Lecture 8.

Localization and mesoscopic effects

- Anderson localization
- Theory of weak localization
- Negative magnetoresistance effect
- Localization in magnetic systems

Anderson localization

Disorder problem:

$$\mathcal{H} = -\frac{\hbar^2}{2m}\Delta + V(\vec{r})$$

$V(\vec{r})$ – impurities, defects: random potential

Physical quantity: averaged over disorder.

$\langle A \rangle$ (Example: conductivity $\langle \sigma_{ij} \rangle$)

In disordered systems:

localized states (P.W. Anderson, 1957)

(is it really so?)

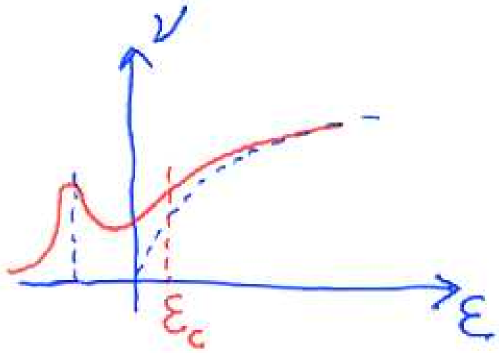
By changing some parameters (e.g., carrier density) we switch between localized and delocalized: metal–insulator (or Anderson) transition.

Role of dimensionality:

3D system: mobility edge

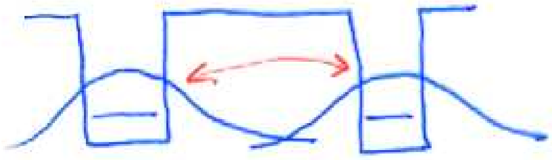
1D: all states are localized

2D: localized (?) asymptotically



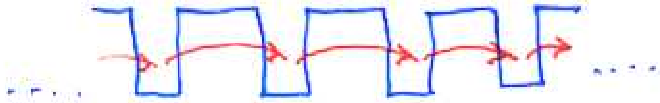
How to realize ?

Two wells (impurities)

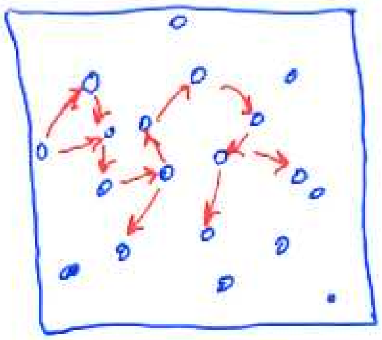


tunneling

Periodic potential (lattice):



Disordered:



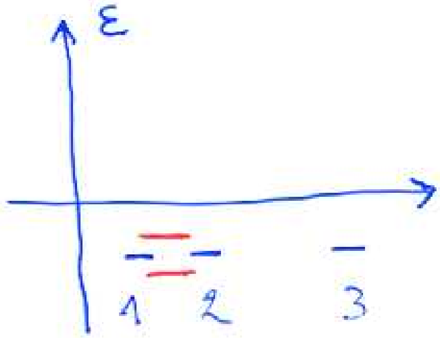
wrong picture

2D

Three impurities:

1 o   o 2

o 3

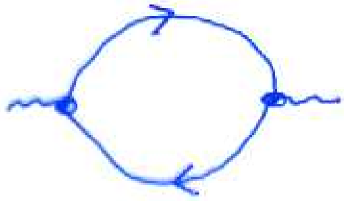


separated !

Back to Kubo formula:

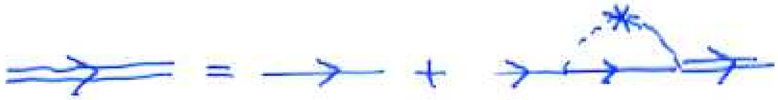
what was missing?

$$\sigma_0 = 2e^2 \cdot \frac{v_F^2}{3} \cdot \frac{1}{2\pi} \int \frac{d^3k}{(2\pi)^3} G^R(k)\, G^A(k)$$



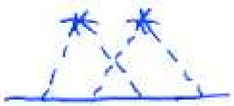
$$\int \ldots = \int \frac{\nu\, d\varepsilon_k}{\left(\mu - \varepsilon_k + \frac{i\hbar}{2\tau}\right)\left(\mu - \varepsilon_k - \frac{i\hbar}{2\tau}\right)}$$

Green functions:

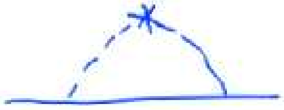


Born approximation

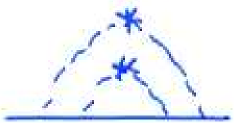
Crossed lines:



$\sim V_0^4$



$\sim V_0^2$



$\sim V_0^4$

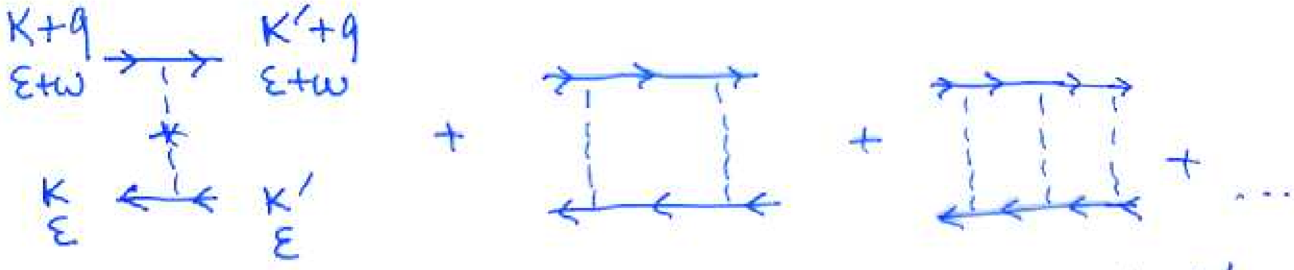
We assume V_0 small

Are many-impurity-line diagrams always small?

$$\int \nu\, d\varepsilon_k\, G^R G^A = 2\pi \nu \tau$$

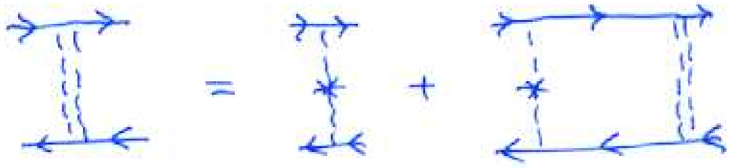
$$\sigma_0 = \frac{ne^2\tau}{m}$$

Diffuson



Ladder diagrams

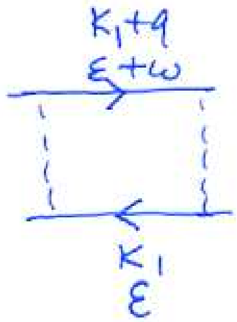
Equation:



$$\mathcal{D}_0(q\omega) = V_0^2 + V_0^2\, \Pi(q\omega)\, \mathcal{D}_0(q\omega)$$

short-range impurities

where $\Pi(q\omega) =$



$$\Pi(q\omega) = \int \frac{d^3K_1}{(2\pi)^3}\, G^R(K_1+q, \varepsilon+\omega)\, G^A(\varepsilon K_1) \simeq$$

$$\simeq \int \frac{d^3K_1}{(2\pi)^3} \frac{1}{\frac{\hbar^2 K_1^2}{2m} + \frac{\hbar^2 \vec{K}_1\cdot\vec{q}}{m} - \mu - \omega - \frac{i\hbar}{2\tau}}$$

$$\times \frac{1}{\frac{\hbar^2 K_1^2}{2m} - \mu + \frac{i\hbar}{2\tau}}$$

we will drop $\hbar$

$$\Pi(q\omega) \simeq \int \frac{\nu\, d\varepsilon_{k_1}}{\left(\varepsilon_{k_1}-\mu-\omega-\frac{i}{2\tau}\right)\left(\varepsilon_{k_1}-\mu+\frac{i}{2\tau}\right)} +$$

$$+ \int \frac{d^3 k_1}{(2\pi)^3} \frac{1}{\left(\varepsilon_{k_1}-\mu-\frac{i}{2\tau}\right)^3} \frac{(\vec{k}_1\cdot\vec{q})^2}{m^2} \frac{1}{\varepsilon_{k_1}-\mu+\frac{i}{2\tau}} =$$

$$= \frac{2\pi\nu\tau}{1-i\omega\tau} + \frac{2q^2}{3m} \int \frac{\nu\, \varepsilon_k\, d\varepsilon_k}{\left(\varepsilon_k-\mu-\frac{i}{2\tau}\right)^3\left(\varepsilon_k-\mu+\frac{i}{2\tau}\right)} \simeq$$

$$\simeq 2\pi\nu\tau\left(1+i\omega\tau-\mathcal{D}q^2\tau\right)$$

$\omega\tau \ll 1$

$\mathcal{D}q^2\tau \ll 1$

Ladder equation:

$$\mathcal{D}_0 = V_0^2 + V_0^2\, \Pi\, \mathcal{D}_0$$

$$\mathcal{D}_0 = \frac{V_0^2}{1-V_0^2\,\Pi} = \frac{V_0^2}{1-\underbrace{V_0^2\cdot 2\pi\nu\tau}\left(1+i\omega\tau-\mathcal{D}q^2\tau\right)}$$

Relaxation time: $\frac{1}{\tau} = 2\pi\nu V_0^2$

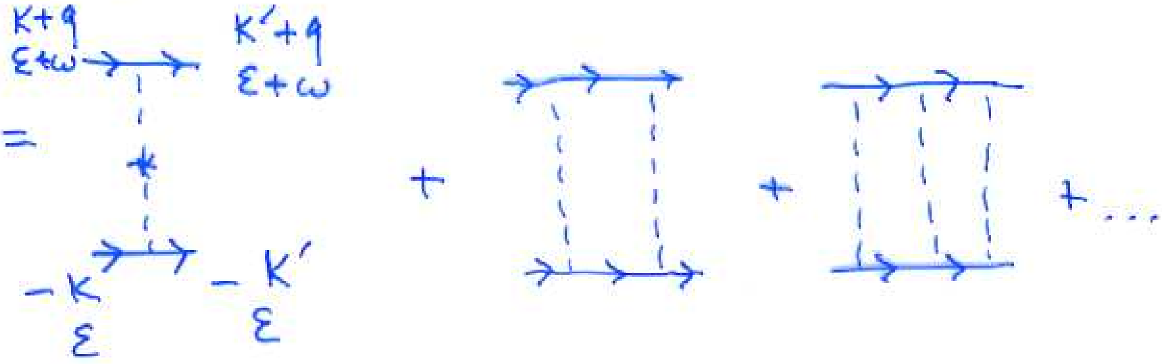
$$\boxed{\mathcal{D}_0(q\omega) = \frac{1}{2\pi\nu\tau^2}\,\frac{1}{-i\omega+\mathcal{D}q^2}}$$

$$-i\omega+\mathcal{D}q^2 \longrightarrow \frac{\partial}{\partial t} - \mathcal{D}\Delta$$

diffusion operator

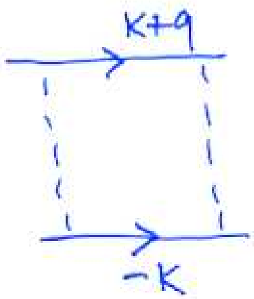
Cooperon

$C_0(q\omega) =$



$$C_0 = V_0^2 + V_0^2 \Pi_1 C_0$$

$\Pi_1(q\omega) =$



$=$

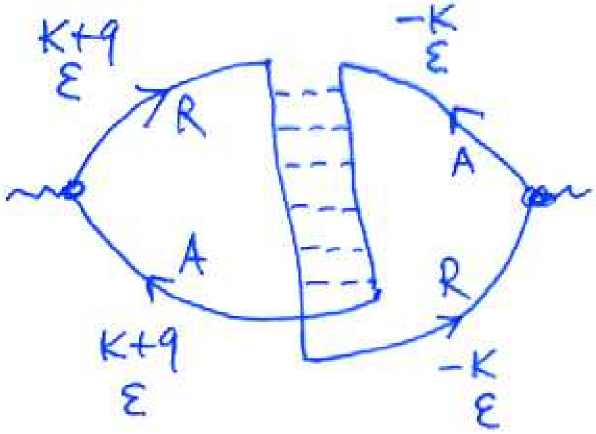
$$= \int \frac{d^3K}{(2\pi)^3} G^R(K+q, \varepsilon+\omega)\, \underbrace{G^A(-K, \varepsilon)}_{(\varepsilon_K = \varepsilon_{-K})}$$

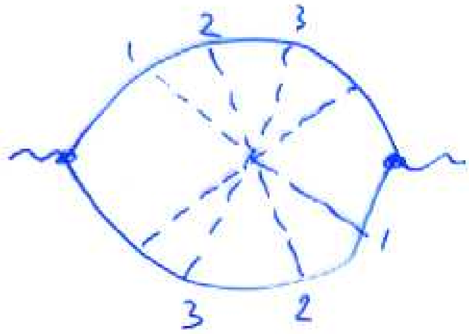
$$\Rightarrow \quad \boxed{C_0(q\omega) = \frac{1}{2\pi\nu\tau^2} \frac{1}{-i\omega + Dq^2 + \frac{1}{\tau_\varphi}}}$$

phase relaxation
(decoherence)

Propagator of superconductivity
density fluctuations

Localization correction





$$\delta\sigma = \frac{e^2 v_F^2 \nu}{3\pi} \int (G^R)^2 (G^A)^2 \, d\varepsilon_K \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2\pi\nu\tau^2} \frac{1}{Dq^2}$$

Dimensionality: $\int \frac{d^3 q}{(2\pi)^3} \to \int \frac{d^d q}{(2\pi)^d}$

2D case

$$\int \frac{d^2 q}{(2\pi)^2} \frac{1}{Dq^2} = \frac{1}{2\pi D} \int_{q_{min}}^{q_{max}} \frac{dq}{q} \simeq \frac{1}{4\pi D} \ln \frac{\tau_\varphi}{\tau}$$

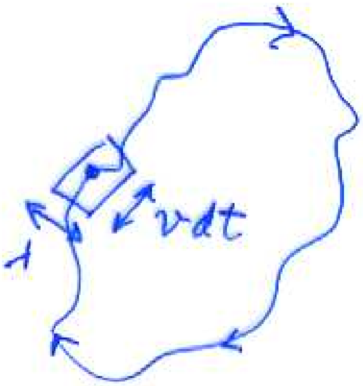
$$D q_{min}^2 \simeq \frac{1}{\tau_\varphi}$$

$$D q_{max}^2 \simeq \frac{1}{\tau}$$

$$\delta\sigma_{2D} \simeq -\frac{e^2}{2\pi^2 \hbar} \ln \frac{\tau_\varphi}{\tau}$$

Localization correction – estimation

$$\frac{\delta\sigma}{\sigma} \sim -\int_{\tau}^{\tau_\varphi} \frac{v\,dt\,\lambda^{d-1}}{(\mathcal{D}t)^{d/2}} =$$



$$= -\frac{v\lambda^{d-1}}{\mathcal{D}^{d/2}} \int_{\tau}^{\tau_\varphi} \frac{dt}{t^{d/2}}$$

$$\lambda = \frac{1}{k_F}$$

$$\mathcal{D} = \frac{v_F^2\tau}{d}$$

$d=2$

$$\frac{\delta\sigma}{\sigma_0} = -\frac{\hbar}{\varepsilon_F\tau}\ln\frac{\tau_\varphi}{\tau}$$

$d=3$

$$\frac{\delta\sigma}{\sigma_0} = -\frac{\hbar}{\varepsilon_F\tau}\left(\frac{\lambda}{l} - \frac{\lambda}{L_\varphi}\right)$$

$$l = v_F\tau$$

$$L_\varphi = \sqrt{\mathcal{D}\tau_\varphi}$$

$d=1$

$$\frac{\delta\sigma}{\sigma_0} = -\frac{\hbar}{\varepsilon_F\tau}\left(\frac{L_\varphi}{\lambda} - \frac{l}{\lambda}\right)$$

Magnetic field: supresses Cooperon

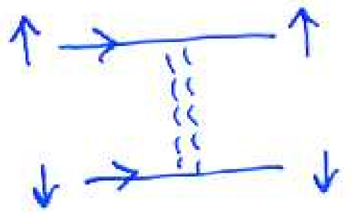
Equation:

$$\left[-i\omega + D\left(-i\nabla - \frac{2e\vec{A}}{c}\right)^2 + \frac{1}{\tau_\varphi}\right] C_0(r\, r') = \frac{\delta(r-r')}{\tau}$$

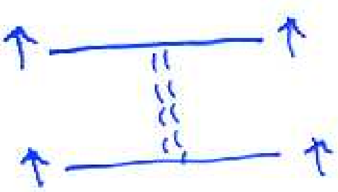
$$C_0 = \frac{1}{2\pi\nu\tau^2}\,\frac{1}{-i\omega + Dq_z^2 + \frac{4eHD}{c}\left(n+\frac{1}{2}\right) + \frac{1}{\tau_\varphi}}$$

$\Rightarrow$ negative magnetoresistance effect

Role of spin



singlet Cooperon



triplet Cooperon

Spin orbit interaction: mixing

In ferromagnet: no singlet Cooperon