Departamento de Física Teórica II. Universidad Complutense de Madrid

Non-ordinary nature Regge Trajectory of the f0(500) or σ-meson

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In collaboration with:

J. Nebreda, A. Szczepaniak and T. Londergan, Phys. Lett. B 729 (2014) 9–14

and J.A. Carrasco in preparation.

EEF70:
Workshop on Unquenched hadron spectroscopy: Non-perturbative Models and Methods of QCD versus experiment
Coimbra 1-5 September 2014

Motivation

- Interest in identification of non-ordinary Quark Model states (non qqbar?)
- “Easy” if quantum numbers are not qqbar -> Exotics!
- Not so easy for cryptoexotics like light scalars.

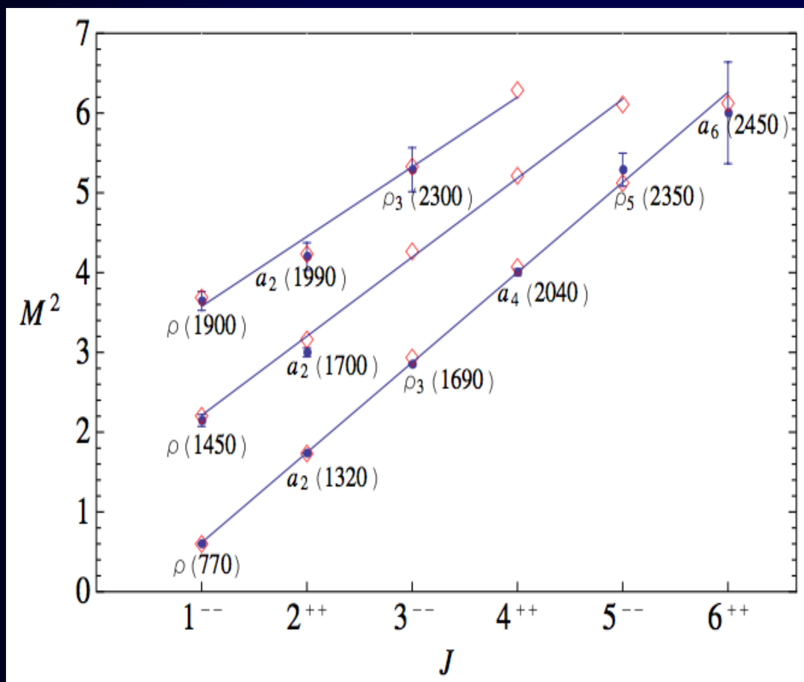
Particularly the f0(500) or σ-meson nature has been debated for over 50 years despite being very relevant for NN attraction, chiral symmetry breaking, Glueball search, lots of decays, etc…

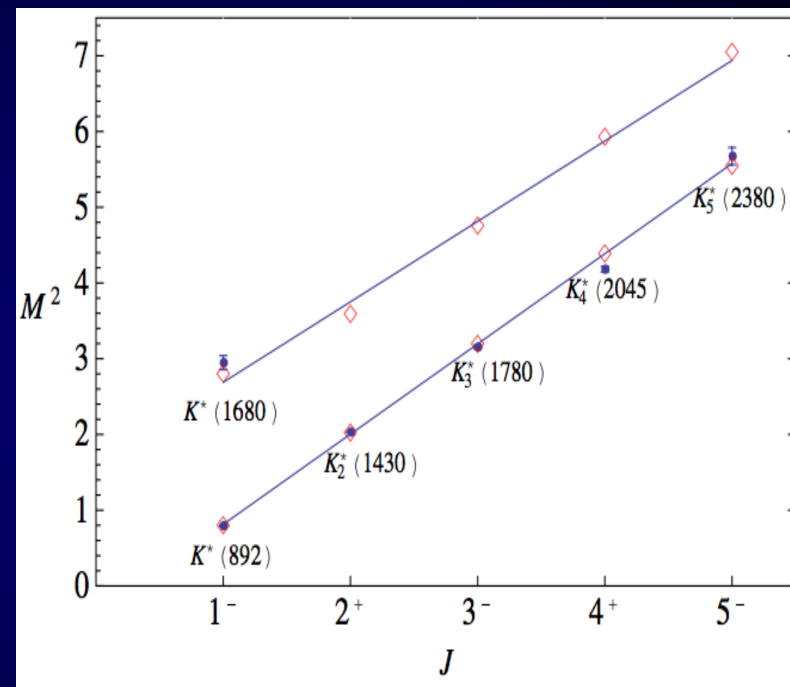
- Hard to tell what a non-ordinary meson resonance is: tetraquark, molecule, glueball…
- Classification in terms of SU(3) multiplets complicated by **mixing.**

But “ordinary” qqbar mesons also follow another classification

Introduction: Regge trajectories

Particles with same quantum numbers and signature ($\tau=(-1)^J$) can be classified in **linear trajectories of** (mass)^2 vs. (spin) with a "**universal" slope of ~1 GeV^{-2}**





Anisovich-Anisovich-Sarantsev-Phys.Rev.D62.051502-4

Warning…, resonances have a width, and this variable here is… " only M^2 "
…some authors use width as uncertainty

In this work we aimed at including the width properly

Let us then dedicate this talk to Eef and, for this talk, play a little with the title and change it to…

“UNQUENCHING “
REGGE TRAJECTORIES

But, hey!! The name is only valid while in Coimbra

Introduction: Regge trajectories

Particles on each trajectory are somehow related by similar dynamics

Quark-antiquark states are well accommodated in these trajectories

A relativistic spinning rod of constant mass/length also has a linear spectrum

A color flux tube between a quark and an antiquark whose energy grows linearly with quark separation (confinement) could mimics this rod and is a crude model for Regge-linear trajectories (see Greensite's textbook) -> string theory

Thus "ordinary" mesons, usually identified as qqbar states**,**

fit within

linear Regge trajectories

But if other resonances have different nature…

…. they do not have to fit well in this scheme

Introduction: Regge trajectories

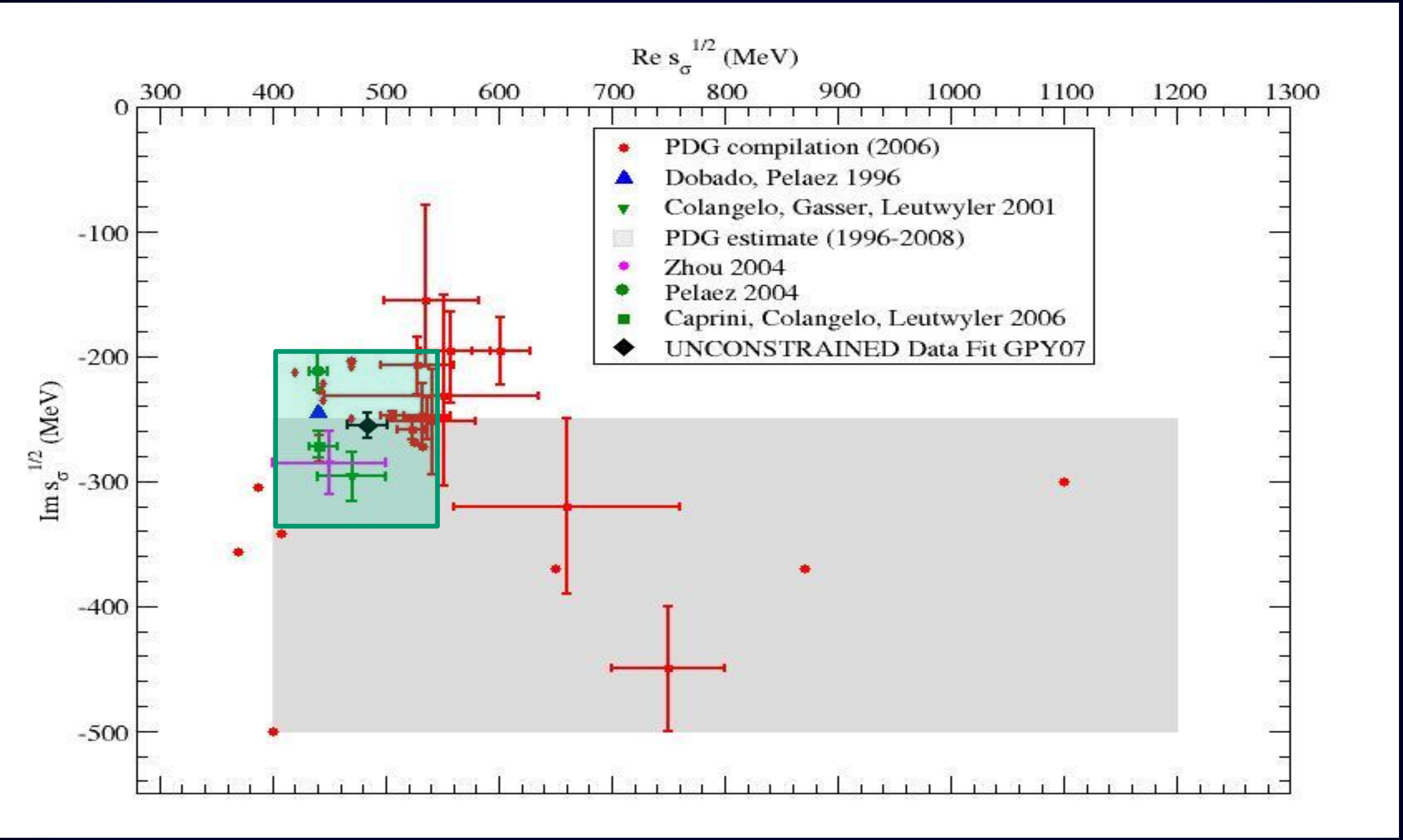
Actually, this happens with the f0(500) or sigma meson

> are doubled due to two flavor components, $n\bar{n}$ and $s\bar{s}$. We do not put the enigmatic σ meson [11–14] on the $q\bar{q}$ trajectory supposing σ is alien to this classification. The broad state

Anisovich-Anisovich-Sarantsev-Phys.Rev.D62.051502-4

Other authors considered that since the f0(500) **had such a big uncertainty in te PDG it could be ignored.**

Introduction: Regge trajectories



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Other authors considered that since the f0(500) **had such a big uncertainty in te PDG it could be ignored.**

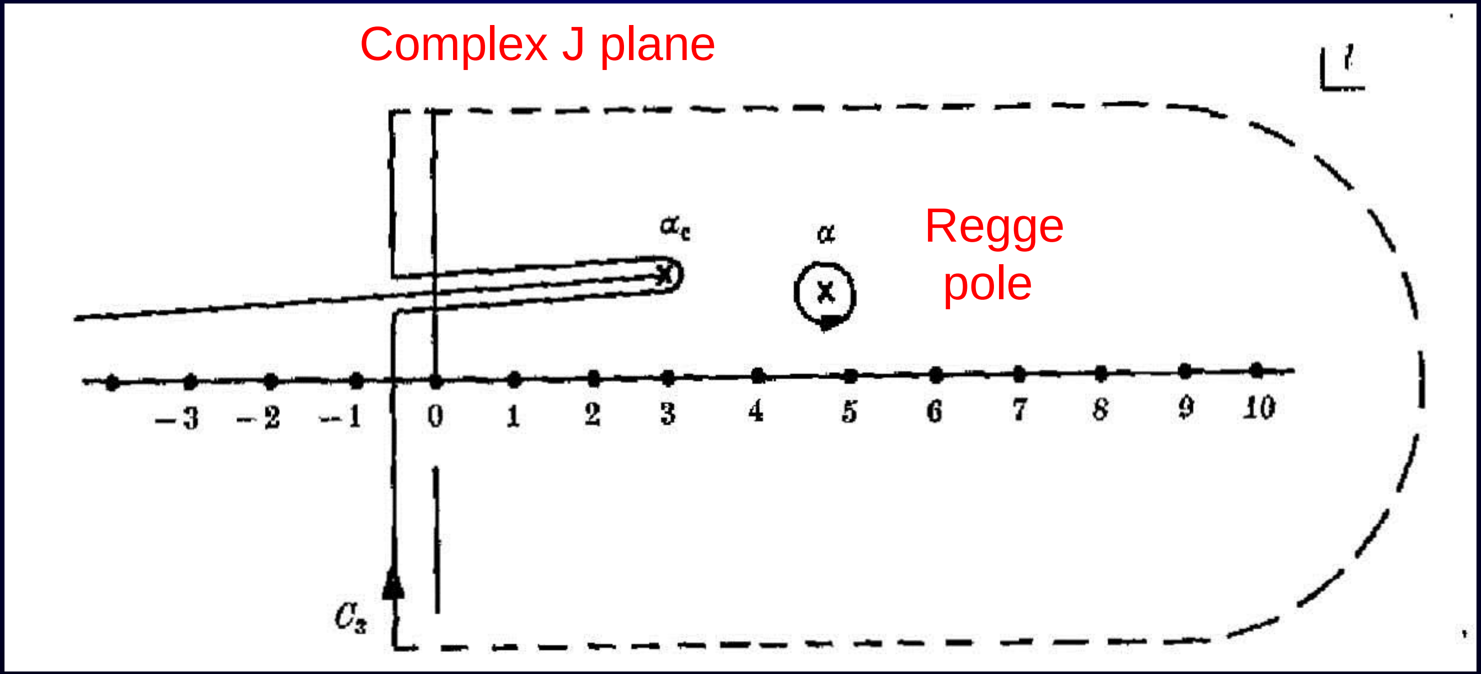
That is no longer an excuse.

One could still think of using the large width as an uncertainty…
But here will study its Regge the trajectory as a complex pole,
thus taking into account its width.

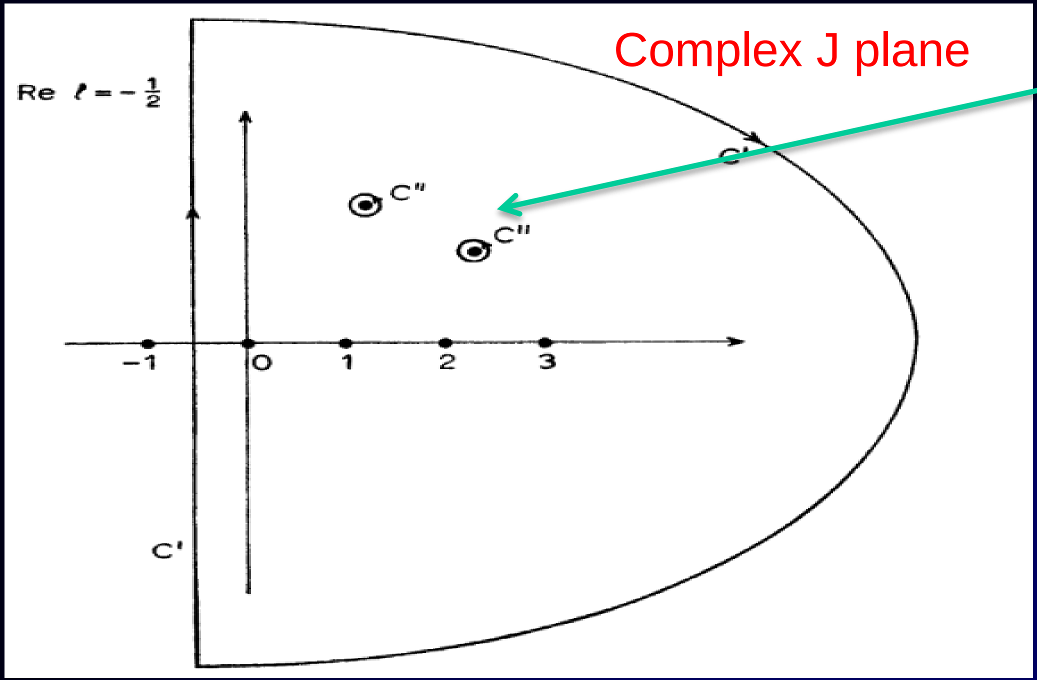
Introduction: Regge Theory

The Regge trajectories can be understood from the analytic extension to the complex angular momentum plane of the partial wave expansion through the Sommerfeld-Watson transform:

$$T(s,t) = \sum_{J=0}^{\infty} (2\,J+1) f_J(s)\, P_J(z) \quad \longrightarrow \quad T(s,t) = -\frac{1}{2\,i}\int_C \frac{(2\,J+1) f(J,s)\, P_J(-z)}{\sin \pi\, J} \mathrm{d}J$$



Introduction: Regge Theory



Regge poles

Position *α(s)*

Residue β*(s)*

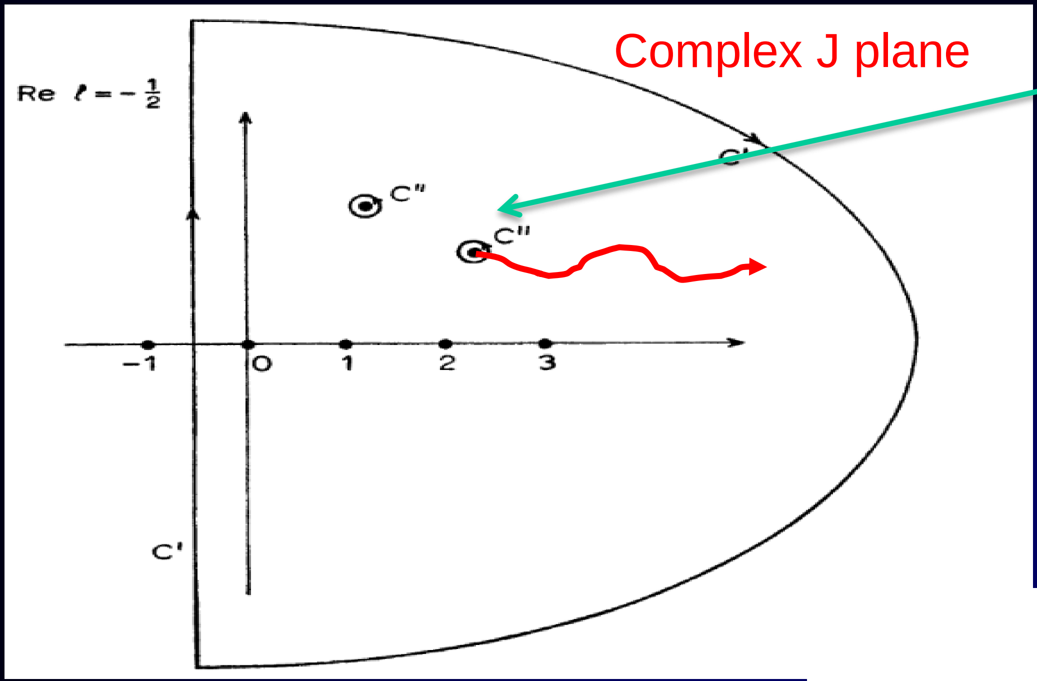
The contribution of a single Regge pole to a partial wave, is shown to be

$$f(J,s) = \hat{f} + \frac{\beta(s)}{J-\alpha(s)}$$

“background” regular function.

Assumption: WE WILL NEGLECT IT in our cases

Introduction: Regge Theory



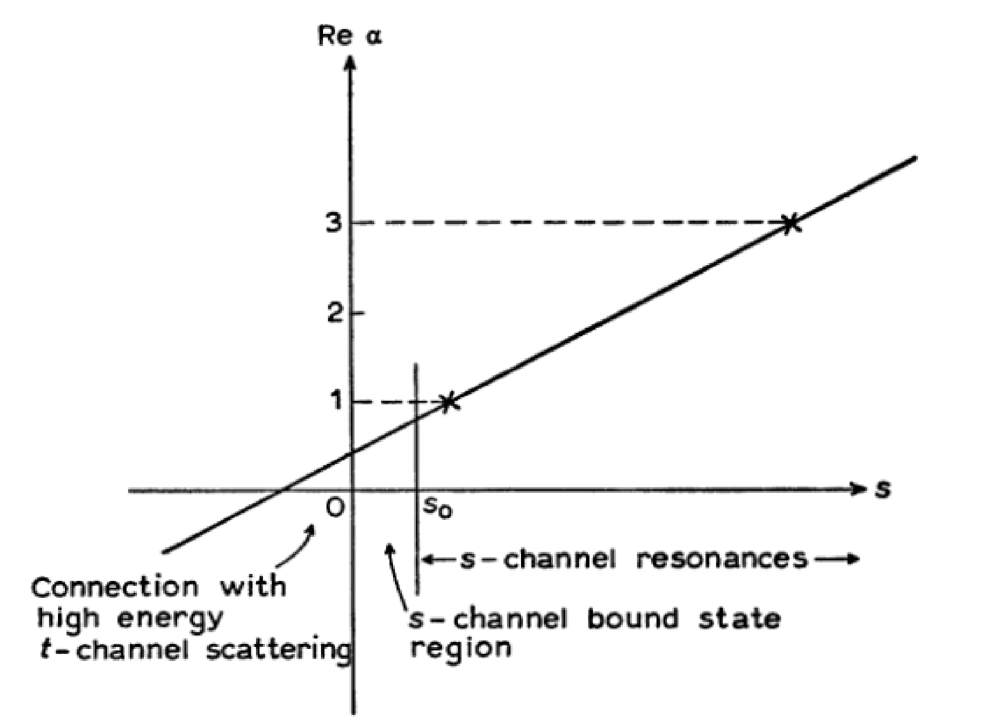
Regge poles

Position $\alpha(s)$

Residue $\beta(s)$

For different s poles move in the complex J plane along **Regge Trajectories**

Linear trajectories of qqbar mesons are just an example of some specific dynamics



Intriduction: Regge Theory

But other dynamics could lead to different trajectories.
<u>However, trajectories and residues cannot be completely arbitrary due to their analytic properties</u> (Collins, Introduction to Regge Theory)

- **Twice-subtracted dispersion relations**

$$\alpha(s) = A + B(s - s_0) + \frac{(s - s_0)^2}{\pi} \int_{\mathrm{thr.}}^{\infty} \frac{\mathrm{Im}\alpha(s')ds'}{(s' - s)(s' - s_0)^2}$$

$$\gamma(s) = g^2 \exp\left\{ C(s - s_0) + \frac{(s - s_0)^2}{\pi} \int_{\mathrm{thr.}}^{\infty} \frac{\phi_\gamma(s')}{(s' - s)(s' - s_0)} ds' \right\}$$

with $$\beta(s) = \frac{\hat{s}^{\alpha(s)}}{\Gamma(\alpha(s) + \frac{3}{2})}\gamma(s) = \frac{\mathrm{Im}\alpha(s)}{\rho(s)}$$

Parametrization of pole dominated amplitudes

Chu, Epstein, Kaus, Slansky, Zachariasen, PR175, 2098 (1968).

Moreover, for ππ scattering:

- Unitarity condition on the real axis implies

$$\mathrm{Im}\ \alpha(s) = \rho(s)\beta(s)$$

$$\rho(s) = \sqrt{1 - 4m_\pi^2/s}$$

- Further properties of β*(s)*

threshold behavior

$$\beta(s) = \frac{\hat{s}^{\alpha(s)}}{\Gamma(\alpha(s) + \frac{3}{2})}\gamma(s)$$

$$\hat{s} = \frac{s - 4m^2}{\tilde{s}}$$

suspress poles
of full amplitude

$$(2\alpha + 1)P_\alpha(z_s) \sim \Gamma(\alpha + \frac{3}{2})$$

analytic function:
β*(s)* real on real axis
⇒ phase of *Y(s)* known
⇒ Omnès-type disp. relation

Thus, the trajectory and residue should satisfy a system of integral equations:

$$
\begin{aligned}
\mathrm{Re}\,\alpha(s) &= \alpha_0 + \alpha' s + \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,\alpha(s')}{s'(s'-s)}, \\
\mathrm{Im}\,\alpha(s) &= \rho(s) b_0 \frac{\hat{s}^{\alpha_0+\alpha' s}}{|\Gamma(\alpha(s)+\frac{3}{2})|} \exp\Big(-\alpha' s[1-\log(\alpha'\tilde{s})] \\
&+ \frac{s}{\pi} PV \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,\alpha(s')\log\frac{\hat{s}}{\hat{s}'} + \arg\Gamma\left(\alpha(s')+\frac{3}{2}\right)}{s'(s'-s)}\Big)
\end{aligned}
$$

THESE ARE THE EQUATIONS WE HAVE TO SOLVE

In order to have a consistent trajectory

In the scalar case a slight modification is introduced (Adler zero)

Our Approach

Fix the subtraction constants JUST from the scattering pole

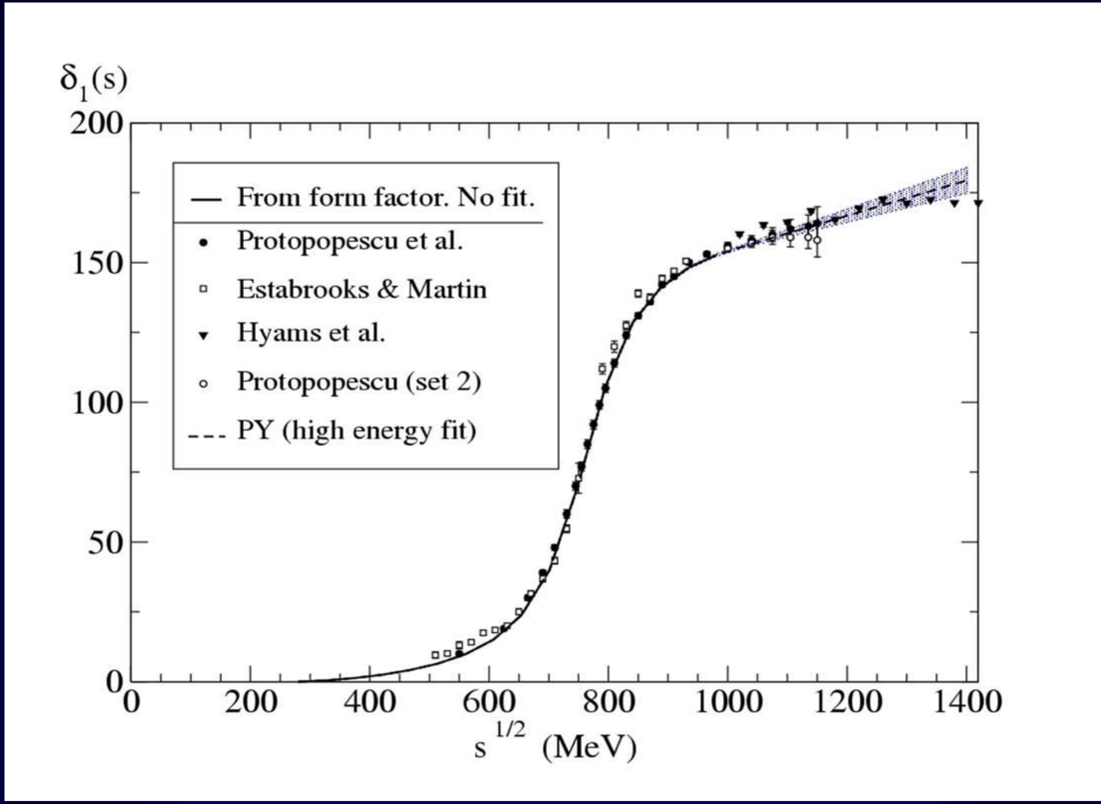
- for a given set of α_0, α' and b_0:
  - solve the coupled equations
  - get α(s) and β(s) in real axis
  - extend to complex *s*-plane
  - obtain pole position and residue

$$f^{II}(J,s) = \hat{f} + \frac{\beta(s)}{J - \alpha^{II}(s)}$$

- fit α_0, α' and b_0 so that the **pole position and residue** coincide with those given by a **dispersive analysis of scattering data**

Garcia-Martin, Kaminski, Pelaez and Ruiz de Elvira, Phys. Rev. Lett. **107**, 072001 (2011)

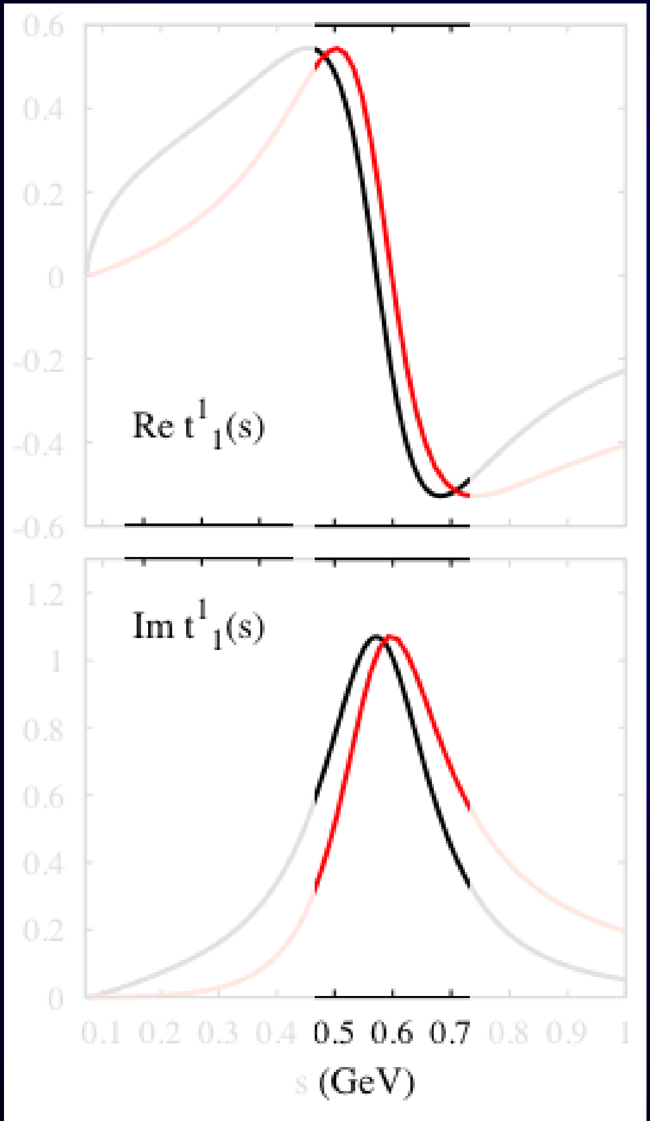
INPUT:Analytic continuation to the complex plane of a dispersive analysis of data



- INPUT for our purposes: **The ρ pole:**

$$\rho_{pole} \approx 763^{+1.7}_{-1.5} - i73.2^{+1.0}_{-1.1}\,\mathrm{MeV} \qquad \left|g\right| = 6.01^{+0.04}_{-0.07}$$

Results: ***ρ case (I = 1, J = 1)***



We (black) recover a fair representation of the partial wave, in agreement with the GKPY amplitude (red)

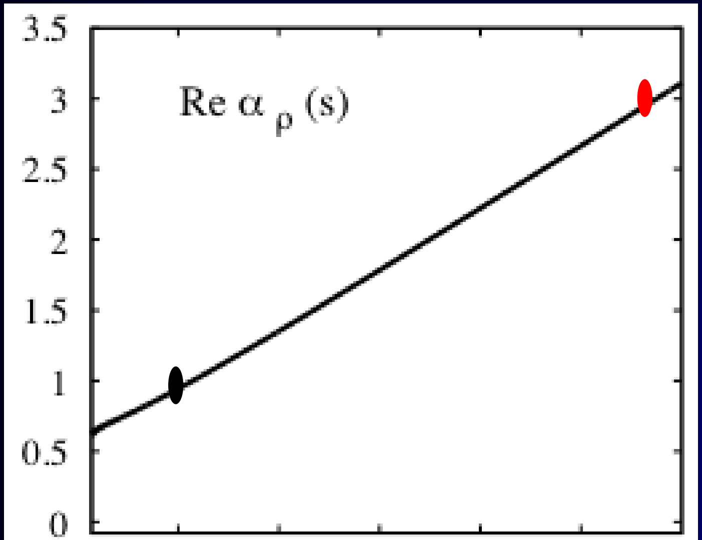
Neglecting the “background” vs. Regge pole gives a 10-15% error.

Particularly in the resonance region

Fair enough to look for the Regge trajectory

Results: ***ρ case (I = 1, J = 1)***

We get a prediction for the ρ Regge trajectory, which is almost real



almost LINEAR $\alpha(s) \sim \alpha_0 + \alpha' s$

intercept $\alpha_0 = 0.52$

slope $\alpha' = 0.913$ GeV^{-2}

Previous studies:

[1] $\alpha_0 = 0.5$	[1] $\alpha' = 0.83$ GeV^{-2}
[2] $\alpha_0 = 0.52 \pm 0.02$	[2] $\alpha' = 0.9$ GeV^{-2}
[3] $\alpha_0 = 0.450 \pm 0.005$	[4] $\alpha' = 0.87 \pm 0.06$ GeV^{-2}

This is a "prediction" for the whole tower of ρ(770) Regge partners:
ρ(1690)
ρ(2350)
....
the LINEAR behavior is a RESULT

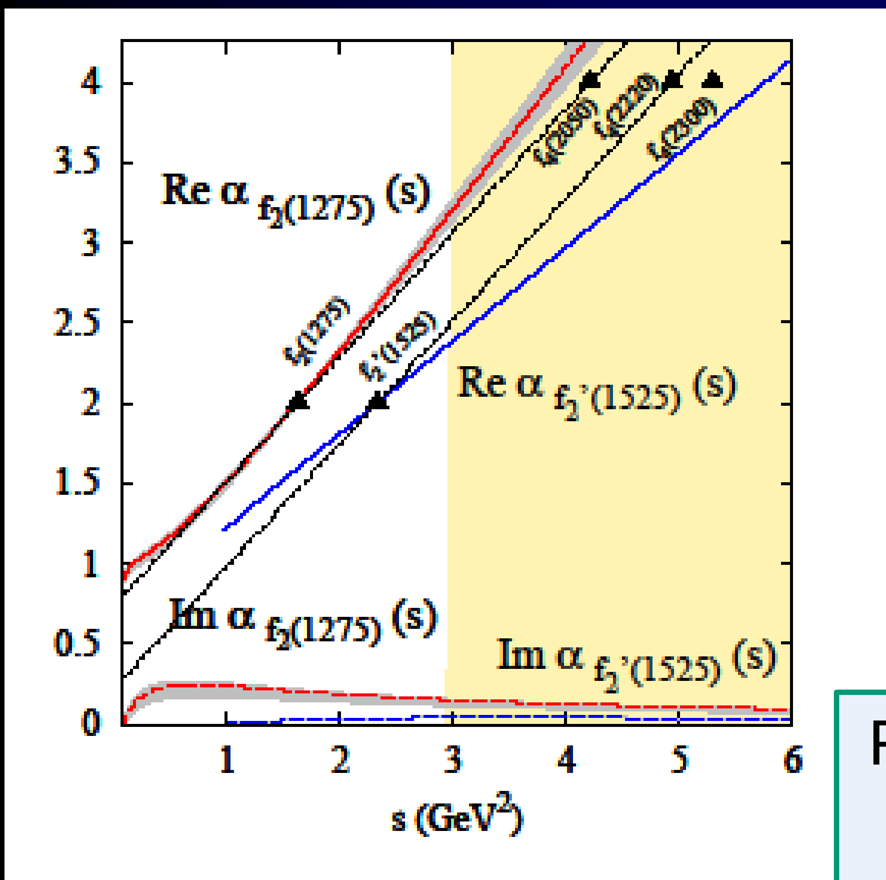
Remarkably consistent with the literature!!,
(taking into account our approximations)

Results: *f_2(1275) and f_2'(1525) cases (I = 0, J = 2)*

These resonances are almost elastic:

f_2(1275) has BR (π π) = 85% and $f_{2'}$(1525) has BR(KK)=90%.

Approximating them with elastic BW-like poles, and solving the integral equations we "predict" again:



almost real and LINEAR $\alpha(s) \sim \alpha_0 + \alpha' s$

For the f_2(1275)

intercept α_0= 0.71	slope α' = 0.83 GeV^{-2}

For the f_2(1525)

intercept α_0= 0.59	slope α' = 0.61 GeV^{-2}

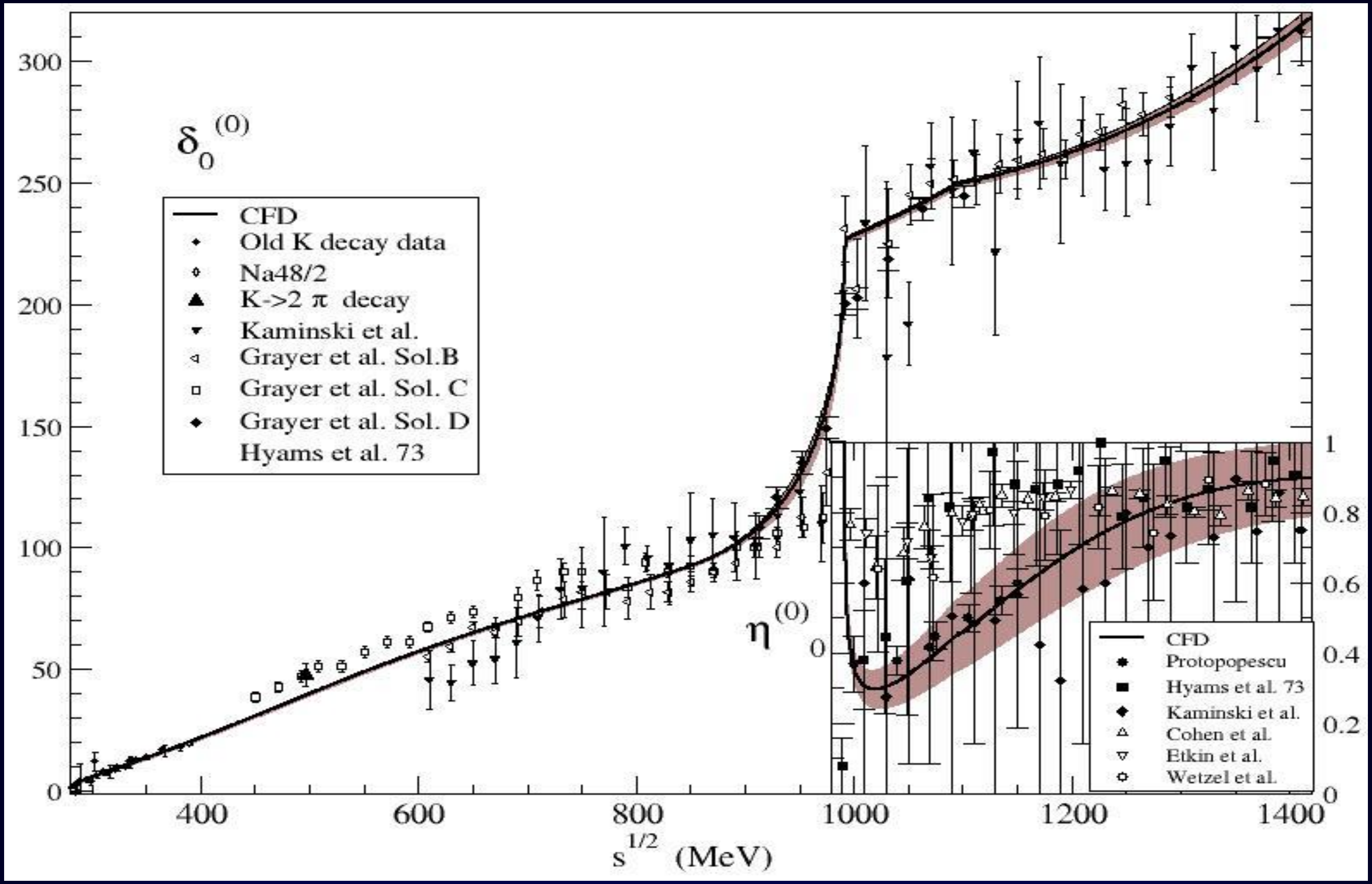
Fair agreement with the literature!!,
(taking into account our approximations)
Remember this is NOT a fit!!

This “prediction” for the rho trajectory
Was known since the 70’s, we have just updated it
and obtained new “predictions” for the f_2 and f_2’

So, once we have checked that our approach
Predicts the established Regge trajectories just from the pole position and residue…

What about the $f_0(500)$?

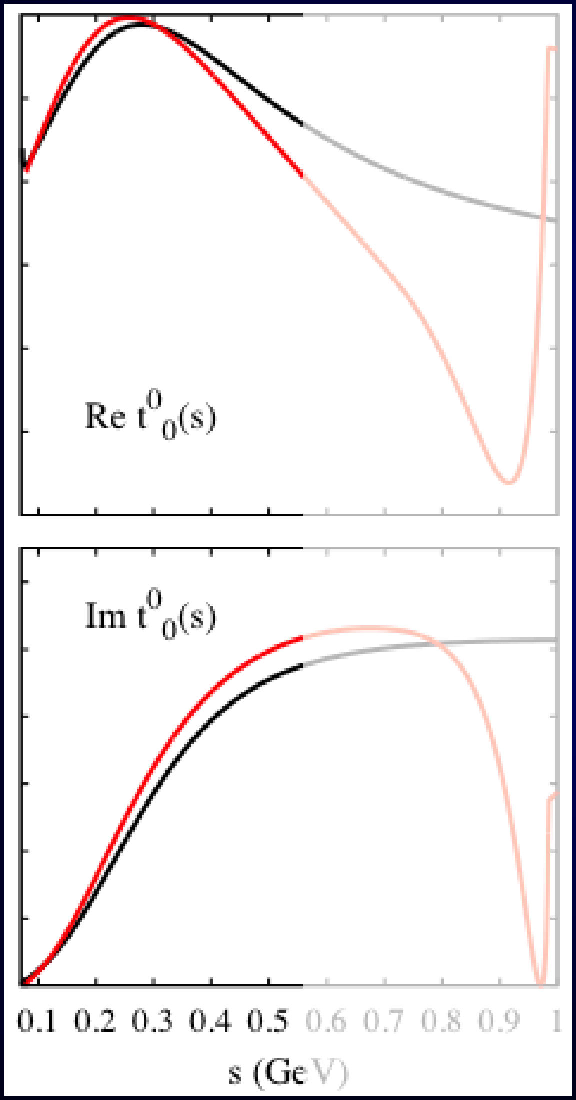
INPUT:Analytic continuation to the complex plane of a dispersive analysis of data



- INPUT for our purposes: **The σ pole:**

$$(457^{+14}_{-15}) - i(279^{+11}_{-7})\text{MeV} \qquad |g| = 3.59^{+0.11}_{-0.13}\ \text{GeV}$$

Results: ***σ case (I = 0, J = 0)***

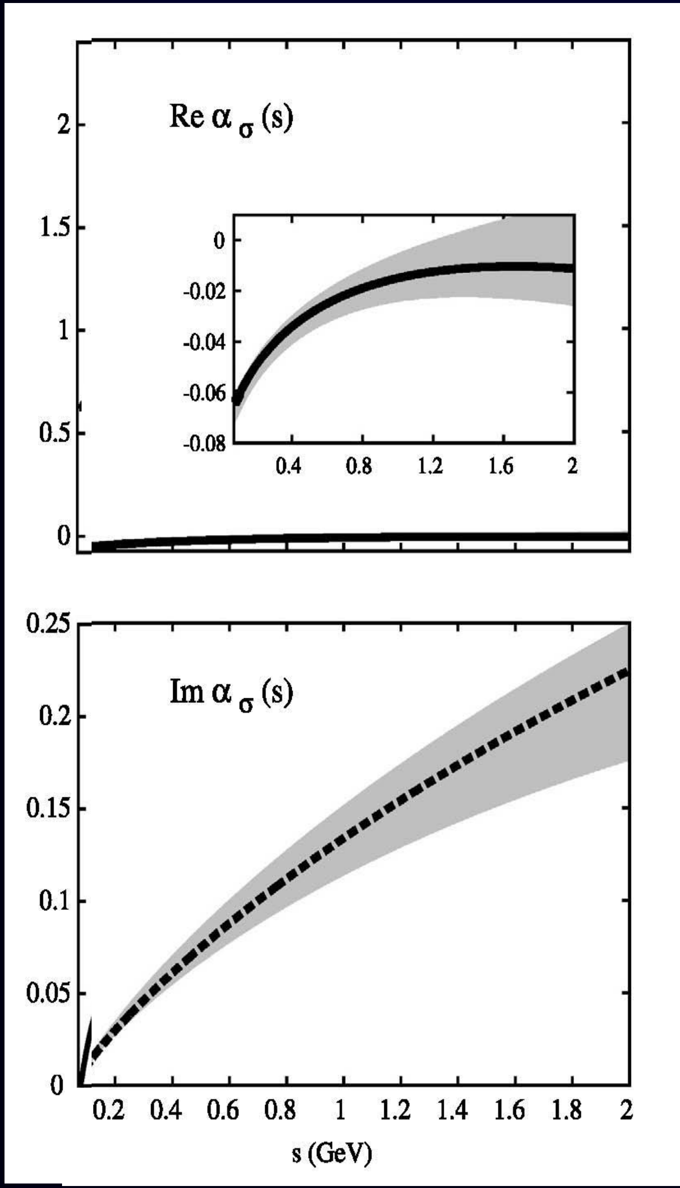


Somewhat better agreement in the resonance region of the Regge pole dominated amplitude with the dispersive amplitude.

So, we apply a similar procedure but now for the f0(500)

Results: ***σ case (I = 0, J = 0)***

The prediction for the *σ* Regge trajectory, is:



- NOT approximately real
- NOT linear

intercept

$$\alpha_\sigma(0) = -0.090^{+0.004}_{-0.012},$$

slope

$$\alpha'_\sigma \simeq 0.002^{+0.050}_{-0.001}\ \mathrm{GeV}^{-2},$$

Compare with the rho result…

α_0= 0.52

α' = 0.913 GeV^{-2}

The sigma does **NOT** fit the ordinary meson trajectory

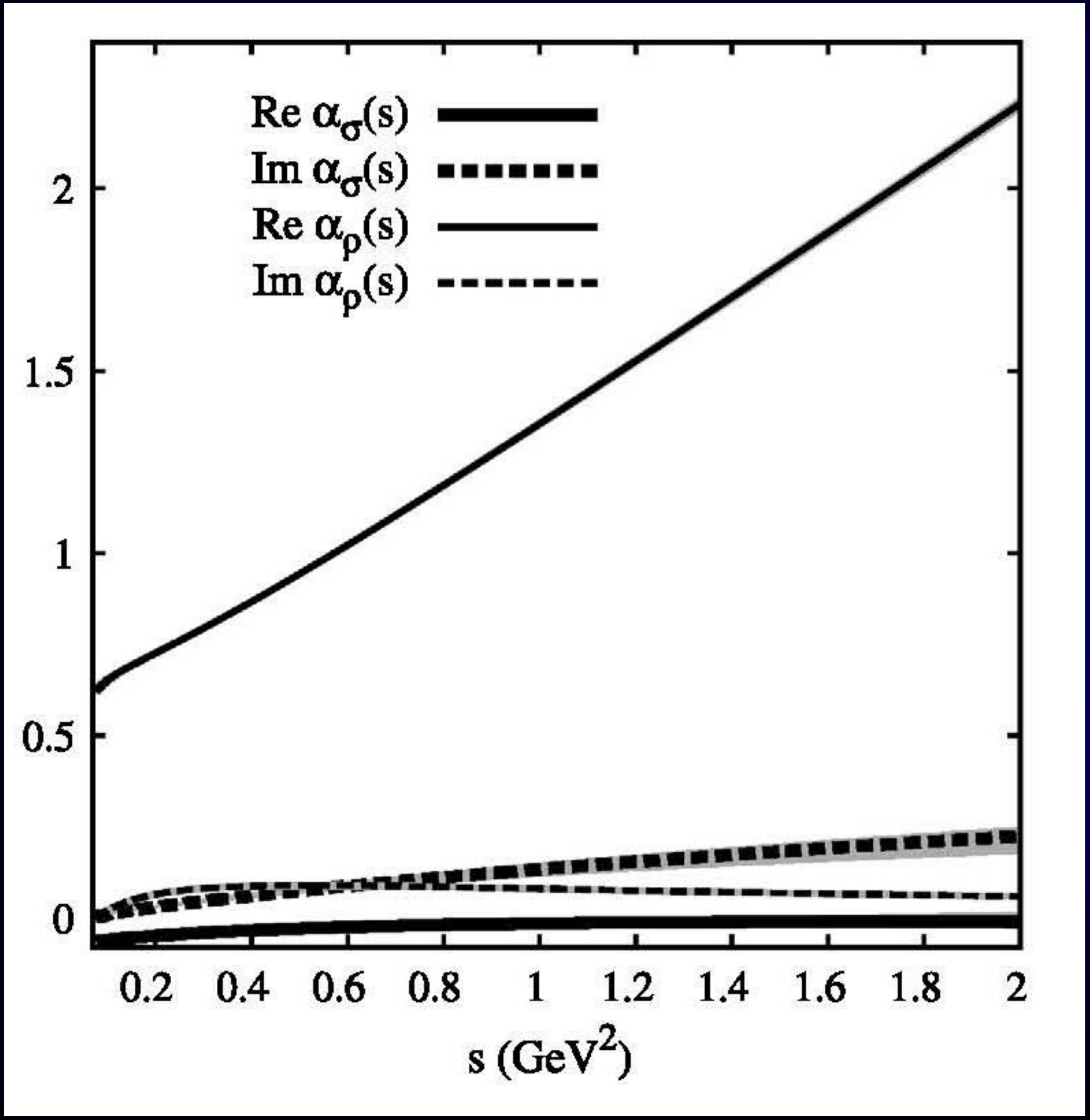
Two orders of magnitude flatter than other hadrons

Typical of meson physics?

F_π , m_π ?

Results: ***σ vs. ρ trajectories***

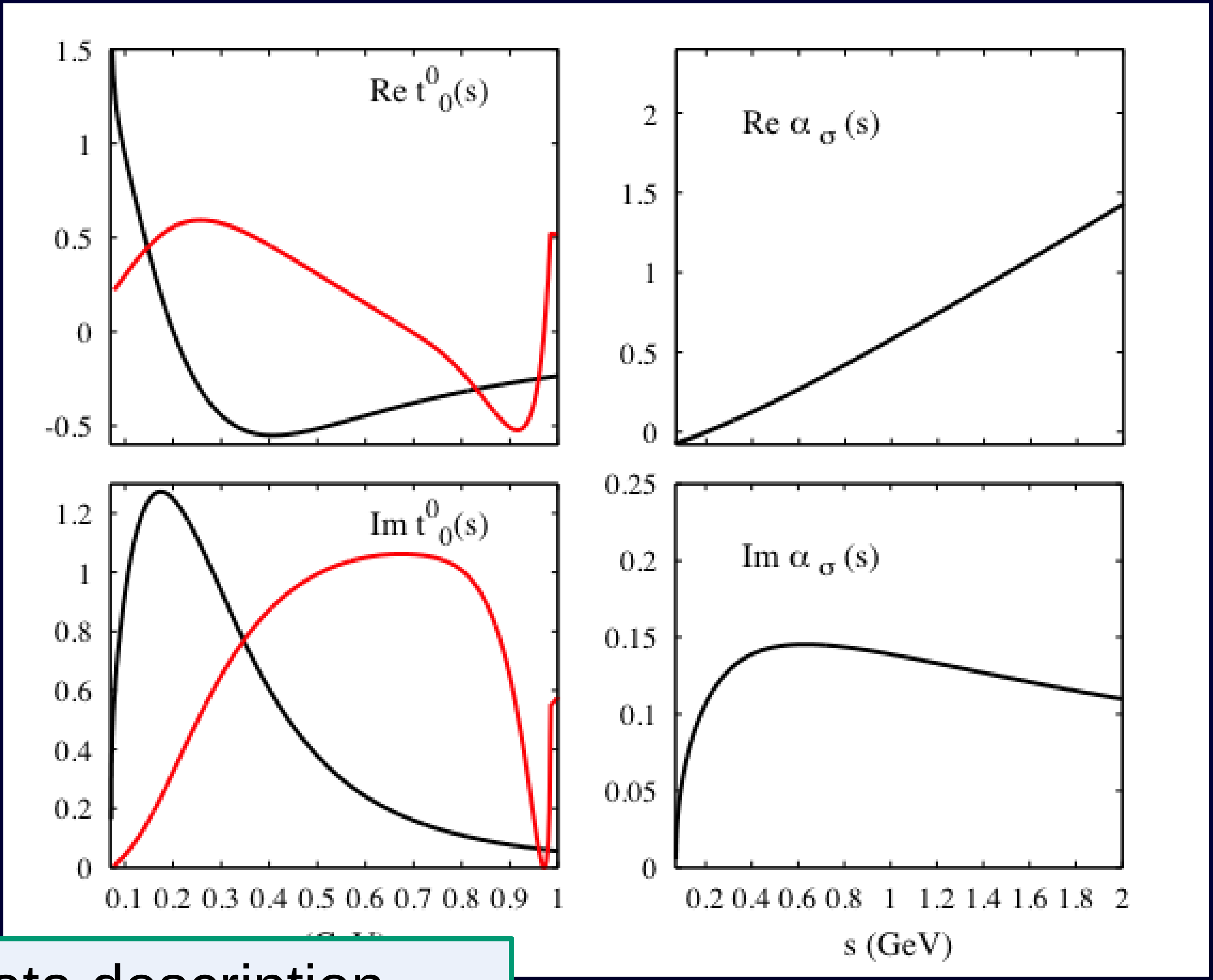
Using the same scale….



No evident Regge partners for the f0(500)

Results: ***σ case (I = 0, J = 0)***

IF WE INSISTED in fixing the α' to an "ordinary" value ~ 1 GeV^{-2}



The data description would be severely spoilt

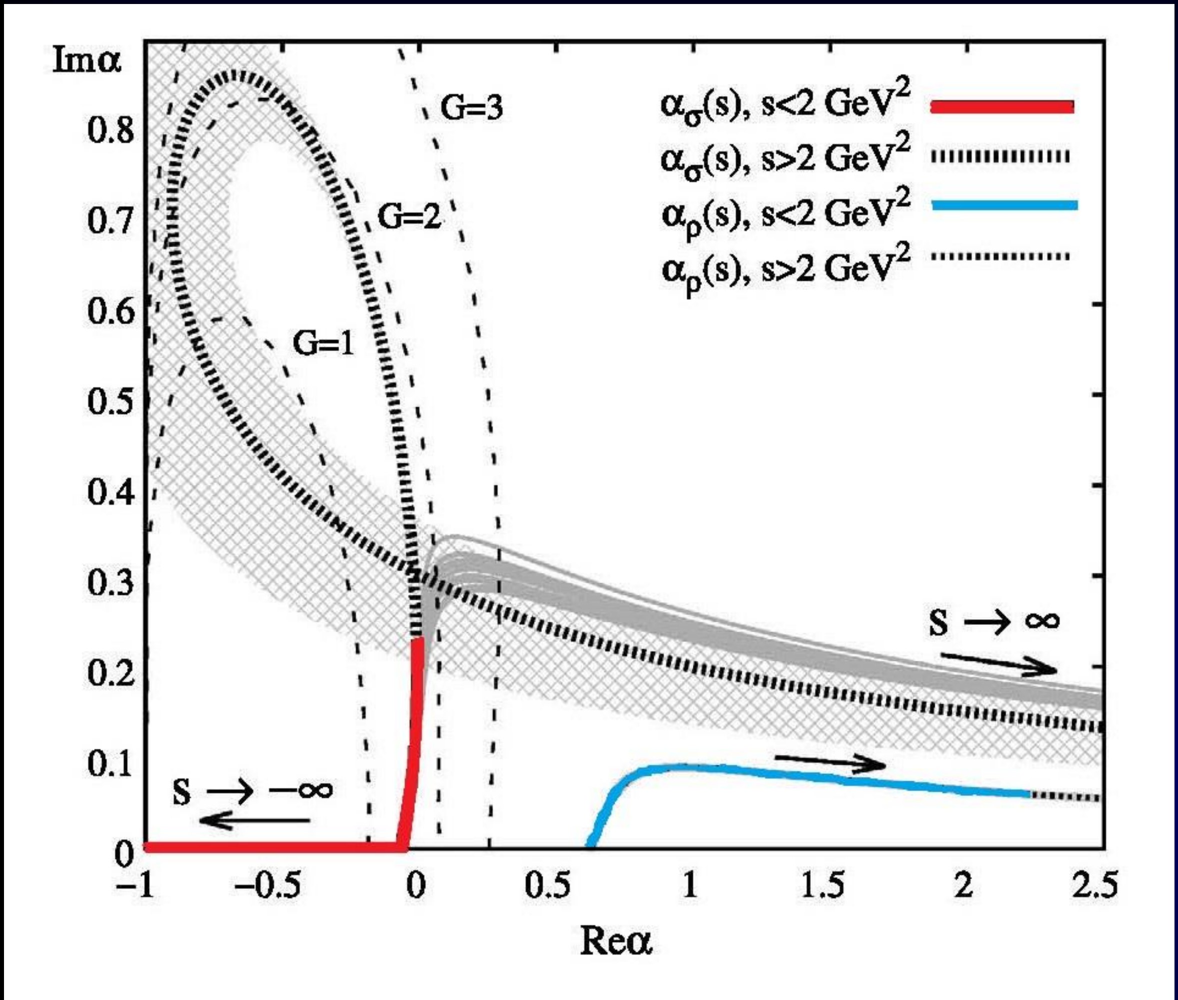
If not-ordinary…

What then?

Can we identify the dynamics of the trajectory?

Not quite yet… but…

Ploting the trajectories in the complex J plane…



Striking similarity with Yukawa potentials at low energy: *V(r)=−Ga exp(−r/a)/r*

Our result is mimicked with a=0.5 GeV^{-1} to compare with S-wave ππ scattering length 1.6 GeV^{-1}

“a” rather small !!!

Non-ordinary σ trajectory

Ordinary ρ trajectory

The extrapolation of our trajectory also follows a Yukawa but deviates at very high energy

Summary

- Analytic constraints on Regge trajectories as integral equations.
- Consistent treatment of the width
- **Fitting JUST the pole position and residue of an isolated resonance**, yields its Regge trajectory parameters

- *ρ, f2 and f2'* trajectories: COME OUT LINEAR, with universal parameters

- *σ* trajectory: NON-LINEAR.

  Trajectory slope **two orders of magnitude smaller**

  No partnerts.

- If we force the *σ* trajectory to have universal slope, data description ruined

- At low energies, striking similarities with trajectories of Yukawa potential

Outlook

- Different mass case (kappa, K*…)
- Meson-nucleon (Delta, etc…)
- Similar non-linear behavior for other non-ordinary states?
- Coupled channels
- Microscopic models, relation to compositeness, etc…

Happy Birthday Eef and congratulations for these *first* 70 years of very fruitful career

Thank you for your advice and personal example of dedication and enjoyment when doing Physics