Lecture 2.

Transport theories of metals and semiconductors

Outline:

- Classical theory of Drude-Lorentz
- Boltzmann kinetic equation
- Magnetoresistance of metals and semiconductors
- Hall effect

Classical Theory of Drude-Lorentz

P. Drude (1900)
H. A. Lorentz (1905)

→ Classical equation of motion

Electron in electric field

$$m\dot{v} + \frac{mv}{\tau} = eE$$

Solution:

$$v = \frac{eE\tau}{m} \qquad \dot{v} = 0$$

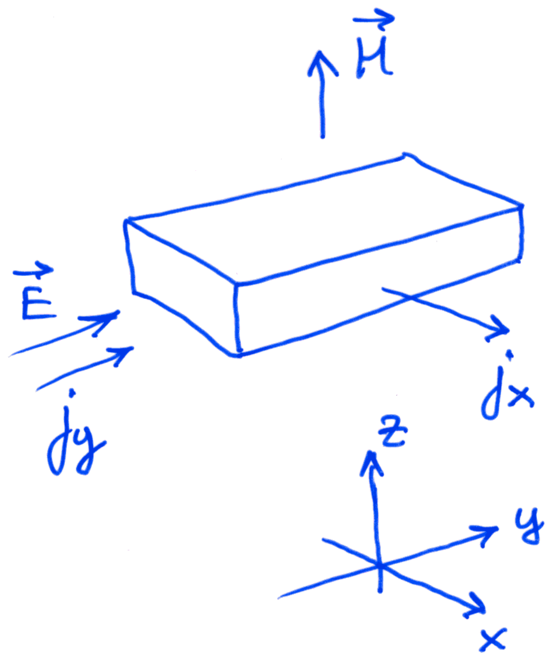
Current:

$$j = env = \frac{ne^2\tau E}{m} = \sigma E$$

$$\boxed{\sigma = \frac{ne^2\tau}{m}}$$

Electron in electric and magnetic field

$$m\dot{\vec{v}} + \frac{m\vec{v}}{\tau} = e\vec{E} + \frac{e}{c}[\vec{v}\vec{H}]$$



$$m\dot{v}_x + \frac{mv_x}{\tau} = \frac{e}{c} v_y H$$

$$m\dot{v}_y + \frac{mv_y}{\tau} = eE - \frac{eH}{c} v_x$$

Define:

$$w = v_x + i v_y$$

$$m\dot{w} + \frac{mw}{\tau} = ieE - \frac{ieH}{c} w$$

$$\left(m\frac{d}{dt} + \frac{m}{\tau} + \frac{ieH}{c}\right) w = ieE$$

Solution:

$$w = Ae^{-i\omega t} + B$$

$$\omega = \frac{eH}{mc} - \frac{i}{\tau} = \omega_c - \frac{i}{\tau}$$

$$B = \frac{eE\tau}{m} \frac{i + \omega_c\tau}{1+(\omega_c\tau)^2}$$

$$\Rightarrow \quad w = Ae^{-i\omega_c t} e^{-t/\tau} + \frac{eE\tau}{m} \frac{i+\omega_c\tau}{1+(\omega_c\tau)^2}$$

$$\boxed{\begin{aligned} v_x &= Ae^{-t/\tau}\cos(\omega_c t) + \frac{eE\tau^2\omega_c}{m}\frac{1}{1+(\omega_c\tau)^2} \\ v_y &= -Ae^{-t/\tau}\sin(\omega_c t) + \frac{eE\tau}{m}\frac{1}{1+(\omega_c\tau)^2} \end{aligned}}$$

τ momentum relaxation time

ω_c cyclotron frequency

For $t < \tau$ motion is <u>ballistic</u>

For $t > \tau$ <u>diffusive</u>

$$j_y = \frac{ne^2\tau}{m} \frac{1}{1+(\omega_c\tau)^2} E$$

$\tau \gg t$
(drift)

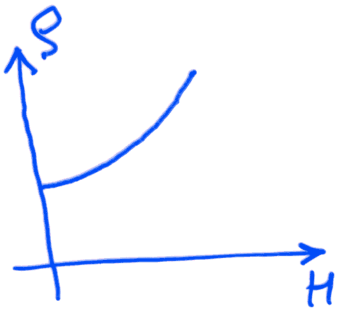
$$\sigma_{yy} = \frac{\sigma_0}{1+(\omega_c\tau)^2} \qquad \sigma_0 = \frac{ne^2\tau}{m}$$

(Conductivity is tensor)

If magnetic field is small, $\omega_c\tau \ll 1$ $\left(H \ll \frac{mc}{e\tau}\right)$

$$\sigma_{yy} \simeq \sigma_0 (1 - \omega_c^2\tau^2)$$

Classical positive MR



$$j_x = \frac{ne^3\tau^2 H}{m^2 c} \frac{E}{1+(\omega_c\tau)^2} =$$

$$= \sigma_0 \frac{\omega_c\tau}{1+(\omega_c\tau)^2} E$$

$$\sigma_{xy} = \sigma_0 \frac{\omega_c\tau}{1+(\omega_c\tau)^2}$$

Small magnetic field, $\omega_c\tau \ll 1$

$$\sigma_{xy} \simeq \sigma_0 \omega_c \tau$$

j_x is Hall current – Hall effect

Boltzmann kinetic equation

Distribution function $f(\vec{k},\vec{r},t) \to f_k$

Equilibrium:

$$f_{0k} = \frac{1}{\exp\frac{\varepsilon_k-\mu}{T}+1} \qquad \varepsilon_k = \frac{k^2}{2m}$$

$$\frac{\partial f_k}{\partial t} + \frac{d\vec{k}}{dt}\frac{\partial f_k}{\partial \vec{k}} + \frac{d\vec{r}}{dt}\frac{\partial f_k}{\partial \vec{r}} = -\,St\, f_k$$

In equilibrium: $\frac{df_k}{dt}=0$

$St\, f_k$ includes all scattering (collision) mechanisms

Stationary homogeneos state with weak electric field and scattering from impurities

Stationary: $\frac{\partial f_k}{\partial t}=0$

Homogeneous: $\frac{\partial f_k}{\partial \vec{r}}=0$

Electric field: $\frac{d\vec{k}}{dt}=e\vec{E}$

$$e\vec{E}\frac{\partial f_k}{\partial \vec{k}} = -\sum_{k'}\left[w_{kk'}f_k(1-f_{k'}) - w_{k'k}f_{k'}(1-f_k)\right]$$

Elastic interactions: $w_{kk'}=w_{k'k}$ and $|\vec{k}|=|\vec{k}'|$

$$\boxed{e\vec{E}\,\frac{\partial f_k}{\partial \vec{k}} = -\sum_{k'} W_{kk'}\,(f_k - f_{k'})}$$

Perturbation theory (linear response):

$$f_k = f_{0k} + \delta f_k$$

$$e\vec{E}\,\frac{\partial f_{0k}}{\partial \vec{k}} = -\sum_{k'} W_{kk'}\,(\delta f_k - \delta f_{k'})$$

1) $\dfrac{\partial f_{0k}}{\partial \vec{k}} = \dfrac{\partial \varepsilon_k}{\partial \vec{k}}\,\dfrac{\partial f_{0k}}{\partial \varepsilon_k} = \vec{v}\,\dfrac{\partial f_{0k}}{\partial \varepsilon_k}$ $\qquad \vec{v} = \dfrac{\vec{k}}{m}$

2) Let $\delta f_k = \vec{k}\cdot\vec{g}_k$, where $\vec{g}_k$ depends on $|\vec{k}|$

$$e\vec{E}\cdot\vec{v}\,\frac{\partial f_{0k}}{\partial \varepsilon_k} = -\sum_{k'} W_{kk'}\,(\vec{k}-\vec{k}')\cdot\vec{g}_k \qquad \vec{g}_k = \vec{g}_{k'}$$

Calculation of $\vec{I}_k = \sum_{k'} W_{kk'}\,\vec{k}'$

$$\vec{I}_k = \sum_{k'} W_{kk'}\,\vec{k}' = \vec{k}\,a_k$$

$$\vec{k}\cdot\vec{I}_k = k^2 a_k$$

$$a_k = \frac{\vec{k}\cdot}{k^2}\sum_{k'} W_{kk'}\,\vec{k}'$$

$$\vec{I}_k = \frac{\vec{k}}{k^2}\sum_{k'} W_{kk'}\,(\vec{k}\cdot\vec{k}') = \vec{k}\sum_{k'} W_{kk'}\cos\theta$$

Back to kin. equation:

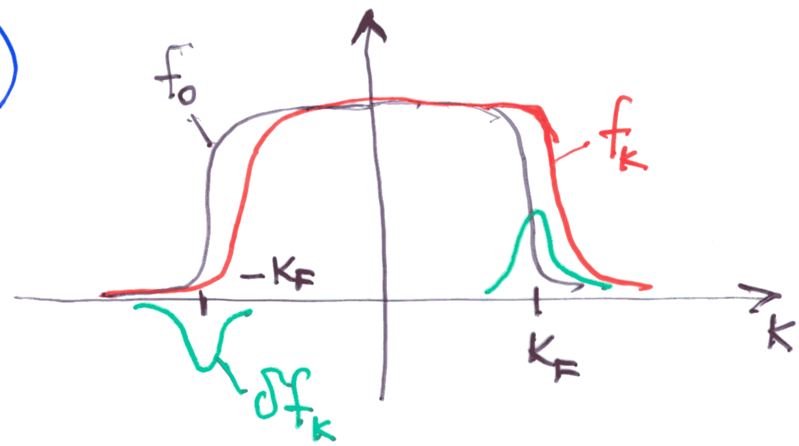
$$e\vec{E}\cdot\vec{v}\,\frac{\partial f_0}{\partial \varepsilon_k} = -\,\delta f_k \underbrace{\sum_{k'} w_{kk'}\,(1-\cos\theta)}_{\frac{1}{\tau_k}}$$

"transport" relaxation time

$$e\vec{E}\cdot\vec{v}\,\frac{\partial f_0}{\partial \varepsilon_k} = -\,\frac{\delta f_k}{\tau_k}$$

(Relaxation-time approximation)

$$\delta f_k = e\vec{E}\cdot\vec{v}\,\tau_k\left(-\frac{\partial f_0}{\partial \varepsilon_k}\right)$$



Current:

$$\vec{j} = 2e\sum_k \vec{v}\,\delta f_k$$

$$\vec{j} = 2e^2\sum_k \vec{v}\,(\vec{v}\cdot\vec{E})\,\tau_k\left(-\frac{\partial f_0}{\partial \varepsilon_k}\right)$$

In a homogeneous system $\vec{j} \parallel \vec{E}$

Let $\vec{E} \parallel z$

$$j = 2e^2\sum_k v_z^2\,\tau_k\left(-\frac{\partial f_0}{\partial \varepsilon_k}\right)E$$

$$= \frac{2e^2}{3}\sum_k v^2\,\tau_k\left(-\frac{\partial f_0}{\partial \varepsilon_k}\right)E$$

$$\boxed{\sigma = \frac{2e^2}{3}\sum_k v^2\,\tau_k\left(-\frac{\partial f_0}{\partial \varepsilon_k}\right)}$$

3D system:

$$2\sum_k \ldots = 2\int \frac{d^3k}{(2\pi\hbar)^3}\ldots = \int \nu(\varepsilon_k)\,d\varepsilon_k$$

Calculation of $\nu(\varepsilon_k)$:

$$2\int \frac{d^3k}{(2\pi\hbar)^3}\ldots = 2\cdot 4\pi\int_0^\infty \frac{k^2dk}{(2\pi\hbar)^3}\ldots =$$

$$= \frac{m}{\pi^2\hbar^3}\int_0^\infty k\,d\frac{k^2}{2m}\ldots = \int_0^\infty \underbrace{\frac{mk}{\pi^2\hbar^3}}_{\nu(\varepsilon_k)}\,d\varepsilon_k$$

$$v^2 = \frac{2\varepsilon_k}{m}$$

A) Suppose $\tau_k \to \tau$ (does not depend on k)

B) or $\mu \gg T$ (metal) $\tau_k \to \tau\,(k = k_F)$

$$\sigma = \frac{2e^2\tau}{3}\int_0^\infty \nu(\varepsilon_k)\,d\varepsilon_k\,\varepsilon_k\left(-\frac{\partial f_0}{\partial \varepsilon_k}\right)$$

Case B (metal):

$$\sigma = \frac{2e^2\tau\nu_F\,\varepsilon_F}{3m} \qquad \nu_F = \frac{mk_F}{\pi^2\hbar^3}$$

$$\varepsilon_F = \frac{k_F^2}{2m}$$

$$\boxed{\sigma = \frac{ne^2\tau}{m}} \qquad n = \frac{k_F^3}{3\pi^2\hbar^3}$$

Big surprise!

Scattering mechanisms:

- impurities
- magnetic impurities
- phonons
- e-e interactions
- magnons (in ferromagnets)

etc.

For impurity scattering:

$$W_{kk'} = \frac{2\pi}{\hbar} |V_{kk'}|^2 \delta(\varepsilon_k - \varepsilon_{k'})$$

What about spin?