FÍSICA da MATÉRIA CONDENSADA

Mestrado em Engenharia Física Tecnológica

Série 2c

1. The Landau-Lifshitz-Gilbert equation describes the Larmor precession of a macrospin in the presence of a magnetic field and coupled to an environment introducing damping. According to Gilbert [1], the damping can be considered adding a viscous force term to the equation of motion, leading to:

$$\frac{d\vec{M}}{dt} = -\gamma_G \vec{M} \times \vec{H} + \frac{\alpha}{M}\vec{M} \times \frac{d\vec{M}}{dt} \tag{1}$$

where $M = |\vec{M}|$ is the magnetization length, defined by its saturation value, which is conserved, and the magnetic field, $\vec{H} = \vec{H}_{ext} + \frac{\partial U}{\partial \vec{M}}$, is the sum of the external field $\vec{H}_{ext}$ with some internal field, given by some anisotropy energy U, describing preferred magnetization directions (easy magnetization axis or plane).

In the presence of an electron current, the macrospin will experience an extra force, described by the Slonczewski torque added to the LLG equation,

$$\frac{d\vec{M}}{dt} = -\gamma_G \vec{M} \times \vec{H} + \frac{\gamma_G}{M}\vec{M} \times (\vec{M} \times \vec{J}) + \frac{\alpha}{M}\vec{M} \times \frac{d\vec{M}}{dt} \tag{2}$$

which can be formally reduced to the form of eq. (1), defining the effective field $\vec{H}_{eff} = \vec{H} - (\vec{M}/M) \times \vec{J}$.

a) Show that the LLG equation, with the Slonczewski torque, can then be obtained in the original Landau-Lifshitz [2] form, taking the cross product with $\vec{M}$ to get $\vec{M} \times d\vec{M}/dt$, substituting it in eq. (1) and solving then for $d\vec{M}/dt$, obtaining

$$\frac{d\vec{M}}{dt} = -\gamma \vec{M} \times \vec{H_{eff}} - \frac{\lambda}{M}\vec{M} \times (\vec{M} \times \vec{H}_{eff}). \tag{3}$$

where $\gamma = \gamma_G/(1+\alpha^2)$ and $\lambda = \alpha\gamma$.

b) Show that the length of the magnetization vector is a constant of motion and therefore it remains on a sphere.

References

[1] Gilbert T L 1955 *Phys. Rev.* **100** 1243.

[2] Landau L D, Lifshitz E M 1935 *Phys. Z. Sowjetunion* **8** 153-164.

2. a) Show that the Landau-Lifshitz-Gilbert equation describing the Larmor precession of a macrospin in the presence of a magnetic field and coupled to an environment introducing damping:

$$\frac{d\vec{M}}{dt} = -\gamma \vec{M} \times \vec{H} - \frac{\lambda}{M} \vec{M} \times (\vec{M} \times \vec{H}) \tag{4}$$

for a fixed magnetic field $\vec{H} = H\vec{e}_z$, takes the form:

$$\begin{aligned} \frac{dM_x}{dt} &= -\gamma H M_y - \frac{\lambda H}{M} M_z M_x \\ \frac{dM_y}{dt} &= \gamma H M_x - \frac{\lambda H}{M} M_z M_y \\ \frac{dM_z}{dt} &= \frac{\lambda H}{M}(M_x^2 + M_y^2) = \frac{\lambda H}{M}(M^2 - M_z^2) \end{aligned}$$

where $M^2 = \vec{M}^2$.

b) Give the physical interpretation of the different terms.

c) Show that the length of the magnetization vector is a constant of motion.

d) Solve these equations of motion for arbitrary initial conditions.

3. a) The Bloch equation describing the Larmor precession of a macrospin in the presence of a constant magnetic field $\vec{H} = H\vec{e}_z$, coupled to an environment introducing damping is:

$$\begin{aligned} \frac{dM_x}{dt} &= -\gamma H M_y - \frac{1}{T_2} M_x \\ \frac{dM_y}{dt} &= \gamma H M_x - \frac{1}{T_2} M_y \\ \frac{dM_z}{dt} &= -\frac{1}{T_1}(M_z - M) \end{aligned}$$

where T_1, T_2 are the longitudinal and transversal relaxation times and M is the saturation value of the magnetization, i.e. its asymptotic value for $t \to \infty$.

a) Give the physical interpretation of the different terms.

b) Verify that the length of the magnetization vector is not a constant of motion.

c) Solve these equations of motion for arbitrary initial conditions.

d) Compare these equations and their solutions with those of the previous problem, on the Landau-Lifshitz-Gilbert equation.