Lecture 6

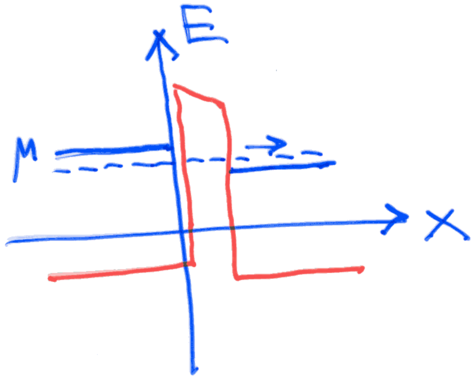
Transport in low-dimensional systems: size-quantization effects (cont)

- Ballistic transport in nanoconstrictions
- Aharonov-Bohm effect in nanorings
- Quantization of Hall conductivity in 2D systems

Conductance in 1D system

model : 1D channel with a barrier

$$j = 2e \int_{\mu - e\varphi/2}^{\mu + e\varphi/2} d\varepsilon_x \, \nu_1(\varepsilon_x)\, v \, T(\varepsilon_x)$$



Linear response:

$$\Rightarrow \quad \sigma = 2e^2 \nu_1(\mu)\, v\, T(\mu)$$

$T(\varepsilon)$ is the transmission coefficient

1D density of states:

$$\int_{-\infty}^{\infty} \frac{dk}{2\pi} = \frac{1}{\pi}\int_0^{\infty} dk = \frac{1}{\pi}\int_0^{\infty} d\left(\frac{\hbar^2 k^2}{2m}\right)\frac{m}{\hbar^2 k} = \frac{1}{\pi\hbar}\int_0^{\infty}\frac{d\varepsilon_x}{v}$$

$$\Rightarrow \quad \underset{\sim\sim\sim}{\nu_1(\varepsilon) = \frac{1}{2\pi\hbar v}}$$

only $k>0$

$$j = \frac{2e}{2\pi\hbar}\int_{\mu - e\varphi/2}^{\mu + e\varphi/2} d\varepsilon_x \, T(\varepsilon_x) = \frac{2e}{h}\int_{\mu - e\varphi/2}^{\mu + e\varphi/2}$$

$$\sigma = \frac{2e^2}{2\pi\hbar} T(\mu) = 2\,\frac{e^2}{h}\,T(\mu)$$

Generalization to N channels:

$$\sigma = \frac{e^2}{2\pi\hbar}\sum_i T_i(\mu)$$

Landauer–Büttiker

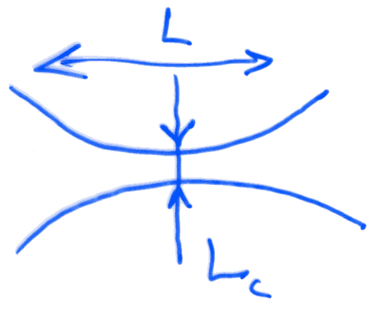
For $T_i = 1$ (no impurities, barriers)

$$\sigma = \frac{e^2}{2\pi\hbar} N$$

N is the number of channels

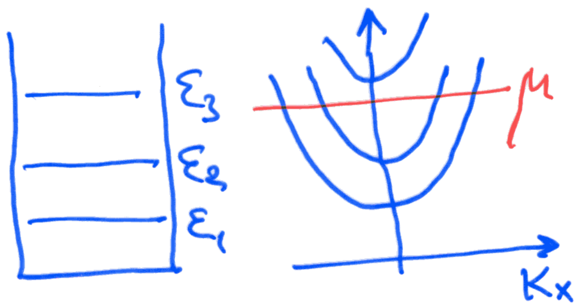
<u>Realization</u> : nanoconstriction

ballistic regime : $L < \ell$
(ℓ is mean free path)



$L_c \sim \lambda$

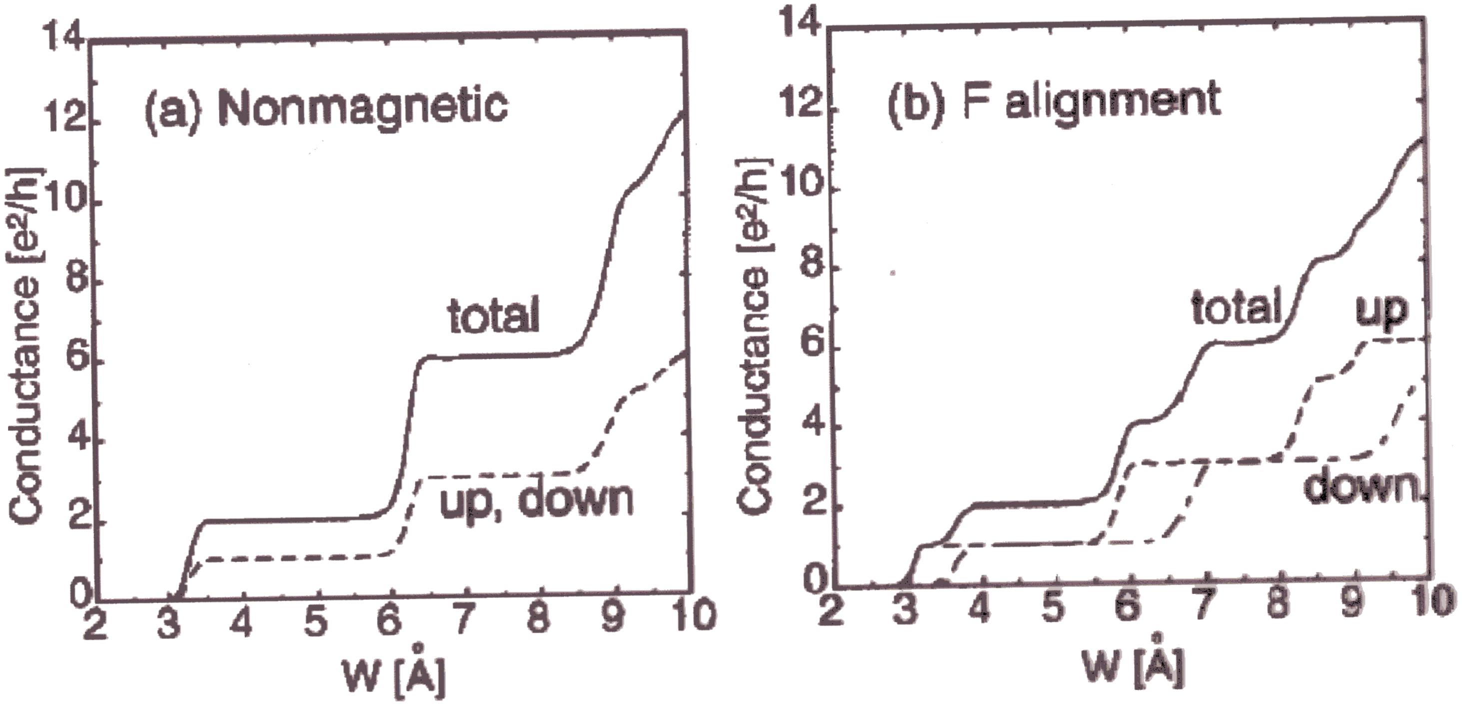
Size-quantization in wire:



⇒ Conductance depends on L_c (wire thickness)

In non-magnetic case each level is spin-degenerate

In magnetic case : no degenerate (spin)



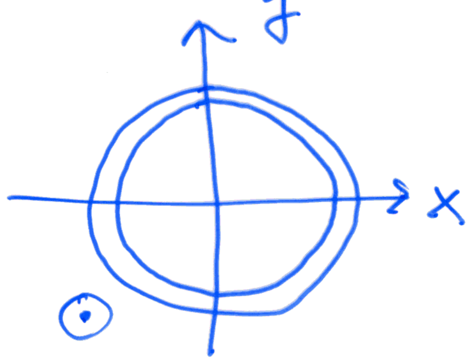
(a) Nonmagnetic
total
up, down
Conductance [e²/h]
W [Å]
(b) F alignment
total
up
down
Conductance [e²/h]
W [Å]

Aharonov-Bohm effect

mesoscopic ring in magnetic field

- ring's size $< l_\varepsilon$ (coherence length)
- ring's thickness $< \lambda$

$\vec{H} \parallel z$



Vector potential

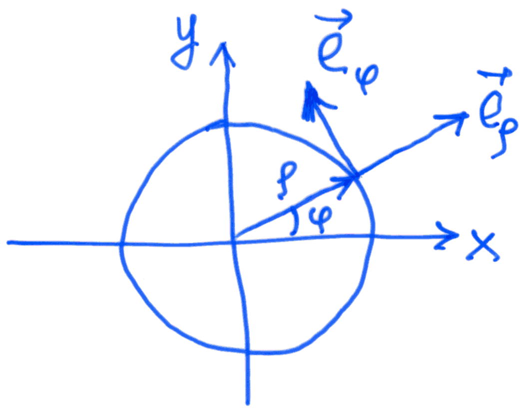
$$\vec{A} = \frac{1}{2}(-Hy, Hx, 0)$$

symmetric gauge

$$H = \partial_x A_y - \partial_y A_x$$

more convenient: cylindric coordinate:

$$\vec{A} = A_\rho \vec{e}_\rho + A_\varphi \vec{e}_\varphi$$



$$A_x = A_\rho \cos\varphi - A_\varphi \sin\varphi$$

$$A_y = A_\rho \sin\varphi + A_\varphi \cos\varphi$$

Our choice:

$$A_\rho = 0$$

$$A_\varphi = \frac{\rho H}{2}$$

$$\partial_x = \cos\varphi \frac{\partial}{\partial\rho} - \frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}$$

$$\partial_y = \sin\varphi \frac{\partial}{\partial\rho} + \frac{\cos\varphi}{\rho}\frac{\partial}{\partial\varphi}$$

Check:

$$H = \left(\cos\varphi \frac{\partial}{\partial\rho} - \frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}\right) A_\varphi \cos\varphi +$$

$$+ \left(\sin\varphi \frac{\partial}{\partial\rho} + \frac{\cos\varphi}{\rho}\frac{\partial}{\partial\varphi}\right) A_\varphi \sin\varphi =$$

$$= \cos^2\varphi \frac{\partial A_\varphi}{\partial\rho} + \frac{\sin^2\varphi}{\rho} A_\varphi + \sin^2\varphi \frac{\partial A_\varphi}{\partial\rho} + \frac{\cos^2\varphi}{\rho} A_\varphi =$$

$$= \frac{\partial A_\varphi}{\partial\rho} + \frac{A_\varphi}{\rho} = H$$

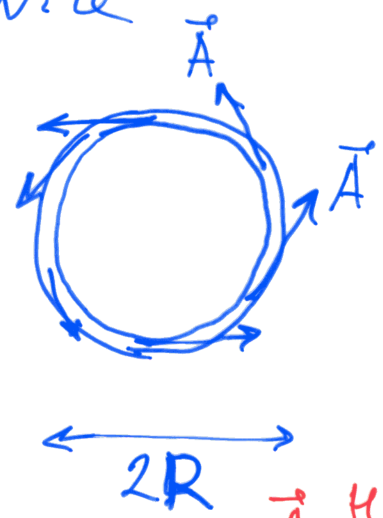
$$A_\varphi = \frac{H\rho}{2}$$

$\Rightarrow$ Vector potential along the wire

In mesoscopic ring

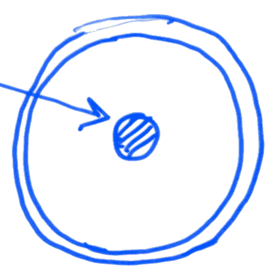
$R \gg \lambda$

$\Rightarrow$ adiabatic motion of electrons in 1D wire



$\vec{A} = \frac{HR}{2}$

Suppose magnetic field is within a small region (tube)



What matters:

<u>Flux</u> $\Phi = \int_{S_0} H\, dS = \oint A_i(r)\, dr_i$

Ring in homogeneous field: $\Phi = \pi R^2 H$

For the flux tube:

effect is topological

(does not depend on ring's shape or size)

Electron in electromagnetic potential

$$\left[\frac{\hbar^2}{2m}\left(-i\nabla - \frac{eA}{\hbar c}\right)^2 - \varepsilon\right]\psi(x) = 0$$

For $A(x)$ smooth: semiclassical approximation

$$\psi \sim e^{i\xi} \qquad |\xi''| \ll (\xi')^2$$

$$\frac{\hbar^2}{2m}\left(K - \frac{eA}{\hbar c}\right)^2 = \varepsilon$$

where $K = K(x) = \dfrac{d\xi(x)}{dx}$

$$\psi \simeq e^{\frac{i}{\hbar}\sqrt{2m\varepsilon}\,x}\; e^{\frac{ie}{\hbar c}\int A(x)\,dx}$$

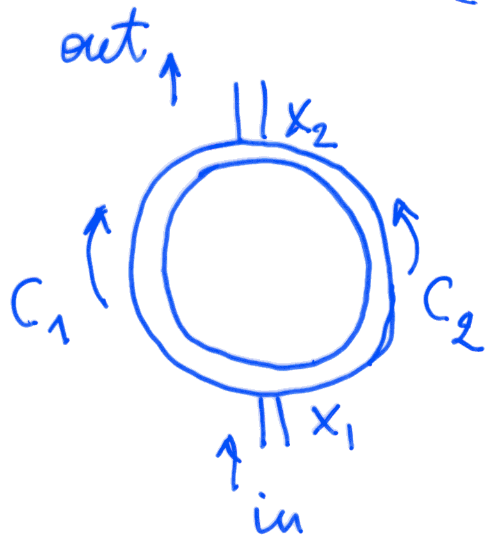
Phase: $\displaystyle \phi = \frac{e}{\hbar c}\int_{x_0}^{x} A(x)\,dx$

For mesoscopic ring:

$$\Phi_0 = \frac{hc}{e}$$

$$\phi = \frac{e}{\hbar c}\,\frac{HR}{2}\cdot 2\pi R = 2\pi\frac{\Phi}{\Phi_0}$$

Conductance



$$\varnothing_1(x_1,x_2) = \frac{e}{\hbar c}\int_{C_1} A(x)\,dx$$

$$\varnothing_2(x_1,x_2) = \frac{e}{\hbar c}\int_{C_2} A(x)\,dx$$

$$\Delta\varnothing_{out} = \varnothing_1 - \varnothing_2 = \frac{e}{\hbar c}\oint A(x)\,dx = 2\pi\frac{\Phi}{\Phi_0}$$

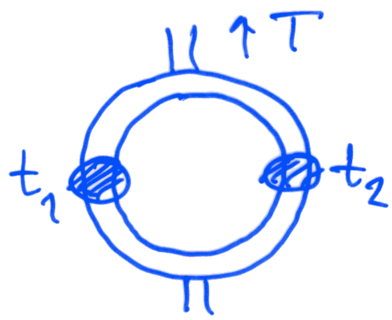
Transmission:

If $\Delta\varnothing_{out} = 2\pi n \quad T = 1$

If $\Delta\varnothing_{out} = \pi n \quad T = 0$

Generalization: barrier in each arm

Yu. Gefen, Y. Imry,
M. Ya. Azbel PRL (1984)

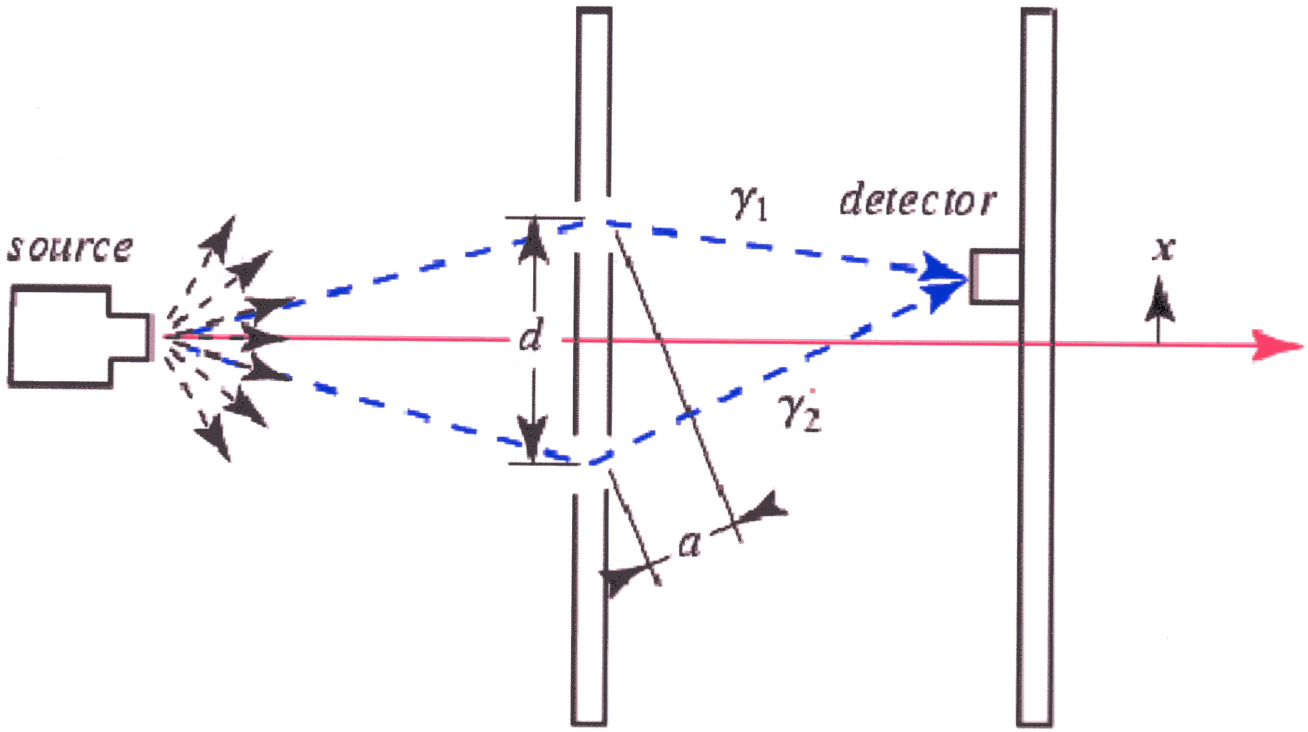


Conductance oscillates as Φ/Φ_0
(normal AB effect)

Experiment:

R. A. Webb et al PRL (1985)

Aharonov-Bohm effect



Y. Aharonov and D. Bohm. Phys. Rev. 115, 485 (1959)

In experiment both Φ/Φ_0 and $2\Phi/\Phi_0$ oscillations were observed

Explanation of $2\Phi/\Phi_0$:

B.L. Altshuler et al. Pis'ma ZhETF (1981)

→ weak localization (quantum interference effect)

Main question:

How electron knows about magnetic field ?

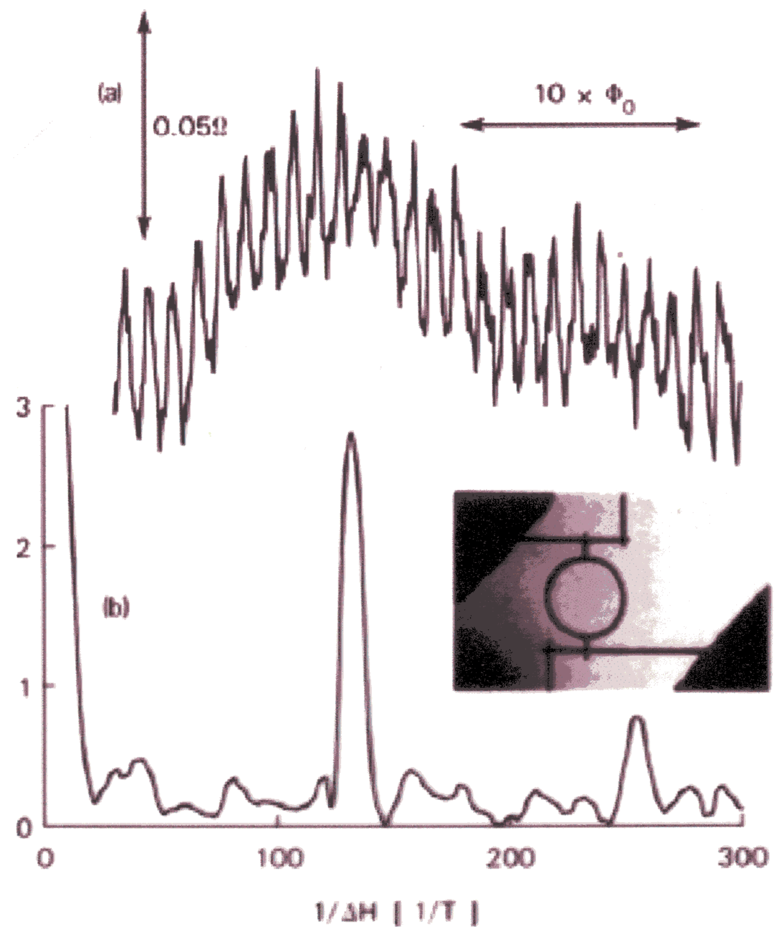
"A" is not really defined (up to the gauge)

New development: gauge potential instead of electromagnetic

→ M.V. Berry (1984)

Observation of h/e Aharonov-Bohm Oscillations in Normal-Metal Rings

R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 27 March 1985)



Persistent current in mesoscopic ring

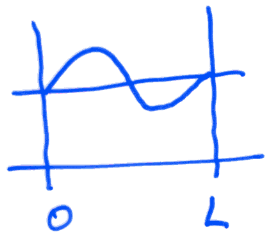
Back to the solution:

$$K = \frac{eA}{\hbar c} + \frac{\sqrt{2m\varepsilon}}{\hbar} \qquad A = \frac{HR}{2}$$

Momentum quantization:

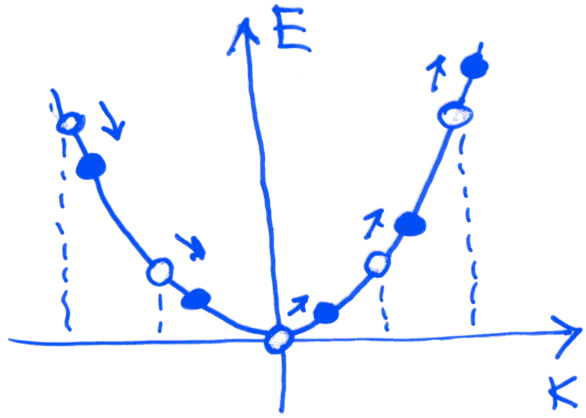
$A=0$: $\quad kL = 2\pi n$, $\qquad L = 2\pi R$

$A \neq 0$: $\left(K - \frac{eA}{\hbar c}\right)\cdot 2\pi R = 2\pi n$



$$K = \frac{1}{R}\left(n + \frac{eHR^2}{2\hbar c}\right)$$

$$\boxed{K = \frac{1}{R}\left(n + \frac{\Phi}{\Phi_0}\right)}$$



$\Rightarrow$ Equilibrium current

Quantum Hall effect

2D electron gas in perpendicular magnetic field

$$\mathcal{H} = -\frac{\hbar^2}{2m}\left(\nabla - \frac{ie\vec{A}}{\hbar c}\right)^2 =$$

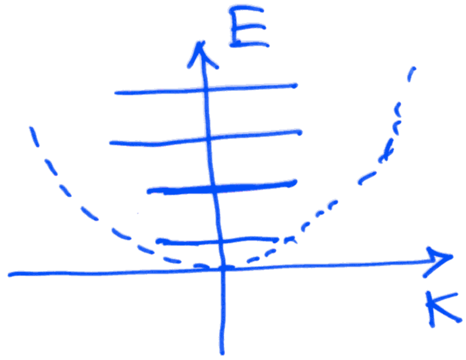
$\vec{A} = (-Hy, 0, 0)$

Landau gauge

$$\rightarrow \frac{m\omega_H^2}{2}(y-y_0)^2 + \frac{\hbar^2}{2m}\frac{d^2}{dy^2}$$

$$y_0 = -\frac{\hbar c K_x}{eH}$$

$$\underset{\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim}{\varepsilon_n = \hbar\omega_H\left(n+\frac{1}{2}\right)}$$



$$K_x^{max} = L_y \frac{eH}{\hbar c}$$

Number of states in one Landau level:

$$N_i = L_x \frac{K_x^{max}}{2\pi} = L_x L_y \frac{eH}{2\pi\hbar c} = \frac{\Phi}{\Phi_0}$$

Hall conductance

Drude-Lorenz: $\cancel{\frac{\vec{v}}{\tau}} = e\vec{E} + \frac{e}{c}[\vec{v}\times\vec{H}]$

small for large H

$$v_H = \frac{c}{H} E$$

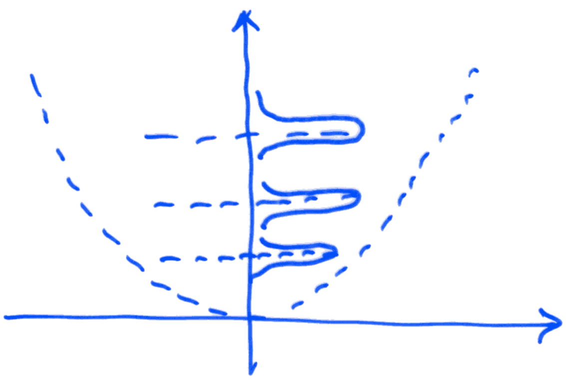
$$j_H = e n v_H = e \cdot \frac{L_x L_y e H}{2\pi\hbar c} N \cdot \frac{cE}{H} =$$

$$= \frac{e^2}{2\pi\hbar} L_x L_y N E$$

$$\boxed{G_H = \frac{e^2}{2\pi\hbar} N}$$

N is the number of filled levels

Real theory should take care of impurities!



Experiment:

K. von Klitzing et al.
PRL (1980)

Theory: R. Laughlin (1981)
D. Thouless (1982)
and many others ...

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New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

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