Lecture 10.

Coulomb interaction and theories of strongly correlated systems (cont)

- Stoner mechanism of ferromagnetism in metals
- Effect of Coulomb blockade

Stoner mechanism of ferromagnetism

materials:
- metals (Fe, Co, Ni)
- magnetic semiconductors (GaMnAs)

What makes them magnetic?
- Coulomb interaction
- indirect exchange (RKKY)

Stoner instability

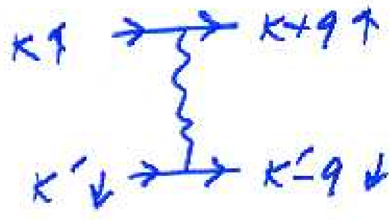
$$\mathcal{H} = \sum_{k} \Big[\varepsilon_k \left(a^+_{k\uparrow} a_{k\uparrow} + a^+_{k\downarrow} a_{k\downarrow} \right) +$$

$$+ g \sum_{qk'} \left(a^+_{k+q\uparrow} a_{k\uparrow} a^+_{k'-q\downarrow} a_{k'\downarrow} \right) +$$

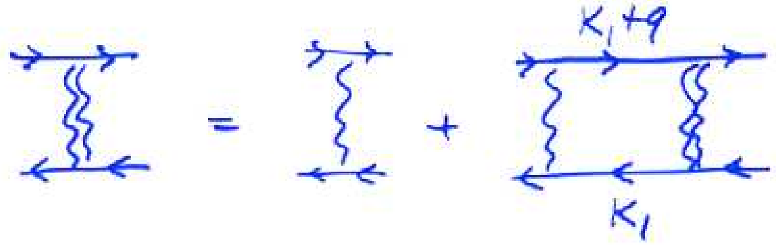
$$+ g_2 \sum_{qk'} \cdots$$

Interaction term (Hubbard)

$$g \sum_{k} \rho_{q\uparrow} \rho_{-q\downarrow}$$



Ladder diagrams:



$$\Gamma = g + g\Pi\Gamma$$

Loop:

$$\Rightarrow \quad \Gamma = \frac{g}{1-g\Pi}$$

$$\Pi(q) = i\int \frac{d\varepsilon}{2\pi}\frac{d^3k}{(2\pi)^3}\, G(\varepsilon, k+q)\, G(\varepsilon, k) =$$

$$= i\int \frac{d\varepsilon}{2\pi}\frac{d^3k}{(2\pi)^3}\, \frac{1}{\varepsilon - \varepsilon_{k+q} + \mu + i\delta\,\mathrm{sign}(\varepsilon_{k+q}-\mu)}$$

$$\times \frac{1}{\varepsilon - \varepsilon_k + \mu + i\delta\,\mathrm{sign}(\varepsilon_k - \mu)} =$$

$$= -\int \frac{d^3k}{(2\pi)^3}\left[\frac{f(\varepsilon_{k+q})}{\varepsilon_{k+q}-\varepsilon_k} + \frac{f(\varepsilon_k)}{\varepsilon_k - \varepsilon_{k+q}}\right] =$$

$$= -\int \frac{d^3k}{(2\pi)^3}\,\frac{f(\varepsilon_{k+q}) - f(\varepsilon_k)}{\varepsilon_{k+q}-\varepsilon_k}$$

$$\xrightarrow{q\to 0} \int \frac{d^3k}{(2\pi)^3}\left(-\frac{\partial f(\varepsilon_k)}{\partial \varepsilon_k}\right) = \nu_F$$

$$\Gamma = \frac{g}{1 - g\nu_F}$$

divergence
instability

Mean field theory

Spin-dependent interaction:

$$H = \int d^3r \left[\psi^+ \left(-\frac{\Delta}{2m} - \mu \right) \psi + \frac{g_s}{2} \left(\psi^+ \vec{\sigma} \psi \right)^2 \right]$$

$$g_s = g_{\parallel} - g_{\perp}$$

Physically: spin-density — spin-density
and density-density

Functional integration method

Partition function:

$$Z = \int \mathcal{D}\psi \, \mathcal{D}\psi^+ \, e^{iF} \quad , \quad F = i \int dt \, L$$

($T=0$ technique)

$$L = \int d^3r \left[\psi^+ \left(i \frac{\partial}{\partial t} + \frac{\Delta}{2m} + \mu \right) \psi - \right.$$

$$\left. - \frac{g_s}{2} \left(\psi^+ \vec{\sigma} \psi \right) \right]$$

Decoupling (Hubbard-Stratonovich):

$$\frac{g_s}{2} \left(\psi^+ \vec{\sigma} \psi \right)^2 \to \left(\psi^+ \sigma_i \psi \right) M_i - \frac{1}{2g_s} M^2$$

$$Z = \int \mathcal{D}\psi\, \mathcal{D}\psi^+ \mathcal{D}\vec{\mathcal{M}} \times$$

$$\times\, e^{i\int dt\,[\psi^+ (i\frac{\partial}{\partial t} - H_0 - \vec{\sigma}\cdot\vec{\mathcal{M}})\psi + \mathcal{M}^2/2g_s]}$$

How to check it: (denominator!)

$$\frac{\mathcal{M}^2}{2g} - \vec{p}\cdot\vec{\mathcal{M}} = \frac{1}{2g}(\vec{\mathcal{M}} - g\vec{p})^2 - \frac{g p^2}{2}$$

$$\vec{p} = \psi^+ \vec{\sigma}\, \psi$$

Saddle point:

$$\frac{\delta L}{\delta \vec{\mathcal{M}}} = 0$$

$$\Rightarrow \quad \vec{\mathcal{M}} = g_s (\psi^+ \vec{\sigma}\, \psi)$$

mean field:

$$\vec{\mathcal{M}} = g_s \langle \psi^+ \vec{\sigma}\, \psi \rangle =$$

$$= -i g_s \, Sp\, \vec{\sigma}\, G(r t; r, t+\delta) =$$

$$= -i g_s \, Sp \int \frac{d\varepsilon}{2\pi} \frac{d^3k}{(2\pi)^3}\, \vec{\sigma}\, G(\varepsilon, \vec{k})\, e^{i\varepsilon\delta}$$

$$G(\varepsilon, k) = \frac{1}{\varepsilon - \varepsilon_k + \vec{\sigma}\cdot\vec{\mathcal{M}} - \mu + i\delta\, \mathrm{sign}\, \varepsilon}$$

Quantization axis along z

$$M = g_s (n_\uparrow - n_\downarrow)$$

Second derivative:

$$\frac{\delta^2 L}{\delta M_\alpha \, \delta M_\beta} = -\chi^0_{\alpha\beta} + \frac{1}{g_s}\delta_{\alpha\beta}$$

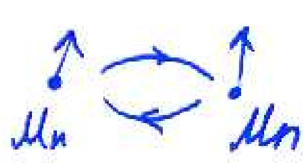
$$\chi^0_{\alpha\beta} = -i \, Sp \int \frac{d\varepsilon}{2\pi} \frac{d^3k}{(2\pi)^3} \sigma_\alpha G(\varepsilon \vec{k}) \sigma_\beta G(\varepsilon \vec{k})$$

susceptibility $(q=0, \omega=0)$

$1 - g_s \chi^0_{zz} = 0$ Stoner criterium

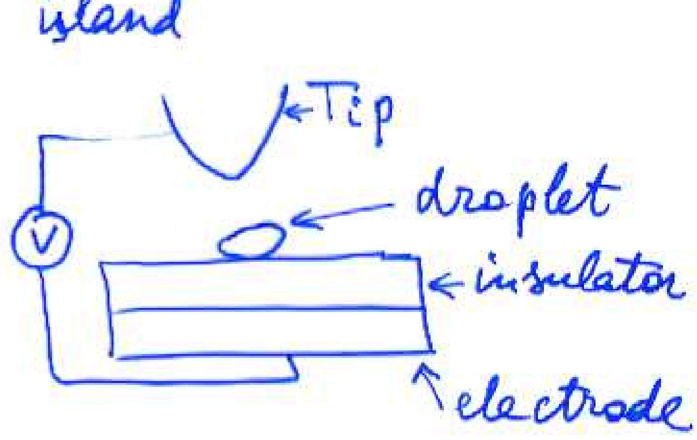
Ferromagnetic semiconductors:

- magnetic impurities
- indirect interaction
- no Coulomb interaction effects ?

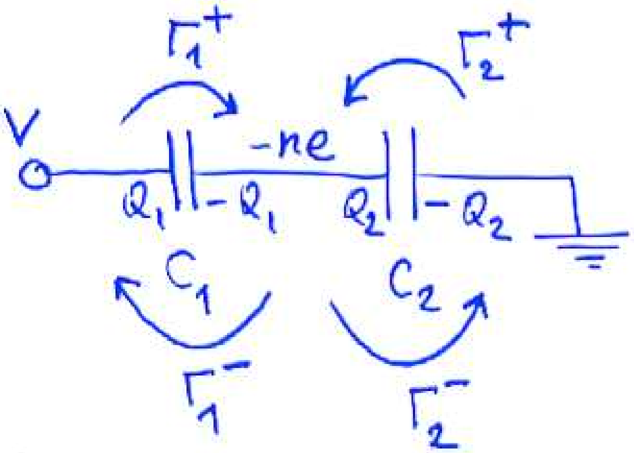


Coulomb blockade

Tunneling through an island



Equivalent circuit:



Tunneling rate:

$$\Gamma_i^{\pm}(n) = \frac{1}{e^2 R_i} \int d\varepsilon_k \, d\varepsilon_p \, f(\varepsilon_k)\left[1 - f(\varepsilon_p)\right] \times$$

$$\times \, \delta\left(\varepsilon_k - \varepsilon_p - E_i^{\pm}(n)\right)$$

where $E_i^{\pm}(n)$ is the electrostatic energy change for the transition of the island $n \to n \pm 1$ by tunneling of electron through i-th junction

Example:

$$E_1^+(n) = F(n+1, \xi+1) - F(n, \xi)$$

Electron tunnels through barrier 1
(taking one source electron)

Free energy: $F(n,\xi) = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} + (Q - e\xi)V$

We use: $n = \frac{Q_1 - Q_2}{e}$

$$\delta Q_1 - \delta Q_2 = e \quad \text{(charge conservation)}$$

$$\delta Q_1/C_1 + \delta Q_2/C_2 = 0 \quad \text{(Kirchhoff's law)}$$

$$\Rightarrow E_1^{\pm}(n) = (1 \pm 2n)\frac{e^2}{2C} \pm \frac{C_2}{C} eV$$

$$E_2^{\pm}(n) = (1 \pm 2n)\frac{e^2}{2C} \pm \frac{C_1}{C} eV$$

$C = C_1 + C_2$
total capacitance
of the island

For $T \neq 0$

$$\Gamma_i^{\pm}(n) = \frac{1}{e^2 R_i} \frac{E_i^{\pm}(n)}{e^{E_i^{\pm}(n)/T} - 1}$$

How many electrons on the island?

Probability to find n excess electrons: $p(n)$

Master equation:

$$\frac{dp(n)}{dt} = p(n-1)\,\Gamma(n-1\to n) + p(n+1)\,\Gamma(n+1\to n) - p(n)\left[\Gamma(n\to n+1) + \Gamma(n\to n-1)\right]$$

where
$$\Gamma(n\to n+1) = \Gamma_1^+(n) + \Gamma_2^+(n)$$
$$\Gamma(n\to n-1) = \Gamma_1^-(n) + \Gamma_2^-(n)$$

Stationary probability:

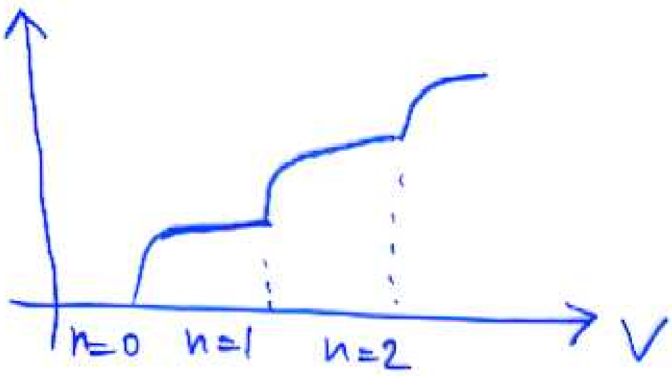
$$p(n)\left[\Gamma_1^+(n) + \Gamma_2^+(n)\right] = p(n+1)\left[\Gamma_1^-(n+1) + \Gamma_2^-(n+1)\right]$$

Normalization: $\sum\limits_{n=-\infty}^{\infty} p(n) = 1$

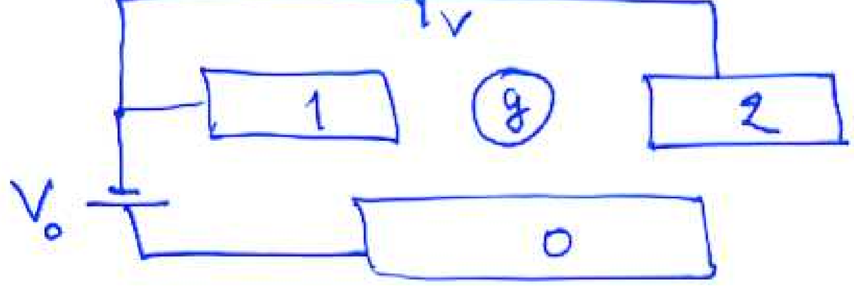
Current:

$$I = -e\sum_{n=-\infty}^{\infty} p(n)\left[\Gamma_1^+(n) - \Gamma_1^-(n)\right] =$$
$$= -e\sum_{n=-\infty}^{\infty} p(n)\left[\Gamma_2^+(n) - \Gamma_2^-(n)\right]$$

Coulomb staircase



Geometry with the gate:

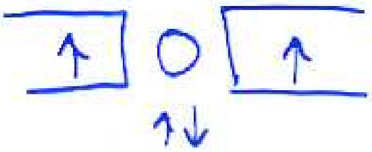


$V \to 0$

- Jumps at I-V characteristics
- Periodic oscillations

$\Rightarrow$ Single-electron transistor

Ferromagnetic junctions:



Ferromagnetic single-electron transistor

$\Rightarrow$ oscillations of TMR with voltage

Scanning-Tunneling-Microscope Observations of Coulomb Blockade and Oxide Polarization in Small Metal Droplets

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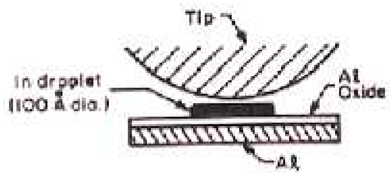

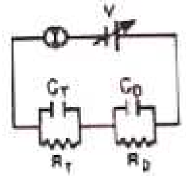

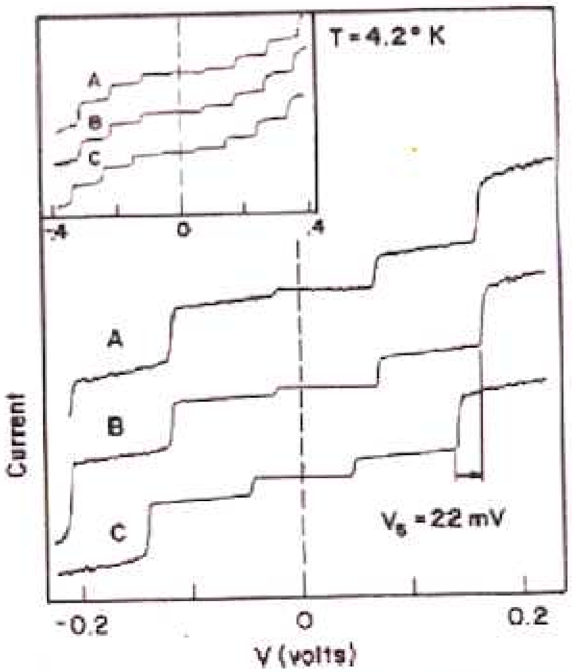

FIG. 2. Curve A is an experimental I-V characteristic from an In droplet in a sample with average droplet size of 300 Å.

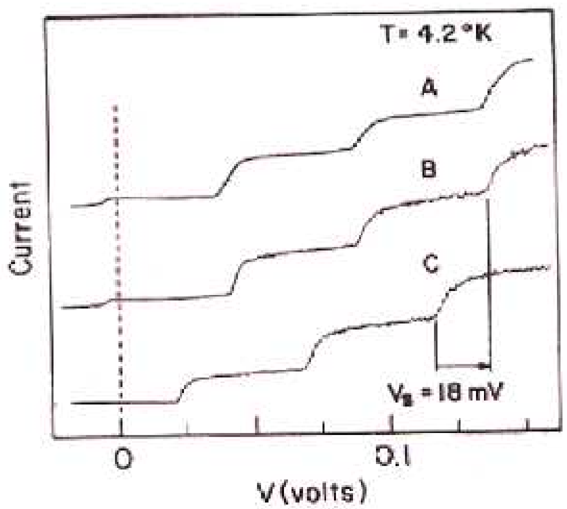

FIG. 3. Curve A is an experimental I-V characteristic from an In droplet in a sample with average size of 100 Å. The