Lecture 7

Transport in magnetic systems

- Spin-dependent scattering
- GMR effect
- Anomalous Hall effect: mechanisms of side-jump and skew scattering

Spin-dependent scattering

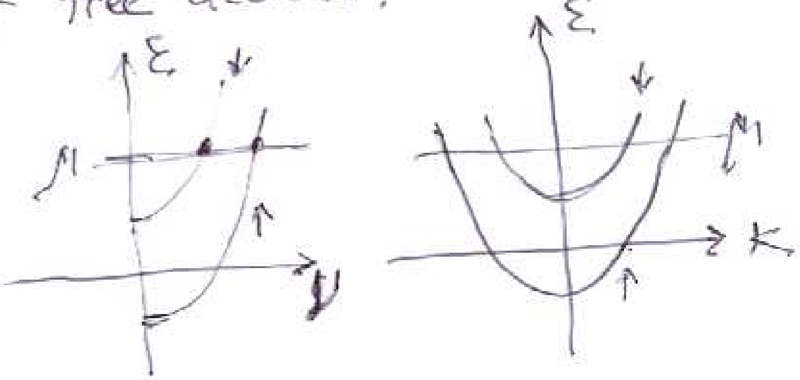
$$\frac{1}{\tau_\uparrow} = \frac{2\pi}{\hbar} \sum_{k'} |\langle k\uparrow | V | k'\uparrow \rangle|^2 \, \delta(\varepsilon_{k\uparrow} - \varepsilon_{k'\uparrow})$$

$$\rightarrow \frac{2\pi \nu_\uparrow |V_\uparrow|^2}{\hbar}$$ – short-range potential

$$\frac{1}{\tau_\downarrow} = \frac{2\pi \nu_\downarrow |V_\downarrow|^2}{\hbar}$$

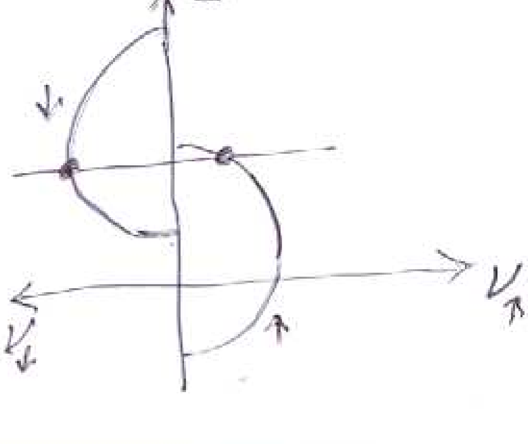
$\tau_\uparrow \neq \tau_\downarrow$ – reasons:
- different density of state
- different matrix elements V_σ

Density of states:
- model of free electons:



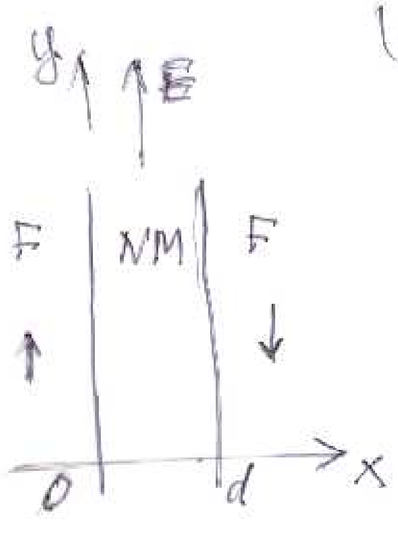
Here $\nu_\uparrow > \nu_\downarrow$

It can be wrong:



Here $\nu_\downarrow > \nu_\uparrow$

Theory of GMR effect



AF ordering
(H=0)

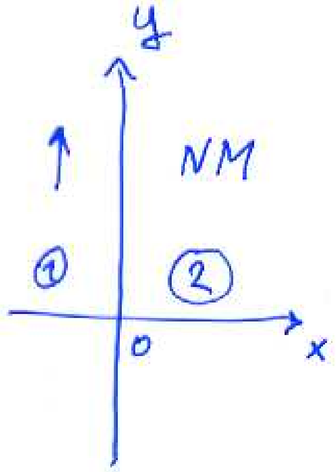
Let $d \gtrsim l$. CIP current in plane

Boltzmann equation

$$eEv_y \frac{\partial f_0}{\partial \varepsilon_k} + v_x \frac{\partial \, \delta f}{\partial x} = -\frac{\delta f}{\tau}$$

$$\tau = \tau_M, \tau_m \text{ in F}$$
$$= \tau \text{ in NM}$$

First interface



$\vec{E}$ along y

(1) $$v_x \frac{\partial \delta f_i}{\partial x} + \frac{\delta f_i}{\tau_i} = -eEv_y \frac{\partial f_0}{\partial \varepsilon_k}$$

$i = M, m$

Homogeneous:

$$\delta f_i = c e^{-x/v_x \tau_i}$$

If $v_x < 0$: $\delta f_i = c e^{x/|v_x|\tau_i}$

If $v_x > 0$: $\delta f_i = 0$

$$\delta f_i = C_1 \theta(-v_x) e^{x/|v_x|\tau_i}$$

Inhomogeneous:

$$\delta f_i = -eEv_y \tau_i \frac{\partial f_0}{\partial \varepsilon_k}$$

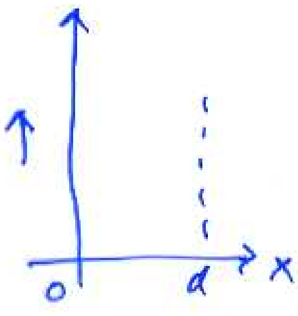
$$\Rightarrow \delta f_i = -eEv_y \tau_i \frac{\partial f_0}{\partial \varepsilon_k} + C_1 \theta(-v_x) e^{x/|v_x|\tau_i}$$

2 $$\delta f_\sigma = -eEv_y\tau\frac{\partial f_0}{\partial \varepsilon} + C_2\,\theta(v_x)e^{-x/v_x\tau}$$

$\sigma = \uparrow, \downarrow$

How to match:

$$\delta f_M(x=0) = \delta f_\uparrow(x=0)$$



$v_x > 0$

$$-eEv_y\tau_M\frac{\partial f_0}{\partial \varepsilon_k} = -eEv_y\tau\frac{\partial f_0}{\partial \varepsilon_k} + C_2$$

$$C_2 = -eE\,v_y(\tau_M - \tau)\frac{\partial f_0}{\partial \varepsilon_k}$$

$v_x < 0$

$$-eEv_y\tau_M\frac{\partial f_0}{\partial \varepsilon_k} + C_1 = -eE\,v_y\,\tau\frac{\partial f_0}{\partial \varepsilon_k}$$

$$C_1 = -eE\,v_y(\tau - \tau_M)\frac{\partial f_0}{\partial \varepsilon_k}$$

$$\delta f_\uparrow = -eEv_y\tau\frac{\partial f_0}{\partial \varepsilon_k}\left[1 - \left(1 - \frac{\tau_M}{\tau}\right)\theta(v_x)e^{-x/v_x\tau}\right]$$

$$\delta f_\downarrow = -eE\,v_y\tau\frac{\partial f_0}{\partial \varepsilon_k}\left[1 - \left(1 - \frac{\tau_m}{\tau}\right)\theta(v_x)e^{-x/v_x\tau}\right]$$

Current in nonmagnetic metal

$$j_\uparrow(x) = e\int\frac{d^3k}{(2\pi)^3}v_y\,\delta f_\uparrow =$$

$$= e^2E\tau\int\frac{d^3k}{(2\pi)^3}\left(-\frac{\partial f_0}{\partial \varepsilon_k}\right)v_y^2\left[1 - \left(1 - \frac{\tau_M}{\tau}\right)\theta(v_x)e^{-x/v_x\tau}\right]$$

Effect of one interface

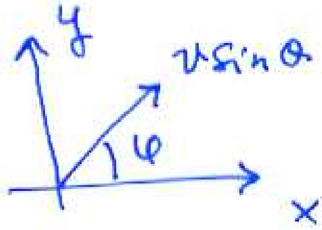
$$j_\uparrow(x) = e^2 E \tau \nu_0 v^2 \frac{1}{4\pi} \int_{-1}^{1} d\cos\theta$$

$$\times \left[\int_0^{2\pi} d\varphi \sin^2\theta \sin^2\varphi - \alpha_{M\uparrow} \int_{-\pi/2}^{\pi/2} d\varphi \sin^2\theta \sin^2\varphi \right.$$

$$\left. \times \exp\left(-\frac{x}{l \sin\theta \cos\varphi}\right)\right]$$

$$\alpha_{M\uparrow} = 1 - \frac{\tau_M}{\tau}$$

$$l = v\tau$$



$$v_x = v \sin\theta \cos\varphi$$
$$v_y = v \sin\theta \sin\varphi$$

$$i_\uparrow^{(1)} = \int_0^d j_\uparrow(x)\, dx$$ effect of left interface

If $d \gg l$

$$i_\uparrow^{(1)} \simeq \frac{\sigma_0 E d}{2} \left[1 - \frac{3\alpha_{M\uparrow}}{4\pi d} \int_0^\infty dx \int_{-1}^{1} d\cos\theta \int_{-\pi/2}^{\pi/2} d\varphi \right.$$

$$\left. \times \sin^2\theta \sin^2\varphi \exp\left(-\frac{x}{l \sin\theta \cos\varphi}\right)\right]$$

$$\int_0^{2\pi} d\varphi \sin^2\varphi = \frac{1}{2}\int_0^{2\pi} d\varphi\,(1 - \cos 2\varphi) = \pi$$

$$\int_{-1}^{1} dt\,(1 - t^2) = 2 - \frac{2}{3} = \frac{4}{3}$$

$$\Rightarrow i_\uparrow^{(1)} = \frac{\sigma_0 E d}{2}\left(1 - \frac{3\lambda_{M+}l}{16 d}\right) = \frac{\sigma_0 E d}{2}\frac{\tilde{\tau}_\uparrow}{\tau}$$

$\tilde{\tau}_\uparrow$ is effective relaxation time

$\tilde{\tau}_\uparrow < \tau$

Taking into account both interfaces

$$\sigma^{AF} = \frac{\sigma_0}{2}\frac{\tilde{\tau}_\uparrow^{AF}}{\tau} + \frac{\sigma_0}{2}\frac{\tilde{\tau}_\downarrow^{AF}}{\tau} = \frac{\sigma_0}{2\tau}\left(\tilde{\tau}_\uparrow^{AF} + \tilde{\tau}_\downarrow^{AF}\right)$$

$$\sigma^{F} = \frac{\sigma_0}{2\tau}\left(\tilde{\tau}_\uparrow^{F} + \tilde{\tau}_\downarrow^{F}\right)$$

$$\frac{1}{\tilde{\tau}_\uparrow^{AF}} = \frac{1}{\tau_\downarrow^{AF}} = \frac{1}{\tau} + \frac{1}{\tilde{\tau}_M} + \frac{1}{\tilde{\tau}_m}$$

$$\frac{1}{\tilde{\tau}_\uparrow^{F}} = \frac{1}{\tau} + \frac{2}{\tilde{\tau}_M}$$

$$\frac{1}{\tilde{\tau}_\downarrow^{F}} = \frac{1}{\tau} + \frac{2}{\tilde{\tau}_m}$$

$$\sigma^F - \sigma^{AF} \neq 0$$

Let $\tilde{\tau}_M \ll \tau \ll \tilde{\tau}_m$

$$\frac{1}{\tau_\uparrow^{AF}} = \frac{1}{\tau_\downarrow^{AF}} \simeq \frac{1}{\tilde{\tau}_M}$$

$$\frac{1}{\tau_\uparrow^F} \simeq \frac{2}{\tilde{\tau}_M}$$

$$\frac{1}{\tau_\downarrow^F} \simeq \frac{1}{\tau}$$

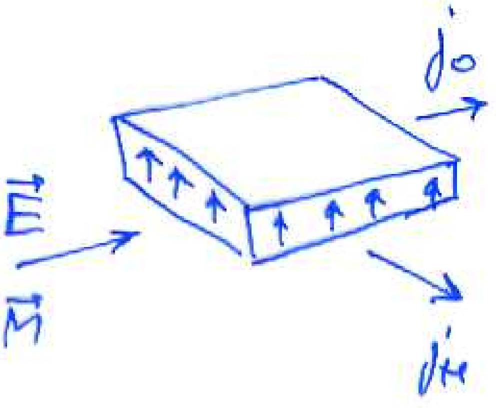
$$\sigma^{AF} - \sigma^F = \frac{\sigma_0}{2\tau}\left(2\tau_\uparrow^{AF} - \tau_\uparrow^F - \tau_\downarrow^F\right) =$$

$$\simeq \frac{\sigma_0}{2\tau}\left(2\tilde{\tau}_M - \frac{\tilde{\tau}_M}{2} - \tau\right) \simeq -\frac{\sigma_0}{2}$$

Effect is large, $\sigma^F > \sigma^{AF}$ – negative magnetoresistance

Anomalous Hall effect in ferromagnets

Hall current – like in case of usual Hall effect



Instead of $\vec{H}$ – magnetization $\vec{M}$

Mechanism?

No Lorentz force

Main theories:

- side jump
- skew scattering

Both related to impurities + SO interaction

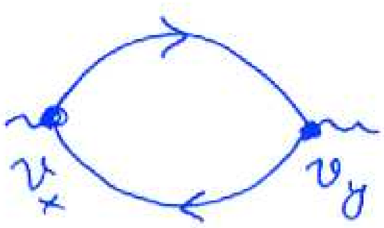
Hamiltonian:

$$\mathcal{H} = -\frac{\hbar^2 \nabla^2}{2m} + M\sigma_z - \frac{i\lambda_0^2}{4}(\vec{\sigma} \times \nabla V)\nabla + V(\vec{r})$$

$$V(\vec{r}) = \sum_i v(\vec{r} - \vec{R}_i)$$

We use Kubo formula

$$\sigma_{xy} = e^2 \sum_k v_x G^R v_y G^A$$



Side jump

Idea: renormalization of vertex

What is $\vec{v}$?

Without EM field: $\vec{v} = \frac{\hbar \vec{k}}{m} = -\frac{i\hbar}{m}\nabla$

Field: $\vec{k} \to \vec{k} - \frac{e\vec{A}}{\hbar c}$

$$\vec{v} = \frac{\hbar}{m}\left(-i\nabla - \frac{e\vec{A}}{\hbar c}\right)$$

More exactly:

$$\vec{v} = -\frac{c}{e}\frac{\delta \mathcal{H}}{\delta \vec{A}}$$

$$\mathcal{H} = \frac{\hbar^2}{2m}\left(-i\nabla - \frac{eA}{\hbar c}\right)^2 + M\sigma_z - \frac{i\lambda_0^2}{4}(\vec{\sigma}\times\nabla V)\left(\nabla - \frac{ieA}{\hbar c}\right)$$

$$\Rightarrow \quad \vec{v} = \frac{\hbar}{m}\left(-i\nabla - \frac{e\vec{A}}{\hbar c}\right) + \frac{\lambda_0^2}{4\hbar}(\vec{\sigma}\times\nabla V)$$

Matrix elements:

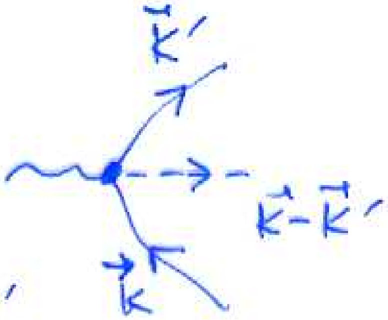
$$v_{\alpha kk'} = \frac{\hbar}{m}\left(k_\alpha - \frac{eA_\alpha}{\hbar c}\right)\delta_{kk'} + \frac{i\lambda_0^2}{4\hbar}\epsilon_{\alpha\beta\gamma}\sigma_\beta(k_\gamma - k'_\gamma)V_{kk'}$$

$$(\nabla_\alpha V)_{kk'} = \int d^3r\, e^{-i\vec{k}\cdot\vec{r}}\left(\nabla_\alpha \sum_q e^{i\vec{q}\cdot\vec{r}} V_q\right) e^{i\vec{k}'\cdot\vec{r}} =$$

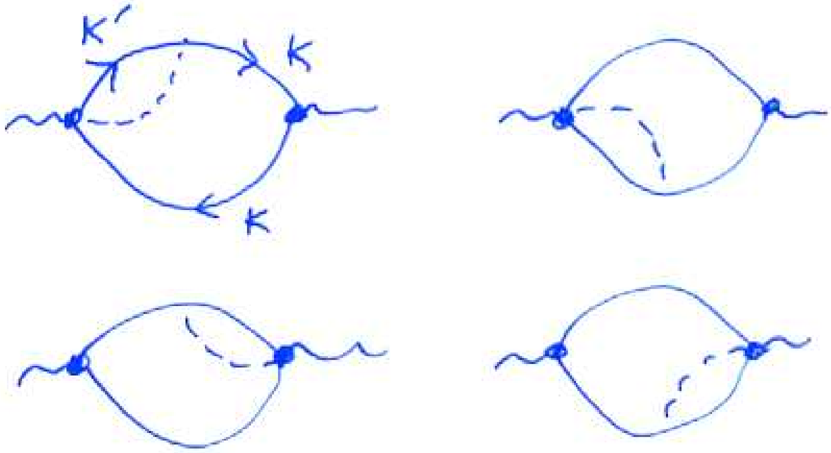
$$= i(k_\alpha - k'_\alpha)V_{k-k'}$$

Anomalous vertex:

$$\frac{i\lambda_0^2}{4\hbar} \epsilon_{\alpha\beta\gamma}\, \sigma_\beta\, (k_\gamma - k'_\gamma) V_{kk'}$$



Averaging over impurities (Born approximation)



$$\Rightarrow \sigma_{xy} = -\frac{ie^2\lambda_0^2 N_i V_0^2}{4\pi m} \mathrm{Tr} \sum_{kk'} \sigma_z k_y^2 G_k^R G_k^A \left(G_{k'}^R - G_{k'}^A\right)$$

$$G_k^{R,A} = \mathrm{diag}\left(\frac{1}{\mu - \varepsilon_{k\uparrow} \pm \frac{i}{2\tau_\uparrow}}, \frac{1}{\mu - \varepsilon_{k\downarrow} \pm \frac{i}{2\tau_\downarrow}} \right)$$

$$\frac{1}{\tau_\sigma} \sim N_i V_0^2$$

$$\Rightarrow \sigma_{xy} = \frac{e^2\lambda_0^2}{6} \left(\nu_\downarrow k_{F\downarrow} v_{F\downarrow} - \nu_\uparrow k_{F\uparrow} v_{F\uparrow} \right)$$

It does not depend on N_i - surprise

Skew scattering

SO in impurity scattering

$$\mathcal{H}_{imp} = V - \frac{i\lambda_0^2}{4}(\vec{\sigma} \times \nabla V)\nabla =$$

$$= V - \frac{i\lambda_0^2}{4}\epsilon_{\alpha\beta\gamma}\sigma_\alpha(\nabla_\beta V)\nabla_\gamma$$

matrix elements:

$$((\nabla_\beta V)\nabla_\gamma)_{kk'} = \int d^3 r\, e^{-i\vec{k}\cdot\vec{r}}\left(\nabla_\beta \sum_q e^{i\vec{q}\cdot\vec{r}} V_q\right)\nabla_\gamma e^{i\vec{k}'\cdot\vec{r}} =$$

$$= -\sum_q \int d^3 r\, e^{-i\vec{k}\cdot\vec{r}} q_\beta V_q e^{i\vec{q}\cdot\vec{r}} k'_\gamma e^{i\vec{k}'\cdot\vec{r}} =$$

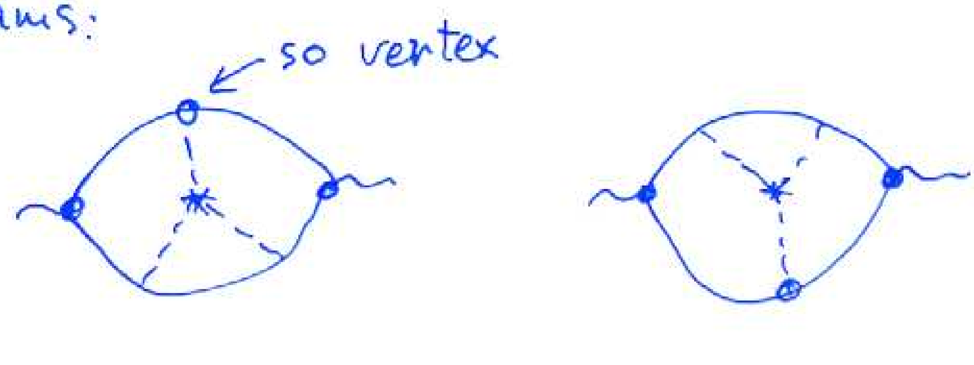
$$= -(k_\beta - k'_\beta)k'_\gamma V_{k-k'} \rightarrow$$

$$\rightarrow -k_\beta k'_\gamma V_{kk'}$$

$$\boxed{V_{kk'} \rightarrow V_{kk'}\left[1 + \frac{i\lambda_0^2}{4}\vec{\sigma}(\vec{k}\times\vec{k}')\right]}$$

SO scattering

Diagrams:



Result:

$$\sigma_{xy} = \frac{\pi e^2 \lambda_0^2}{18\hbar}\left(k_{F\downarrow}^2 \nu_\downarrow v_{F\downarrow}^2 \tau_\downarrow \frac{\nu_\downarrow \gamma_3}{\gamma_2} - k_{F\uparrow}^2 \nu_\uparrow v_{F\uparrow}^2 \tau_\uparrow \frac{\nu_\uparrow \gamma_3}{\gamma_2}\right)$$

γ_2 and γ_3 from

$$\langle V(\bar{r}_1) V(\bar{r}_2)\rangle = \gamma_2\, \delta(\bar{r}_1 - \bar{r}_2)$$

$$\langle V(r_1) V(r_2) V(r_3)\rangle = \gamma_3\, \delta(\bar{r}_1 - \bar{r}_2)\, \delta(\bar{r}_2 - \bar{r}_3)$$

<u>Resistivity tensor</u>

$$j_\alpha = \sigma_{\alpha\beta} E_\beta \qquad \sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

$$E_\alpha = \rho_{\alpha\beta} j_\beta$$

$$\rho = \sigma^{-1}$$

$$\Rightarrow \quad \rho_{xy} \simeq \frac{\sigma_{xy}}{\sigma_{xx}^2} \qquad (\sigma_{xy} \ll \sigma_{xx})$$

Which mechanism is dominant?

Side jump: $\rho_{xy} \sim \frac{1}{\tau^2} \sim R^2$ $\qquad (\sigma_{xx} \sim \tau)$

Skew scattering: $\rho_{xy} \sim \frac{\tau}{\tau^2} \sim R$

$\Rightarrow$ For large R – Side jump

Small R – skew scattering