Non-ordinary nature Regge Trajectory of the f0(500) or σ-meson

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Motivation

- Interest in identification of non-ordinary Quark Model states (non $qq\bar{q}$?)
- “Easy” if quantum numbers are not $qq\bar{q}$ -> Exotics!
- Not so easy for cryptoexotics like light scalars.

Particularly the f0(500) or $\sigma$-meson nature has been debated for over 50 years despite being very relevant for NN attraction, chiral symmetry breaking, Glueball search, lots of decays, etc…

- Hard to tell what a non-ordinary meson resonance is: tetraquark, molecule, glueball…
- Classification in terms of SU(3) multiplets complicated by mixing.

But “ordinary” $qq\bar{q}$ mesons also follow another classification
Particles with same quantum numbers and signature \((\tau=(-1)^J)\) can be classified in **linear trajectories** of \((\text{mass})^2\) vs. \((\text{spin})\) with a "universal" slope of \(\sim 1 \text{ GeV}^{-2}\)

*Introduction: Regge trajectories*

Warning…, resonances have a width, and this variable here is… “only \(M^2\)” …some authors use width as uncertainty
In this work we aimed at including the width properly.

Let us then dedicate this talk to Eef and, for this talk, play a little with the title and change it to...

"UNQUEENCHING " REGGE TRAJECTORIES

But, hey!! The name is only valid while in Coimbra.
Introduction: Regge trajectories

Particles on each trajectory are somehow related by similar dynamics.

Quark-antiquark states are well accommodated in these trajectories.

A relativistic spinning rod of constant mass/length also has a linear spectrum.

A color flux tube between a quark and an antiquark whose energy grows linearly with quark separation (confinement) could mimics this rod and is a crude model for Regge-linear trajectories (see Greensite’s textbook) -> string theory.

Thus “ordinary” mesons, usually identified as qqbar states, fit within

linear Regge trajectories.

But if other resonances have different nature…

…. they do not have to fit well in this scheme.
Actually, this happens with the f0(500) or sigma meson are doubled due to two flavor components, \( nn \) and \( ss \). We do not put the enigmatic \( \sigma \) meson [11–14] on the \( qq \) trajectory supposing \( \sigma \) is alien to this classification. The broad state

Other authors considered that since the f0(500) had such a big uncertainty in the PDG it could be ignored.
Introduction: Regge trajectories
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Actually, this happens with the f0(500) or sigma meson are doubled due to two flavor components, $nn$ and $ss$. We do not put the enigmatic $\sigma$ meson [11–14] on the $qq$ trajectory supposing $\sigma$ is alien to this classification. The broad state

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Other authors considered that since the f0(500) had such a big uncertainty in the PDG it could be ignored.

That is no longer an excuse.

One could still think of using the large width as an uncertainty… But here will study its Regge the trajectory as a complex pole, thus taking into account its width.
The Regge trajectories can be understood from the analytic extension to the complex angular momentum plane of the partial wave expansion through the Sommerfeld-Watson transform:

\[ T(s, t) = \sum_{J=0}^{\infty} (2J + 1) f_J(s) P_J(z) \rightarrow T(s, t) = -\frac{1}{2i} \int_C \frac{(2J + 1) f(J, s) P_J(-z)}{\sin \pi J} dJ \]
The contribution of a single Regge pole to a partial wave, is shown to be

\[ f(J, s) = \hat{f} + \frac{\beta(s)}{J - \alpha(s)} \]

"background" regular function.

Assumption: WE WILL NEGLECT IT in our cases
Introduction: Regge Theory

Regge poles
Position $\alpha(s)$
Residue $\beta(s)$

For different $s$ poles move in the complex $J$ plane along Regge Trajectories.

Linear trajectories of $qq\bar{q}$ mesons are just an example of some specific dynamics.
But other dynamics could lead to different trajectories. However, trajectories and residues cannot be completely arbitrary due to their analytic properties (Collins, Introduction to Regge Theory)

- Twice-subtracted dispersion relations

\[
\alpha(s) = A + B(s - s_0) + \frac{(s - s_0)^2}{\pi} \int_{\text{thr.}}^{\infty} \frac{\text{Im} \alpha(s') ds'}{(s' - s)(s' - s_0)^2}
\]

\[
\gamma(s) = g^2 \exp \left\{ C(s - s_0) + \frac{(s - s_0)^2}{\pi} \int_{\text{thr.}}^{\infty} \frac{\phi(s') ds'}{(s' - s)(s' - s_0)} \right\}
\]

with

\[
\beta(s) = \frac{\hat{s} \alpha(s)}{\Gamma(\alpha(s) + \frac{3}{2})}, \quad \gamma(s) = \frac{\text{Im} \alpha(s)}{\rho(s)}
\]
Moreover, for $\pi\pi$ scattering:

- Unitarity condition on the real axis implies
  \[ \text{Im} \alpha(s) = \rho(s)\beta(s) \]
  \[ \rho(s) = \sqrt{1 - 4m_{\pi}^2 / s} \]

- Further properties of $\beta(s)$
  \[ \beta(s) = \frac{s\alpha(s)}{\Gamma(\alpha(s) + \frac{3}{2})} \gamma(s) \]
  \[ \hat{s} = \frac{s - 4m^2}{\tilde{s}} \]

- Suppress poles of full amplitude
  \[ (2\alpha + 1)P_\alpha(z_s) \sim \Gamma(\alpha + \frac{3}{2}) \]

- Threshold behavior
  \[ \beta(s) \text{ real on real axis} \]
  \[ \Rightarrow \text{phase of } \Upsilon(s) \text{ known} \]
  \[ \Rightarrow \text{Omnès-type disp. relation} \]
Thus, the trajectory and residue should satisfy a system of integral equations:

\[
\Re \alpha(s) = \alpha_0 + \alpha's + \frac{s}{\pi} \text{PV} \int_{4m^2}^{\infty} ds' \frac{\Im \alpha(s')}{s'(s' - s)},
\]

\[
\Im \alpha(s) = \rho(s)b_0 \frac{\hat{s}^\alpha_0 + \alpha's}{|\Gamma(\alpha(s) + \frac{3}{2})|} \exp \left( -\alpha's \left[1 - \log(\alpha'\hat{s}) \right] \right)
\]

\[
+ \frac{s}{\pi} \text{PV} \int_{4m^2}^{\infty} ds' \frac{\Im \alpha(s') \log \frac{\hat{s}}{\hat{s}'} + \text{arg} \Gamma(\alpha(s') + \frac{3}{2})}{s'(s' - s)}
\]

THESE ARE THE EQUATIONS WE HAVE TO SOLVE

In order to have a consistent trajectory

In the scalar case a slight modification is introduced (Adler zero)
Our Approach

Fix the subtraction constants JUST from the scattering pole

- for a given set of $a_0$, $a'$ and $b_0$:
  - solve the coupled equations
  - get $\alpha(s)$ and $\beta(s)$ in real axis
  - extend to complex $s$-plane
  - obtain pole position and residue

\[
 f^{II}(J, s) = \hat{f} + \frac{\beta(s)}{J - \alpha^{II}(s)}
\]

- fit $a_0$, $a'$ and $b_0$ so that the pole position and residue coincide with those given by a dispersive analysis of scattering data

INPUT: Analytic continuation to the complex plane of a dispersive analysis of data.

**INPUT for our purposes:** The $\rho$ pole:

$$\rho_{pole} \approx 763^{+1.7}_{-1.5} - i73.2^{+1.0}_{-1.1} \text{ MeV} \quad \left| g \right| = 6.01^{+0.04}_{-0.07}$$
Results: \textit{\( \rho \) case (}\( l = 1, J = 1 \)\textit{)}

We (black) recover a fair representation of the partial wave, in agreement with the GKPY amplitude (red).

Neglecting the “background” vs. Regge pole gives a 10-15\% error.

Particularly in the resonance region

Fair enough to look for the Regge trajectory
Results: $\rho$ case ($I = 1, J = 1$)

We get a prediction for the $\rho$ Regge trajectory, which is almost real

almost LINEAR $\alpha(s) \sim \alpha_0 + \alpha'$

intercept $\alpha_0 = 0.52$
slope $\alpha' = 0.913 \text{GeV}^{-2}$

Previous studies:

[1] $\alpha_0 = 0.5$
[2] $\alpha_0 = 0.52 \pm 0.02$
[3] $\alpha_0 = 0.450 \pm 0.005$
[4] $\alpha' = 0.87 \pm 0.06 \text{GeV}^{-2}$

Remarkably consistent with the literature!!,
Results: $f_2(1275)$ and $f_2'(1525)$ cases ($l = 0, J = 2$)

These resonances are almost elastic:

- $f_2(1275)$ has BR ($\pi \pi$) = 85% and $f_2'(1525)$ has BR(KK)=90%.

Approximating them with elastic BW-like poles, and solving the integral equations we “predict” again:

- almost real and LINEAR $\alpha(s) \sim \alpha_0 + \alpha' s$

For the $f_2(1275)$

- intercept $\alpha_0 = 0.71$
- slope $\alpha' = 0.83$ GeV$^{-2}$

For the $f_2'(1525)$

- intercept $\alpha_0 = 0.59$
- slope $\alpha' = 0.61$ GeV$^{-2}$

Fair agreement with the literature!!, (taking into account our approximations) Remember this is NOT a fit!!
This “prediction” for the rho trajectory
Was known since the 70’s, we have just updated it
and obtained new “predictions” for the $f_2$ and $f_2’$

So, once we have checked that our approach
Predicts the established Regge trajectories just from the pole
position and residue…

What about the $f_0(500)$?
INPUT: Analytic continuation to the complex plane of a dispersive analysis of data

- **INPUT for our purposes: The σ pole:**

\[(457^{+14}_{-15}) - i(279^{+11}_{-7})\text{MeV}\]

\[|g| = 3.59^{+0.11}_{-0.13}\text{GeV}\]
Somewhat better agreement in the resonance region of the Regge pole dominated amplitude with the dispersive amplitude.

So, we apply a similar procedure but now for the f0(500)
The prediction for the $\sigma$ Regge trajectory, is:

- NOT approximately real
- NOT linear

\[
\begin{align*}
\alpha_\sigma (0) & = -0.090^{+0.004}_{-0.012} \\
\alpha_\sigma' & \simeq 0.002^{+0.050}_{-0.001} \text{ GeV}^{-2}
\end{align*}
\]

Compare with the rho result…

$\alpha_0 = 0.52 \quad \alpha' = 0.913 \text{ GeV}^{-2}$

The sigma does **NOT** fit the ordinary meson trajectory

**Two orders of magnitude** flatter than other hadrons

Typical of meson physics? $F_\pi$, $m_\pi$?
Results: $\sigma$ vs. $\rho$ trajectories

Using the same scale....

No evident Regge partners for the f0(500)
IF WE INSISTED in fixing the $\alpha'$ to an “ordinary” value $\sim 1$ GeV$^{-2}$

**Results:** $\sigma$ case ($l = 0, J = 0$)

The data description would be severely spoilt
If not-ordinary…

What then?
Can we identify the dynamics of the trajectory?

Not quite yet… but…
Ploting the trajectories in the complex J plane…

Striking similarity with Yukawa potentials at low energy: \( V(r) = -Ga \exp(-r/a)/r \)

Our result is mimicked with \( a = 0.5 \text{ GeV}^{-1} \) to compare with S-wave \( \pi\pi \) scattering length 1.6 \text{ GeV}^{-1}

“\( a \)” rather small !!!

The extrapolation of our trajectory also follows a Yukawa but deviates at very high energy.

Non-ordinary \( \sigma \) trajectory

Ordinary \( \rho \) trajectory
Analytic constraints on Regge trajectories as integral equations.

Consistent treatment of the width

Fitting JUST the pole position and residue of an isolated resonance, yields its Regge trajectory parameters

$\rho$, $f_2$, and $f_2'$ trajectories: **COME OUT LINEAR**, with universal parameters

$\sigma$ trajectory: **NON-LINEAR**.

Trajectory slope **two orders of magnitude smaller**

No partners.

If we force the $\sigma$ trajectory to have universal slope, data description ruined

At low energies, striking similarities with trajectories of Yukawa potential
Outlook

- Different mass case (kappa, K*…)
- Meson-nucleon (Delta, etc…)
- Similar non-linear behavior for other non-ordinary states?
- Coupled channels
- Microscopic models, relation to compositeness, etc…
Happy Birthday Eef and congratulations for these first 70 years of very fruitful career

Thank you for your advice and personal example of dedication and enjoyment when doing Physics