}), \tag{43}$$

where

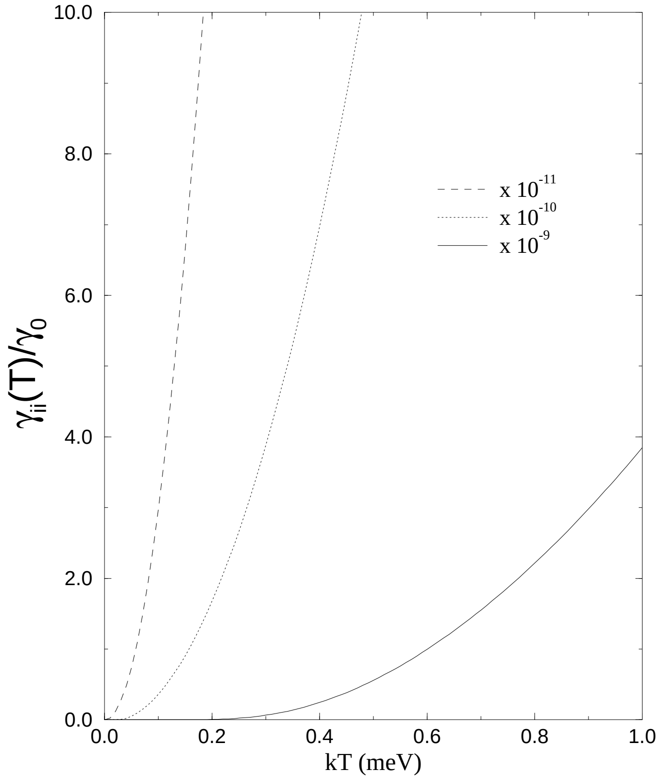
$$e_m = E_m - \frac{g\mu_B B}{2\pi}, \tag{44}$$

involves the potential

$$U(r) = \frac{\alpha' l_0^2}{r^2} - \frac{2^5 \alpha' \lambda^2 l_0^2}{(r^2 + 4\lambda^2)^2} - \frac{2g\mu_B B\lambda^2}{\pi(r^2 + 4\lambda^2)}. \tag{45}$$

Solution

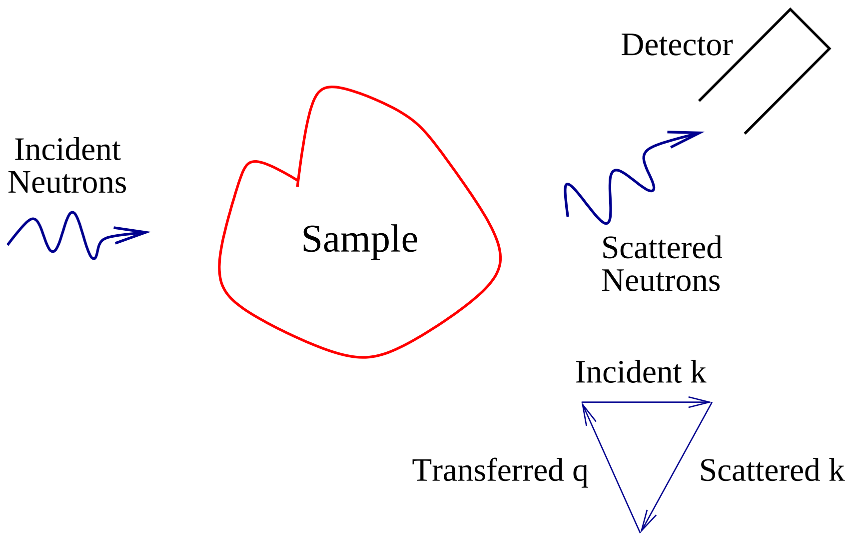
$$u_{mn} \propto \frac{1}{2}\{H^{(1)}_{|m|}(k_{mn}r) + e^{-2i\delta_m} H^{(2)}_{|m|}(k_{mn}r)\}e^{im\phi}. \tag{46}$$



Mobility of the skyrmion as a function of the temperature.

Experimental proposal

- Neutron scattering



Dynamical correlation function

$$\mathcal{S}(rt; r'0) = \frac{1}{Z} \sum_{\psi_i} e^{-\beta E_i} \langle \psi_i | \widehat{S}(r,t) \widehat{S}(r',0) | \psi_i \rangle, \quad (47)$$

$$Z = \sum_i e^{-\beta E_i}. \quad (48)$$

For non-interacting skyrmions

$$S(rt; r'0) = N \langle \cos[\theta(r - R_n - g(t)V_n)] \cos[\theta(-R_n)] \rangle \quad (49)$$

Solution of the non-linear sigma model

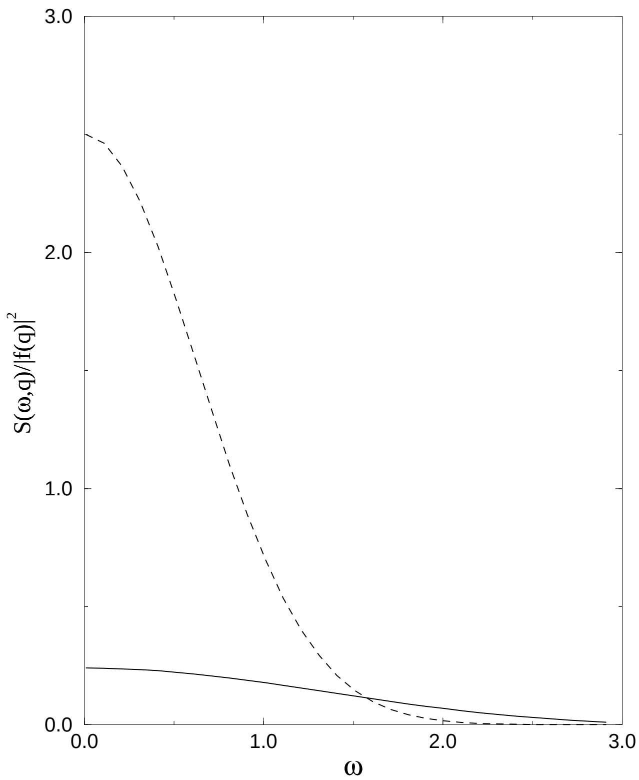
$$\cos(\theta(r, R)) = \frac{(x - X)^2 + (y - Y)^2 - 4\lambda^2}{(x - X)^2 + (y - Y)^2 + 4\lambda^2}, \quad (50)$$

Ballistic motion

$$\mathcal{S}(q,\omega) = 4\pi^{3/2} n_s \frac{|f(q)|^2}{q\overline{u}} e^{-\omega^2/\overline{u}^2 q^2}. \tag{51}$$

Dissipative motion

$$\begin{aligned}\mathcal{S}(q,\omega) \quad &= \quad 2\pi n_s |f(q)|^2 \left(\left(1 - \frac{q^2\overline{u}^2}{4\gamma^2} \right) \delta(\omega) \right. \\ &\left. + \frac{q^2\overline{u}^2}{4\gamma^2} \sqrt{\frac{\pi}{\gamma}} \left(2e^{-\omega^2/8\gamma} - \frac{1}{2} \right) e^{-\omega^2/8\gamma} \right) \end{aligned} \tag{52}$$



The dynamical correlation function. The continuous line corresponds to the dissipative motion.

Correct expression for many kinks

$$\mathcal{S}(\mathbf{r},t) = \langle \int d^3x S_z(\mathbf{r}-\mathbf{x}) S_z(\mathbf{x}) \tilde{\rho}(\mathbf{x},t) \rangle \qquad (53)$$

where $\tilde{\rho}(\mathbf{x},t)$ is the reduced density operator for the collective coordinete only and $\langle\rangle$ stands for a classical average over the initial positions and momenta of the kinks.

Results

· Generalization of the method to study the mobility of topological excitations in two dimensions[1]

· Continuum model of the domain walls and the mobility for the cases π and 2π [2].

· Model for the study of the temperature dependence of the mobility of the QHS [1]

Prospects

· Effects of the dissipative regime for the antiferromagnet TMMC [3].

· Effects of the dissipative regime for systems described by the model of Dzyaloshinski-Moriya [3].

· Study of the many skyrmion system as applicable to quantum Hall systems and high temperature superconductors

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[4] A. Villares Ferrer, PhD thesis (in portuguese), IFGW october 1999.

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[6] R. Rajaraman in "Solitons and Instantons," (North Holland,Amsterdam, 1992).