Lecture 9.

Coulomb interaction and theories of strongly correlated systems

- Landau theory of the Fermi liquid
- Coulomb interaction in 1D system
- Bosonization method

Landau theory of Fermi liquid

e-e interaction problem:

- electron energy spectrum ?
- effective e-i interaction ? (screening)
- transport in metals - low T
- low-dimensional systems !
- localization versus e-e

In metals

$$r_s = \frac{e^2}{r} \frac{m r^2}{\hbar^2} = \frac{r}{a_B} \sim n^{-1/3}$$

In semiconductors

$$r_s = \frac{e^2}{r} \frac{1}{T} \sim n^{1/3}$$

Why e-e interaction looks like irrelevant ?

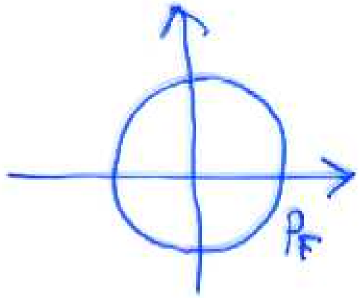
Theory of Landau (normal Fermi liquid)

Particles → quasiparticles obeying Fermi statistics

$n(p)$ - Fermi distribution function ? (spin matrix)

$$\delta \mathcal{E} = \int \varepsilon(p)\, \delta n(p) \frac{d^3p}{(2\pi\hbar)^3}$$

$\varepsilon(p)$ – quasiparticle spectrum



$$\varepsilon(p) - \varepsilon_F \simeq v_F (p - p_F)$$

$$\vec{v}_F = \frac{d\varepsilon(p)}{d\vec{p}}\Big|_{p_F}$$

$$m^* = \frac{p_F}{v_F}$$

Interaction:

$$\delta\varepsilon(p) = \int f(\vec{p}, \vec{p}\,')\, \delta n(\vec{p}\,') \frac{d^3p'}{(2\pi\hbar)^3}$$

$f(\vec{p}, \vec{p}\,')$ – interaction function

$$f(\vec{p}, \vec{p}\,') = f(\vec{p}\,', \vec{p})$$

$$\underbrace{\varepsilon(p) - \varepsilon_F \simeq v_F(p - p_F) + \int f(\vec{p}, \vec{p}\,')\, \delta n(\vec{p}\,') \frac{d^3p'}{(2\pi\hbar)^3}}$$

Relation between m^* and $f(\vec{p}, \vec{p}\,')$

$$\underbrace{\int \vec{p}\, n \frac{d^3p}{(2\pi\hbar)^3}}_{\text{transferred momentum}} = \underbrace{\int m\, \vec{v}\, n(\vec{p}) \frac{d^3p}{(2\pi\hbar)^3}}_{\text{flux density of mass}} \qquad \vec{v} = \frac{\partial \varepsilon(p)}{\partial \vec{p}}$$

Variation over $\delta n(\mathbf{p})$:

$$\int \vec{p}\, \delta n(p) \frac{d^3p}{(2\pi\hbar)^3} = m \int \frac{\partial f(\vec{p},\vec{p}')}{\partial \vec{p}} \delta n(\vec{p}')\, n(p) \frac{d^3p\, d^3p'}{(2\pi\hbar)^6}$$

$$+ m \int \frac{\partial \varepsilon(p)}{\partial \vec{p}} \delta n(p) \frac{d^3p}{(2\pi\hbar)^3}$$

$$\Rightarrow \quad \frac{\vec{p}}{m} = \frac{\partial \varepsilon(p)}{\partial \vec{p}} - \int f(\vec{p},\vec{p}') \frac{\partial n(p')}{\partial \vec{p}'} \frac{d^3p'}{(2\pi\hbar)^3}$$

For $T \to 0$:

$$n(p) = \theta(p)$$

$$\frac{\partial n(p)}{\partial \vec{p}} = -\frac{\vec{p}}{p}\delta(p - p_F)$$

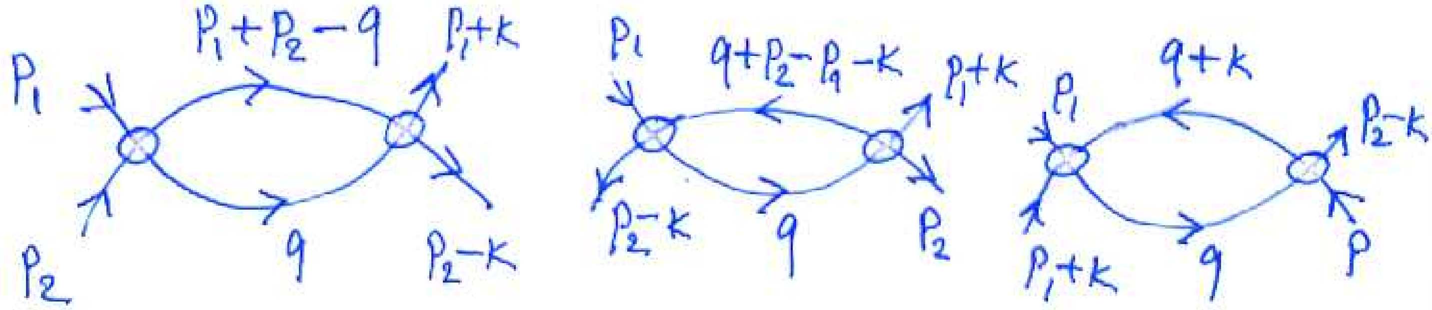
$$\boxed{\frac{1}{m} = \frac{1}{m^*} + \frac{p_F}{(2\pi\hbar)^3} \int f(\theta) \cos\theta \, d\Omega}$$

$d\Omega = 2\pi \sin\theta \, d\theta$

Diagrams in Landau theory

Green function: $\underline{\Gamma(P_1, P_2, k)}$

$$G(\vec{p}, \varepsilon) = \frac{a}{\varepsilon - v(p - p_F) + i\delta \,\mathrm{sign}\, \varepsilon}$$



Cooper $(P_1+P_2=0)$ 0-sound

0-sound channel $(k \to 0, \omega \to 0)$

$$\Gamma(P_1, P_2, k) = \Gamma^{(1)}(P_1, P_2) - i \int \Gamma^{(1)}(P_1, q)\, G(q)\, G(q+k$$

$$\times \Gamma(q, P_2, k) \frac{d^4 q}{(2\pi)^4}$$

Ladder equation

$$G(q)\, G(q+k) = \frac{2\pi i a^2}{v} \frac{\vec{k}\cdot\vec{v}}{\omega - \vec{k}\cdot\vec{v}} + \varphi(q)$$

regular part

where $\vec{v} = v\vec{q}/q$

$$\Rightarrow \quad G(q)\,G(q+k) \equiv i\Phi + \varphi$$

1) Limit $\omega \to 0$, $k/\omega \to 0$ (first $k \to 0$)

$$\Gamma^{\omega}(p_1, p_2) = \Gamma^{(1)}(p_1, p_2) - i\int \Gamma^{(1)}(p_1, q)\varphi(q) \times \Gamma^{\omega}(q, p_2)\,\frac{d^4q}{(2\pi)^4}$$

Operator form:

$$\begin{cases} \Gamma = \Gamma^{(1)} - i\Gamma^{(1)}(i\Phi + \varphi)\Gamma \\ \Gamma^{\omega} = \Gamma^{(1)} - i\Gamma^{(1)}\varphi\Gamma^{\omega} \end{cases}$$

$$(1 + i\Gamma^{(1)}\varphi)\Gamma^{\omega} = \Gamma^{(1)}$$

$$\Gamma = (1 + i\Gamma^{(1)}\varphi)^{-1}\Gamma^{(1)} + (1 + i\Gamma^{(1)}\varphi)^{-1}\Gamma^{(1)}\Phi\Gamma$$

$$\Rightarrow \quad \Gamma = \Gamma^{\omega} + \Gamma^{\omega}\Phi\Gamma$$

$$\Gamma(p_1, p_2, k) = \Gamma^{\omega}(p_1, p_2) + \frac{a^2 p_F^2}{(2\pi)^3 v}\int \Gamma^{\omega}(p_1, q) \times \frac{\vec{v}\cdot\vec{k}}{\omega - \vec{v}\cdot\vec{k}}\,\Gamma(q, p_2)\,d\Omega$$

2) Limit $K \to 0$, $\omega/K \to 0$ (first $\omega \to 0$)

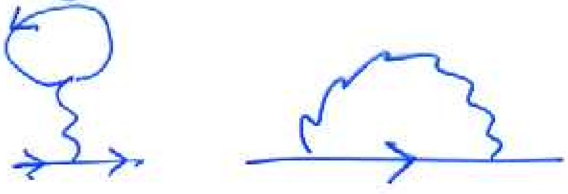
$$\Gamma^K(p_1, p_2) = \Gamma^\omega(p_1, p_2) - \frac{p_F^2 a^2}{(2\pi)^3 v} \int \Gamma^\omega(p_1, q)\, \Gamma^K(q, p_2)\, d\Omega$$

$a^2 \Gamma(p_1, p_2, K)$ – scattering amplitude of quasiparticles

Γ^ω and Γ^K define completely the other parameters of Fermi liquid

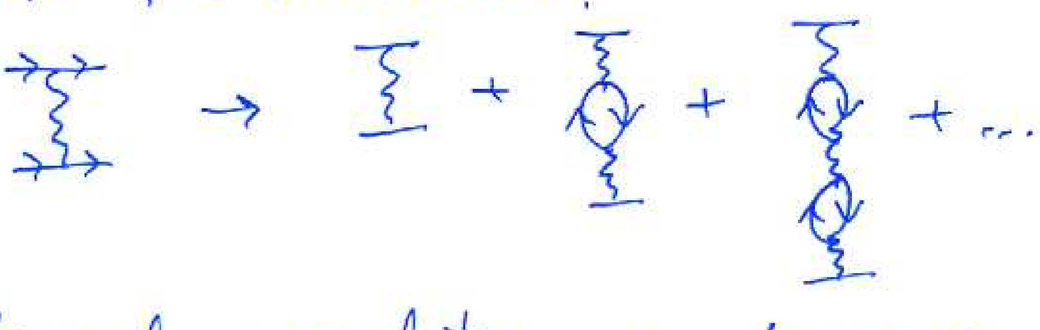
Other methods and theories

– Perturbation theory (Hartree – Fock ...)



self-energy

– RPA for interaction:

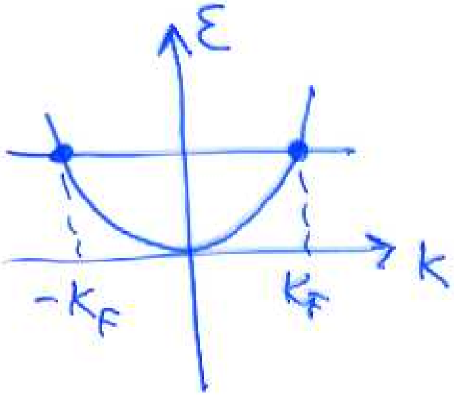


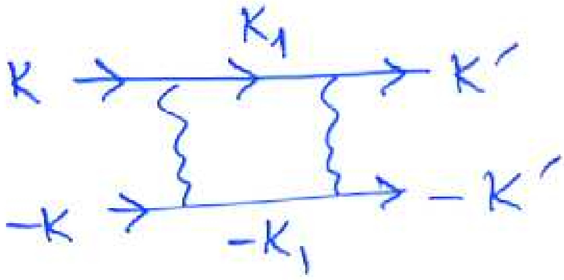
– Strongly correlated – constrained theories (e.g., slave bosons)

. . . .

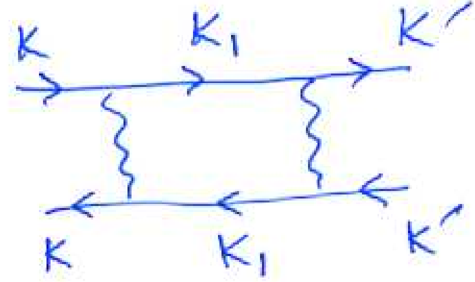
1D metals

Main channels:

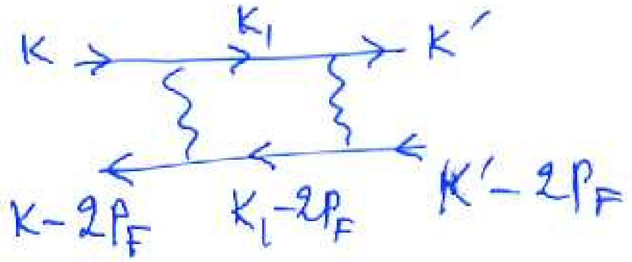




Cooper



0-sound



Peierls

Cooper:

$$-i\int \frac{dk_1}{2\pi}\frac{d\omega_1}{2\pi} G(k_1,\omega_1)\, G(-k_1,-\omega_1+\omega) = -\frac{1}{2\pi v_F}\left(\ln\frac{\omega}{\varepsilon_0} - \frac{i\pi}{2}\right)$$

Peierls:

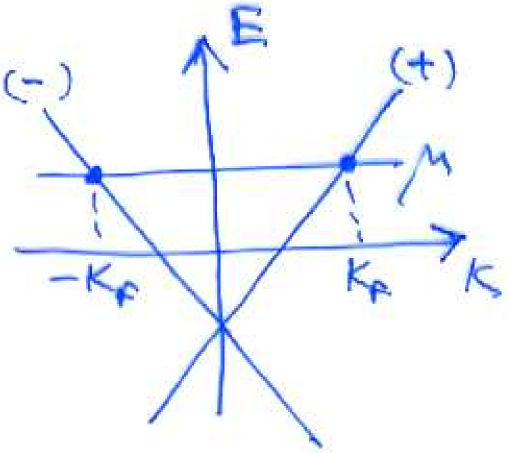
$$-i\int \frac{dk_1}{2\pi}\frac{d\omega_1}{2\pi} G(k_1,\omega_1)\, G(k_1-2p_F,\omega_1-\omega) = \frac{1}{2\pi v_F}\left(\ln\frac{\omega}{\varepsilon_0} - \frac{i\pi}{2}\right)$$

Approximate summation (e.g., parquet)

$$\Gamma \sim \frac{1}{1-\frac{g}{\pi v_F}\ln\frac{\omega}{\varepsilon_0}} \quad \text{- unphysical}$$

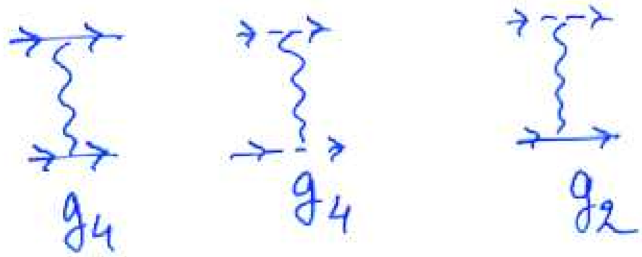
Exact solution: Tomonaga-Luttinger model

- no backscattering
- linear spectrum

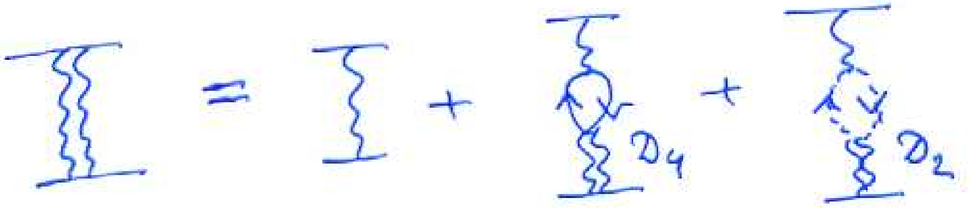


$$G^0_\pm = \frac{1}{\omega - v_F(\pm k - k_F) + i\delta\,\mathrm{sign}\,\varepsilon}$$

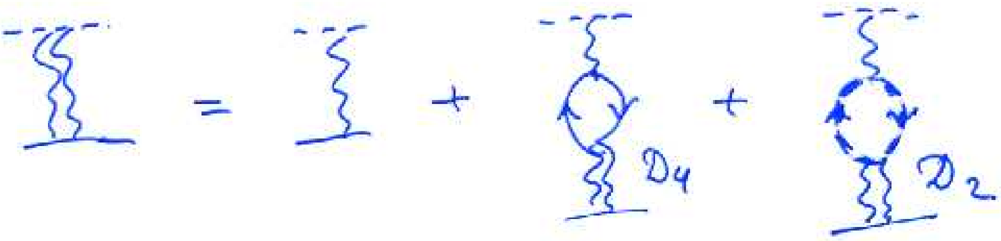
Interactions:

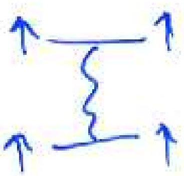


$D_4 =$



$D_2 =$

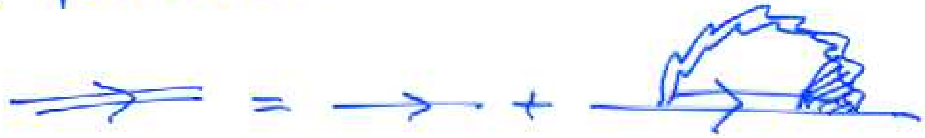




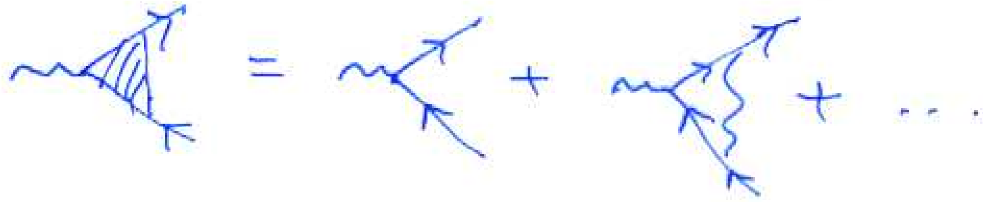
$$D_{4\parallel} = (\omega - v_F k)\left[\frac{A}{\omega - u_\sigma k + i\delta\,\mathrm{sign}\,k} + \frac{B}{\omega + u_\sigma k - i\delta\,\mathrm{sign}\,k} + \frac{C}{\omega - u_\rho k + i\delta\,\mathrm{sign}\,k} + \frac{D}{\omega + u_\rho k - i\delta\,\mathrm{sign}\,k}\right]$$

etc.

Green function:



Vertex function:



$\Rightarrow$ Equation for Green function:

$$[\varepsilon - v_F(p - K_F)]\, G_+(p,\varepsilon) = 1 + i \int \frac{dk\, d\omega}{(2\pi)^2} \frac{D_{4\parallel}(K,\omega)}{\varepsilon - vK} G_+(p-k, \varepsilon-\omega)$$

In real space-time:

$$G_+(x,t) = \frac{1}{2\pi} \frac{x - v_F t + i/\Lambda(t)}{x - v_F t + i\delta(t)}$$

$$\times \frac{\left[\Lambda^2 (x - u_\sigma t + i/\Lambda(t))(x + u_\sigma t - i/\Lambda(t))\right]^{-\alpha_\sigma}}{\left[x - u_\sigma t + i/\Lambda(t)\right]^{1/2}}$$

$$\times \frac{\left[\Lambda^2 (x - u_\rho t + i/\Lambda(t))(x + u_\rho t - i/\Lambda(t))\right]^{-\alpha_\rho}}{\left[x - u_\rho t + i/\Lambda(t)\right]^{1/2}}$$

$\Lambda(t) = \Lambda \,\mathrm{sign}\, t$

Λ - cutoff for K

$\Rightarrow$ In (K,ω)-representation the Green function does not have simple form of Landau theory

Bosonization in Tomonaga-Luttinger model

Luttinger spectrum:

$$H_0 = \sum_{k\alpha} v_F (k - k_F)\, a^+_{k\alpha} a_{k\alpha} + \sum_{k\alpha} v_F(-k-k_F)\, b^+_{k\alpha} b_{k\alpha}$$

$$H_{int} = \frac{1}{L}\sum_{p\alpha} (g_{2\parallel} + g_{2\perp})\left[\rho_1(p)\rho_2(-p) + \rho_1(-p)\rho_2(p)\right] +$$

$$+ \frac{1}{L}\sum_{p\alpha} (g_{2\parallel} - g_{2\perp})\left[\sigma_1(p)\sigma_2(-p) + \sigma_1(-p)\sigma_2(p)\right] +$$

$$+ \frac{1}{L}\sum_{p\alpha} (g_{4\parallel} + g_{4\perp})\left[\rho_1(p)\rho_1(-p) + \rho_2(-p)\rho_2(p)\right] +$$

$$+ \frac{1}{L}\sum_{p\alpha} (g_{4\parallel} - g_{4\perp})\left[\sigma_1(p)\sigma_1(-p) + \sigma_2(-p)\sigma_2(p)\right]$$

where $\rho_{1\alpha} = \sum_k a^+_{k+p\,\alpha} a_{k\alpha}$

$$\rho_{2\alpha} = \sum_k b^+_{k+p\,\alpha} b_{k\alpha}$$

Spin and charge densities:

$$\rho_i = \frac{1}{\sqrt{2}}(\rho_{i\uparrow} + \rho_{i\downarrow})$$

$$\sigma_i = \frac{1}{\sqrt{2}}(\rho_{i\uparrow} - \rho_{i\downarrow})$$

H_0 can be also presented using density operators!

$$H_0 = \frac{2\pi v_F}{L}\sum_p \left[\rho_1(p)\rho_1(-p) + \rho_2(-p)\rho_2(p)\right]$$

$$+ \frac{2\pi v_F}{L}\sum_p \left[\sigma_1(p)\sigma_1(-p) + \sigma_2(-p)\sigma_2(p)\right]$$

After diagonalization:

$$\tilde{H} = \frac{2\pi u_\sigma}{L}\sum_p \left[\sigma_1(p)\sigma_1(-p) + \sigma_2(-p)\sigma_2(p)\right] +$$

$$+ \frac{2\pi u_\rho}{L}\sum_p \left[\rho_1(p)\rho_1(-p) + \rho_2(-p)\rho_2(p)\right]$$