Lecture 5

Transport in low-dimensional systems: size quantization effects

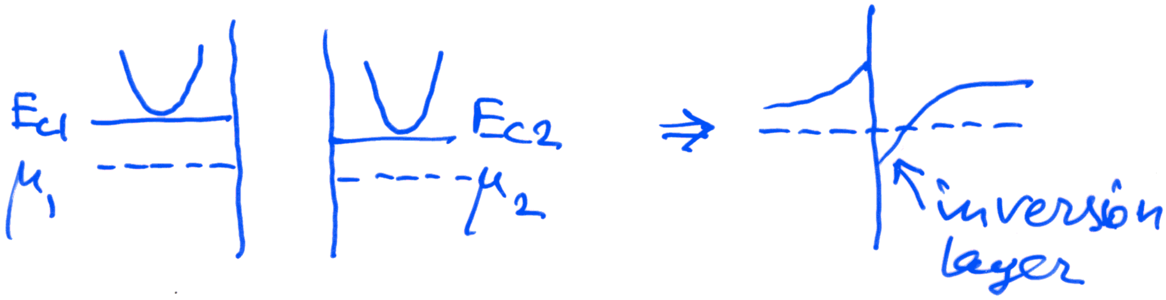
- Two-dimensional electron gas
- Semiconductor quantum wells
- Quantum wires
- Quantum dots
- Spin-orbit interaction in low-dimensional systems

Transport in low-dimensional systems

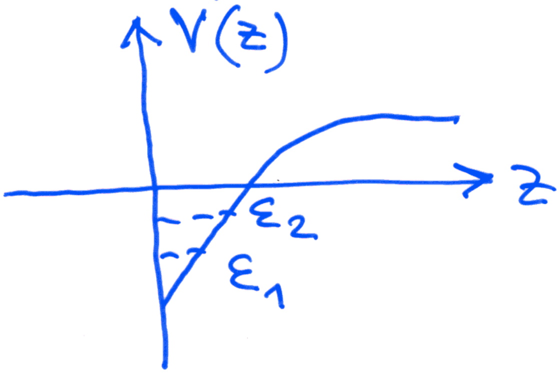
Two-dimensional electron gas (2DEG)

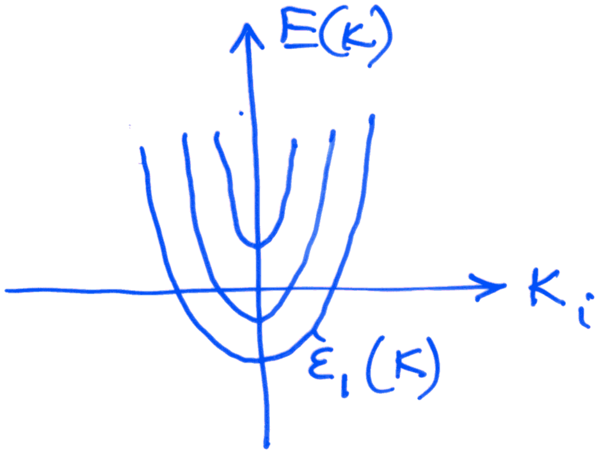
- Si-inversion layers (MOSFET technology)
- semiconductor interfaces (InGaAs-InAlAs)
- semiconductor quantum wells (QW)
- electrons on liquid helium

Contact of two semiconductors:

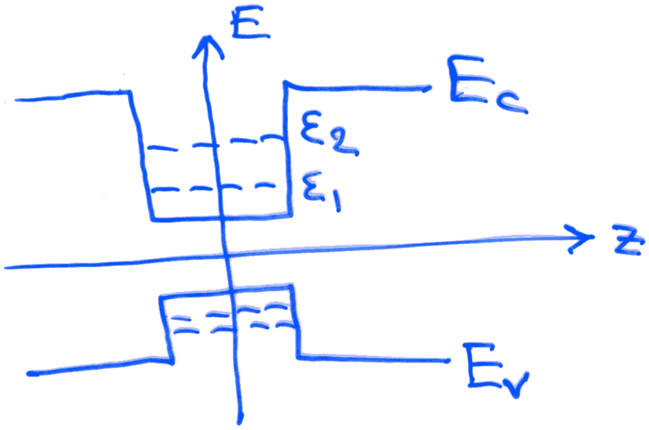


Potential profile





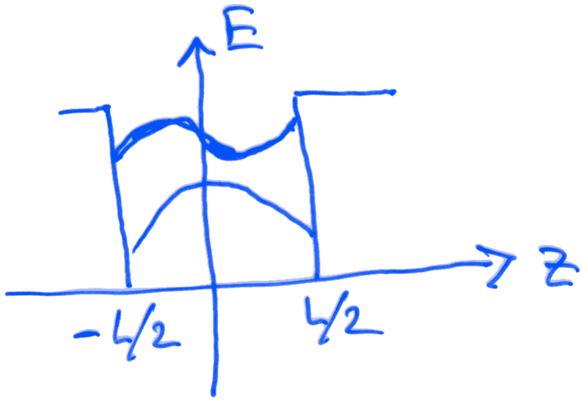
Semiconductor QWs



Doping and scattering are separated (L. Esaki)

Si/SiGe, GaAlAs/GaAs
PbTe/PbSnTe, etc.

rectangular model potential



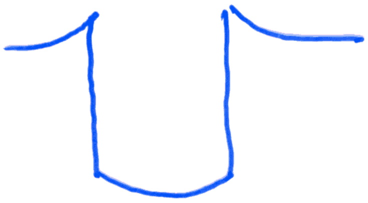
Deep QW:

$$kL = n\pi$$

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

$L \sim \lambda$ - condition of size-quantization

Real profile:
(example)



Transport in 2D system: usual classical methods including Drude-Lorentz formula, Kubo formalism

Problems:

- self-consistent potential profile
- scattering from impurities (screened potential)
- complex energy spectrum of semiconds.

Estimations:

metals: $\lambda \sim a_0$ $\qquad n = 2\frac{4\pi k_F^3}{3(2\pi)^3}$

Semiconductors:

Let $n = 10^{15}\ cm^{-3}$

$k = (3\pi^2 n)^{1/3} \simeq 3\cdot 10^5\ cm^{-1}$

$\lambda \simeq 10^{-5}\ cm = 100\ nm$

Nanotechnology $L \sim 10\ nm$

Low-dimensionality of a different type:

$L \sim l$ (mean free path)

In semiconductors:

$l = v\tau \simeq 10^7\ \frac{cm}{s}\cdot 10^{-13}\ s = 10^{-6}\ cm = 10\ nm$

$(n \simeq 10^{18})$

In metals:

$l = 10^8\ \frac{cm}{s}\cdot 10^{-13} = 10^{-5}\ cm = 100\ nm$

Effect of magnetic field in QW:

- parallel
- perpendicular

In perpendicular field:

- classically calculated Hall conductivity and magnetoresistance
- strong field: QHE

In parallel field:

- if $l_H \lesssim$ magn. field is strong
- otherwise: weak corrections

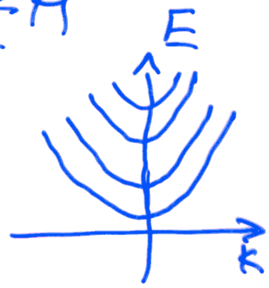
$$l_H = \sqrt{\frac{\hbar c}{eH}}$$

In perpendicular field:

magn. field effect is separated

$$\mathcal{E}_n(k) = \frac{\hbar^2 k^2}{2m} + \hbar\omega_H\left(n+\frac{1}{2}\right) \pm \frac{g\mu_B}{2}H$$

$$\omega_H = \frac{eH}{mc}$$



More complicated quasi 2D systems

- multilayers
- hybrid structures
- layered crystals (GaSe, InSe)

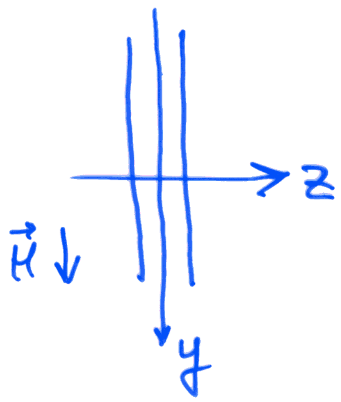
Energy spectrum of QW in parallel magnetic field

Schödinger eq:

$$\left[-\frac{\hbar^2}{2m}\left(\nabla - \frac{ie\vec{A}}{\hbar c}\right)^2 + \frac{m\omega_0^2 z^2}{2}\right]\Psi = \varepsilon\Psi$$

parabolic QW

Let $\vec{H} \parallel y$:



$$\vec{A} = (Hz, 0, 0) \quad \text{Landau gauge}$$

$$\left[-\frac{\hbar^2}{2m}\left(\partial_x - \frac{ieHz}{\hbar c}\right)^2 - \frac{\hbar^2}{2m}\left(\partial_y^2 + \partial_z^2\right) + \frac{m\omega_0^2 z^2}{2} - \varepsilon\right]\Psi = 0$$

ω_0 is classical frequency:

$$m\ddot{z} = -\omega_0^2 z m$$

$$z \sim e^{-i\omega_0 t}$$

$$V(z) = \frac{m\omega_0^2 z^2}{2}$$

$$\Psi \sim e^{ik_x x + ik_y y}$$

$$\left[\frac{\hbar^2(k_x^2 + k_y^2)}{2m} - \frac{\hbar k_x eHz}{mc} + \frac{e^2H^2z^2}{2mc^2} - \frac{\hbar^2}{2m}\frac{d^2}{dz^2} + \frac{m\omega_0^2 z^2}{2} - \varepsilon\right]\Psi = 0$$

$$\left[\frac{\hbar^2(k_x^2+k_y^2)}{2m}+\frac{m\omega^2}{2}\left(z-\frac{\omega_B \hbar k_x}{\omega^2 m}\right)^2-\right.$$

$$\left.-\frac{\hbar^2\omega_B^2 k_x^2}{2m\omega^2}-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}-\varepsilon\right]\psi=0$$

$\omega^2=\omega_B^2+\omega_0^2$

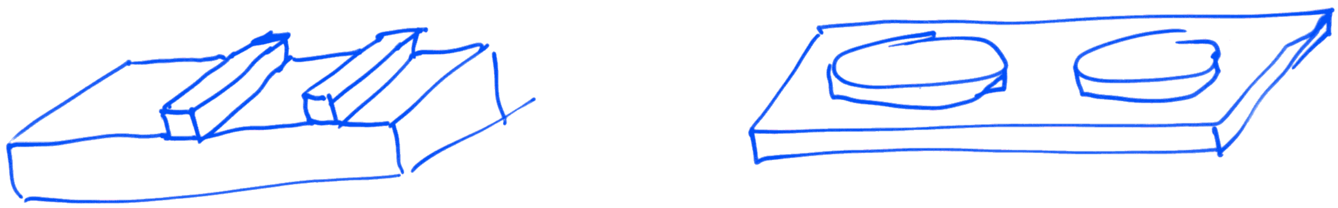
$$\left[\frac{\hbar^2 k_x^2}{2m}\frac{\omega_0^2}{\omega^2}+\frac{\hbar^2 k_y^2}{2m}+\right.$$

$$\left.+\frac{m\omega^2}{2}(z-z_0)^2-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}-\varepsilon\right]\psi=0$$

$$E(k)=\frac{\hbar^2 k_y^2}{2m}+\frac{\hbar^2 k_x^2}{2m}\frac{\omega_0^2}{\omega^2}+\hbar\omega\left(n+\frac{1}{2}\right)$$

Other samples of low-dimensional objects:

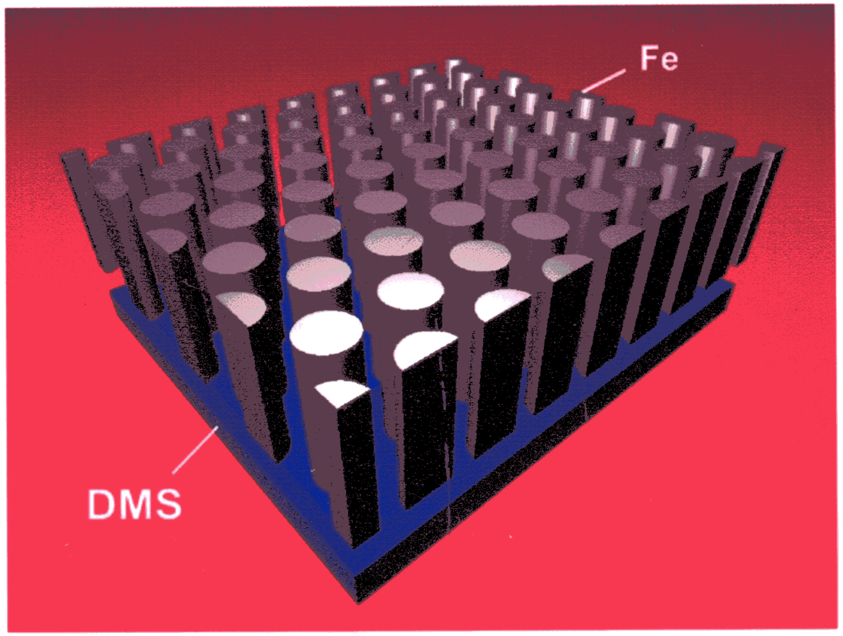
- nanowires (semiconductor nanowires, carbon nanotubes)
- quantum dots

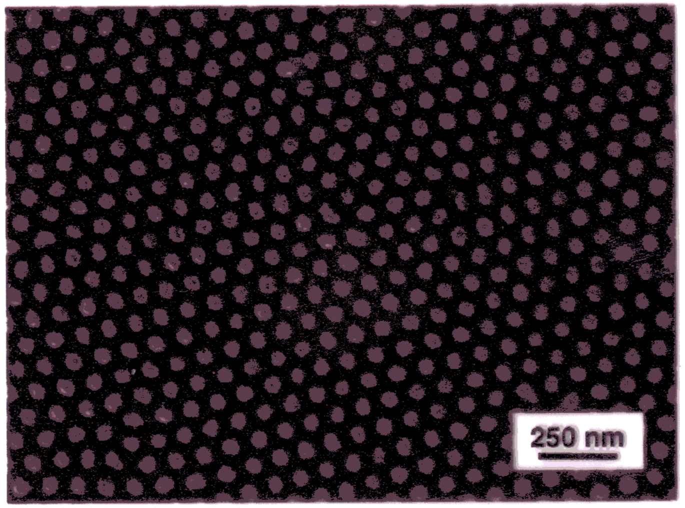


- nanowire lattice
- arrays of quantum dots

etc.

Magnetic nanowires





K. Nielsch et al. APL (2001)

2DEG with Rashba SO interaction

$$H = \frac{\hbar^2 k^2}{2m} + \alpha(\sigma_x k_y - \sigma_y k_x) =$$

$$= \begin{pmatrix} \varepsilon_k & i\alpha k_- \\ -i\alpha k_+ & \varepsilon_k \end{pmatrix}$$

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

(electrons in x-y plane)

$$k_\pm = k_x \pm i k_y$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Schrödinger eq:

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

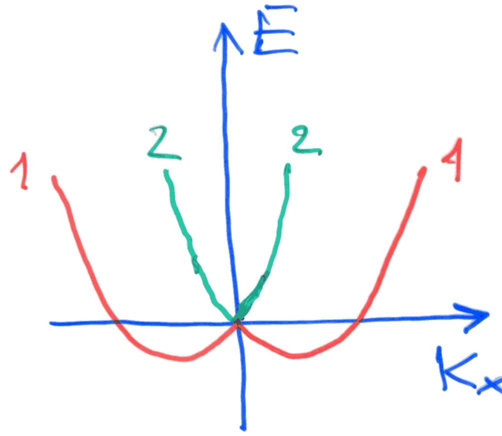
$$\begin{pmatrix} \varepsilon_k - E & i\alpha k_- \\ -i\alpha k_+ & \varepsilon_k - E \end{pmatrix} \begin{pmatrix} \varphi \\ \chi \end{pmatrix} = 0$$

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$$

$$\begin{cases} (\varepsilon_k - E)\varphi + i\alpha k_- \chi = 0 \\ -i\alpha k_+ \varphi + (\varepsilon_k - E)\chi = 0 \end{cases}$$

$$\det = (\varepsilon_k - E)^2 + \alpha^2 k^2 = 0$$

$$\boxed{E_{1,2}(k) = \varepsilon_k \mp \alpha k}$$



Eigenfunctions:

1) For $E_1(k) = \varepsilon_k - \alpha k$

$$\chi = \frac{i\alpha k_+}{\varepsilon_k - E_1}\varphi = \frac{i k_+}{k}\varphi$$

$$\psi_1 = N \begin{pmatrix} k \\ ik_+ \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \frac{ik_+}{k} \end{pmatrix}$$

If $\vec{k} \parallel x$: $\psi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$

Eigenfunction of σ_y:

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

2) $\underline{E_2(k) = \varepsilon_k + \alpha k}$

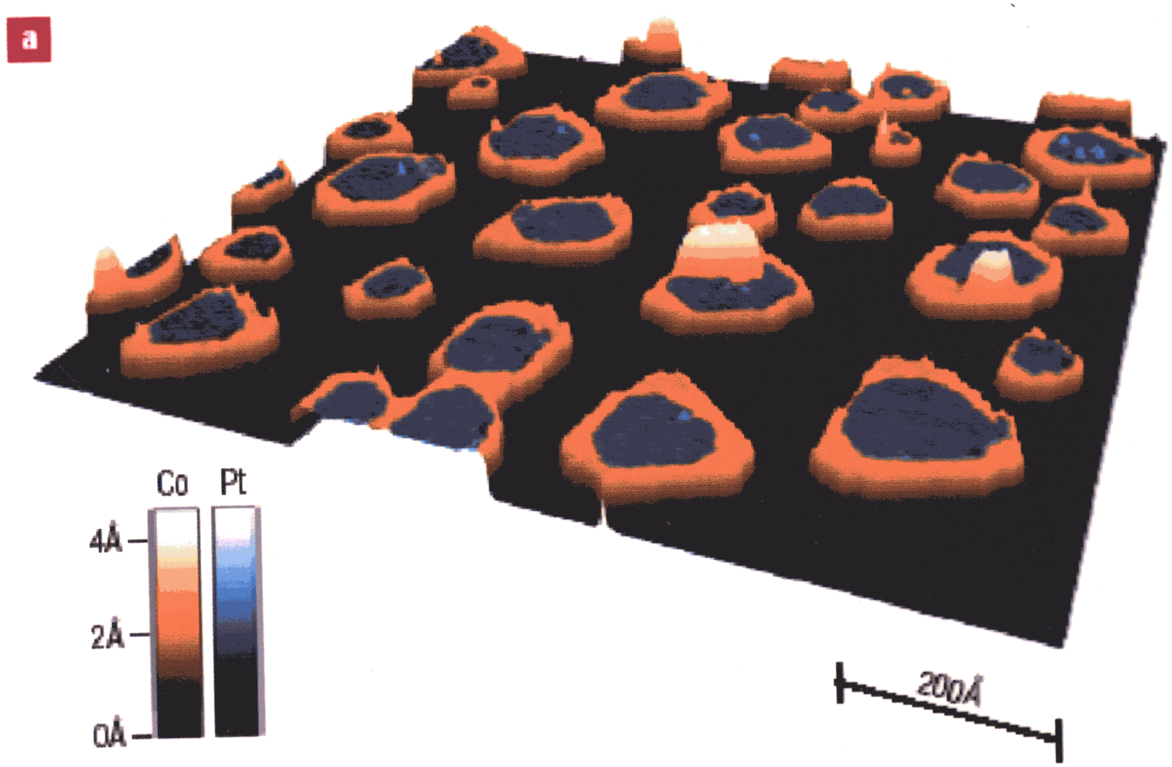
$$\varphi = \frac{i\alpha k_-}{E_2 - \varepsilon_k} = \frac{ik_-}{k} \chi$$

$$\psi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{ik_-}{k} \\ 1 \end{pmatrix}$$

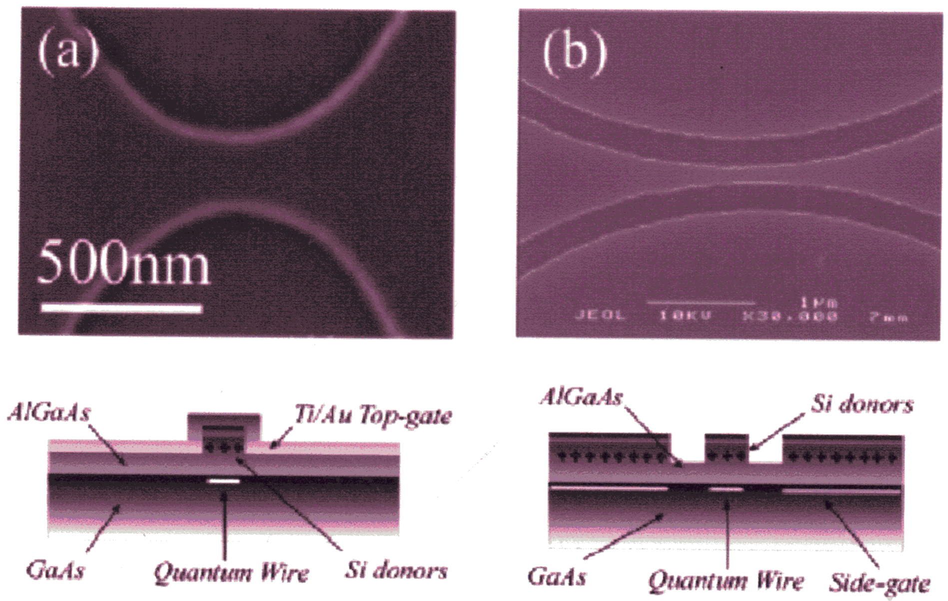
If $\vec{k} \parallel x$: $\psi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix}$

It is also eigenfunction of σ_y:

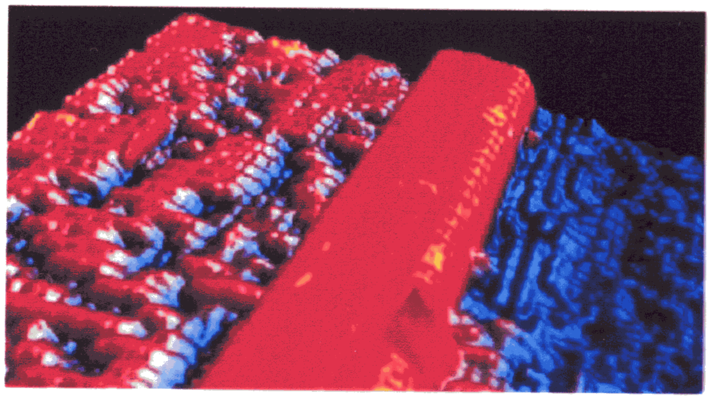
$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix} = -\begin{pmatrix} i \\ 1 \end{pmatrix}$$

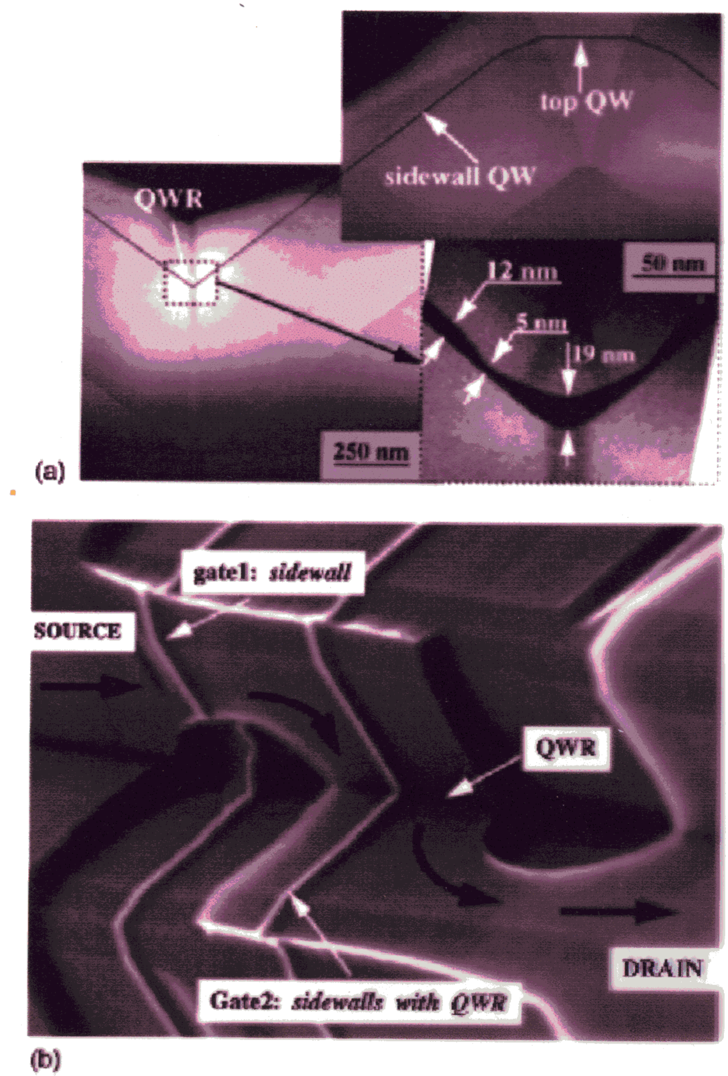


Tailoring bimetallic islands
S. Rusponi et al. Nature (2003)



(a)
500nm
(b)
JEOL 10KV X30,000 1μm 7mm
AlGaAs
Ti/Au Top-gate
GaAs
Quantum Wire
Si donors
AlGaAs
Si donors
GaAs
Quantum Wire
Side-gate





QWR
sidewall QW
top QW
50 nm
12 nm
5 nm
19 nm
250 nm
(a)
gate1: sidewall
SOURCE
QWR
DRAIN
Gate2: sidewalls with QWR
(b)