Scalar mesons in isospin violating τ decay

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Workshop on unquenched hadron spectroscopy: Non-perturbative models and methods of QCD vs. experiment

with: *Sébastien Descotes-Genon, arXiv:1404.0251*

*** EEF70 *** Coimbra, Sept. 1-5, 2014

Introduction

- Reconsider: isospin violating mode: $\tau \to \eta\pi\nu$
- First discussed: [Bramon, Narison, Pich, PL B196 (1987) 543]
  - → Scalar resonance contribution (from $a_0(980)$) could be dominant
  - → Size of contribution sensitive to $\bar{q}q$ versus $\bar{q}q\bar{q}q$: measure

$$\langle 0|\bar{u}d|a_0(980)\rangle$$

- Here: Approach based on unitarity/analyticity, chiral symmetry constraints, link to $\eta\pi \to \eta\pi$ scattering amplitude
- Analogies with $\tau \to K\pi\nu$ and $K\pi \to K\pi$ scattering

- Light scalar resonances in the PDG:

$f_0(500)$	M=$(400-450)-i(200-350)$	
$K_0^*(800)$	M=682 ± 29	$\Gamma=547\pm 24$
$f_0(980)$	M=990 ± 20	$\Gamma=40-100$
$a_0(980)$	M=980 ± 20	$\Gamma=50-100$

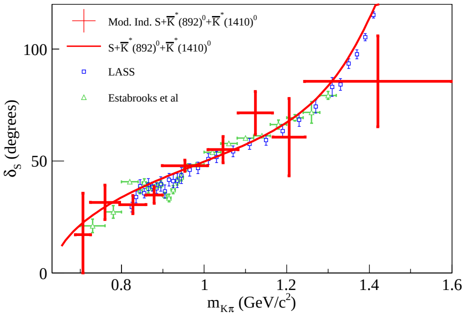
→ Light broad resonances now seem accepted

→ In particular $f_0(500)$ (σ-meson):
[Experimental progress in $\pi\pi$ scattering: New precise K_{l4}, $K\to 3\pi$ from NA48/2, pionium from DIRAC, +new analysis: see talks: Kaminski, Pelaez]

Illustration: $a_0^0=0.2220\pm 0.0140$

→ $K_0^*(800)$? existence would suggest $f_0(500)$ not glueball.

- πK scattering ? New data also exist
  a) $\pi - K$ Phase-shifts extracted from D_{l4} decays [$D^+ \to K^- \pi^+ l \nu$] for the first time: [Babar coll., PR D83 (2011)072001].



[Not yet precise enough]

b) $\pi - K$ atom observed, lifetime measurement for first time: [DIRAC coll., arXiv:1403.0845]
Difference of $J = 0$ scattering lengths:

$$a^{\frac{1}{2}} - a^{\frac{3}{2}} = (0.33^{+0.27}_{-0.12})\ m_\pi^{-1}$$

[Relatively precise]

c) τ decay mode: $\tau \to K\pi\nu$:
First measurements of $K\pi$ energy distribution [ALEPH, EPJ C11 (1999) 599]
New results from [Belle coll., PL B654 (2007) 65]
($\sim 3 \times 10^3$ more τ's)

More on $\tau \to K\pi\nu$ and $K\pi$ scattering

- Consider $\tau \to K^+\pi^0\nu_\tau$

$$\sqrt{2}\langle K^+(p_K)|\bar{u}\gamma^\mu s|\pi^0(p_\pi)\rangle = f_+^{K\pi}(t)\,(p_K+p_\pi)^\mu + f_-^{K\pi}(t)\,(p_K-p_\pi)^\mu$$

with $t = (p_K - p_\pi)^2$. Introduce:

$$f_0^{K\pi}(t) = f_+^{K\pi}(t) + \frac{t}{m_K^2 - m_\pi^2}\, f_-^{K\pi}(t)$$

$f_0^{K\pi}(t)$: Scalar form factor ($K\pi$ scattering in $J = 0$)
$f_+^{K\pi}(t)$: Vector form factor ($K\pi$ scattering in $J = 1$)

- Properties of form factors:
  - → Functions of one variable, Analytic in t with only a right-hand cut
  - → Discontinuity (=im. part) from unitarity relation
  - → **Note:** Analyticity derives from QCD (confinement)
  - → Asymptotic behaviour in QCD: $1/t \log(t)$

- $K\pi$ scattering: Most recent phase-shift/inelasticity determinations from LASS coll. [Aston et al., NP B296 (1988) 493].

- $J = 0$ scattering elastic (effectively) in large range (up to $K\eta'$ threshold).

- Elastic unitarity relation for form factor

$$\boxed{\mathrm{Im}\, f_0^{K\pi}(t) = \sigma_{K\pi}(t)\,\left(T_0^{K\pi}(t)\right)^*\, f_0^{K\pi}(t)}$$

→ $\phi^{K\pi} \equiv \mathrm{Phase}[f_0^{K\pi}] = \delta^{K\pi}$ (Fermi-Watson)

→ Phase dispersive representation:

$$f_0^{K\pi}(t) = f_0^{K\pi}(0)\, \exp\left[\frac{t}{\pi}\int_{(m_K+m_\pi)^2}^{\infty} \frac{dt'\,\phi^{K\pi}(t')}{t'(t'-t)}\right]$$

→ QCD: $\phi^{K\pi}(t)_{t\to\infty} \sim \pi + \dfrac{\pi}{\log\frac{t}{\Lambda^2}} + \cdots$

- Beyond elastic unitarity $\longrightarrow$ model for $\phi^{K\pi}$ [Jamin, Oller, Pich NP B622 (2002) 279]. Proposed for $\pi\pi$: [Donoghue, Gasser, Leutwyler NP B343 (1990) 341] .

- (Coupled) integral equations for form factors: $F_1 \equiv f_0^{K\pi}$, $F_2 \equiv f_0^{K\eta'}$

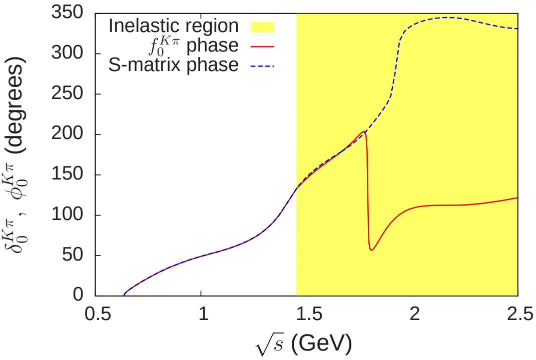
$$\begin{pmatrix} F_1(t) \\ F_2(t) \end{pmatrix} = \frac{1}{\pi} \int_{(m_K+m_\pi)^2}^{\infty} \frac{dt'}{t'-t} \mathbf{T}_0^*(t') \Sigma(t') \begin{pmatrix} F_1(t') \\ F_2(t') \end{pmatrix}$$

- Can be arranged to have a unique solution, once $f_0^{K\pi}(0)$, $f_0^{K\eta'}(0)$ given (or: $f_0^{K\pi}(m_K^2 - m_\pi^2) = F_K/F_\pi + O(m_\pi^2)$ instead of $f_0^{K\eta'}(0)$)

- Asymptotic phase-shifts: $\delta_{K\pi} + \delta_{K\eta'} \to 2\pi$ (Asymptotic QCD)

- FLAG values: $f_0^{K\pi}(0) = 0.9667(23)(23)$, $F_K/F_\pi = 1.194(4)$ (lattice QCD $N_f = 2+1$)

- Result for form factor phase vs phase-shift



- $\phi^{K\pi}$ displays sharp dip

- Vector form factor $f_+^{K\pi}$: analogous construction should hold[BM, EPJ C53 (2008) 401]

- Inelasticity in $J=1$ dominated by quasi-two-body channels[LASS coll., NP B247(1984)261, NP B292 (1987)693, PL B201(1988)169]

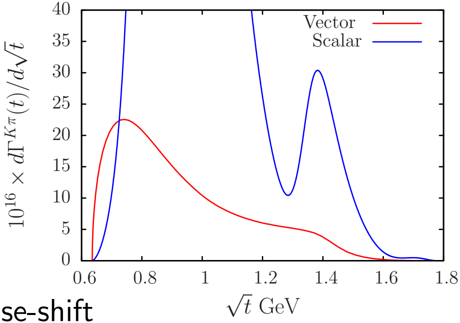
Three-channel T-matrix: $\begin{bmatrix} K\pi \\ K^*\pi \\ K\rho \end{bmatrix}$ Three form factors: $\begin{bmatrix} f_+^{K\pi} \equiv H_1 \\ f_V^{K^*\pi} \equiv H_2 \\ f_V^{K\rho} \equiv H_3 \end{bmatrix}$

- Values $H_2(0)$, $H_3(0)$: input from $\rho \to \pi\gamma$, $K^* \to K\gamma$ detrmined in terms of one parameter a (assuming linear flavour symmetry breaking).

- Results for τ decay:

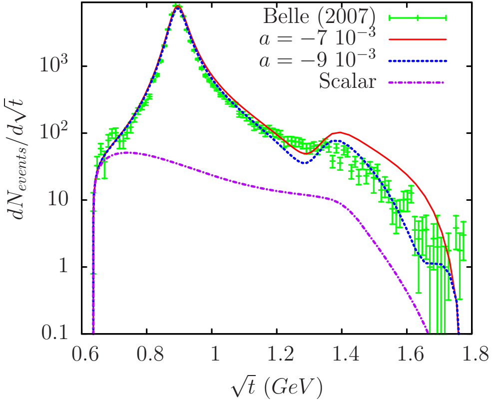
$$\frac{d\Gamma^{K\pi}}{d\sqrt{t}} = \frac{G_F^2 V_{us}^2 m_\tau^2}{128\pi^3} q_{K\pi}(t) \left(1 - \frac{t}{m_\tau^2}\right) \left[\left(1 + \frac{2t}{m_\tau^2}\right) \frac{4q_{K\pi}^2(t)}{t} |f_+^{K\pi}(t)|^2 + \frac{3(m_K^2 - m_\pi^2)}{t^2} |f_0^{K\pi}(t)|^2 \right]$$

→ Vector and Scalar contributions:



→ Low-energy "peak" generated from phase-shift

- Comparison with experiment:

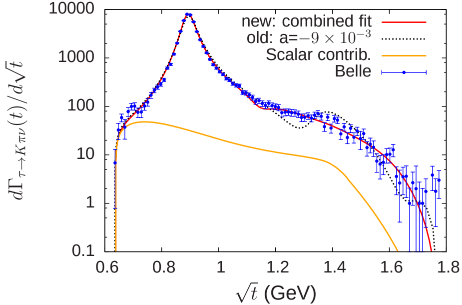


→ Data confirms scalar form factor at low energy

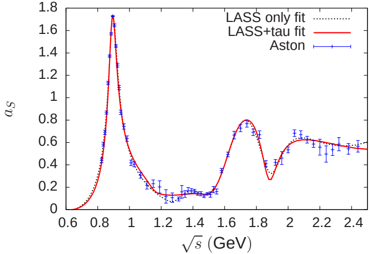
→ Vector resonances: $K^*(892)$ narrower in τ decay, $K^*(1410)$ broader

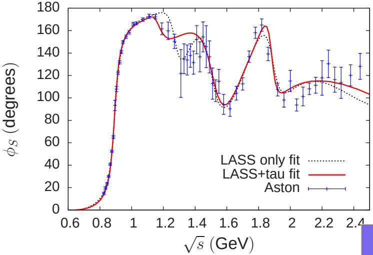
→ More detailed testing if $K\pi$ angular distribution measured.

- Improving $J = 1$ πK phase-shift using τ decays: make a combined fit of Belle and LASS
- Result for τ decay



- Comparison with LASS: $T^{K^+\pi^-} = a_S exp(i\phi_S)$





τ decay mode: $\tau \to \eta\pi\nu$

- So-called second-class amplitude [Weinberg, PR 112 (1958) 1375] (forbidden in isospin limit)

  → Vector current $\bar{u}\gamma^{\mu}d$ even under G-parity transformation

  → $\eta\pi$ odd G-parity eigenstate

- Contributions:
  a) from outside the SM (e.g. from charged Higgses)

  b) (dominant) from the SM via isospin breaking. QCD: $O((m_d - m_u)/m_s)$, QED: $O(e^2)$

- Kinematics (analogous to $K\pi$)

$$\langle \eta\pi^+ | j_\mu^{ud} | 0 \rangle = -\sqrt{2}\, [\, f_+^{\eta\pi}(s)\, (p_\eta - p_\pi)^\mu + f_-^{\eta\pi}(s)\, (p_\eta + p_\pi)^\mu]$$

with: $j_\mu^{ud} = \bar{u}\gamma^\mu d, \quad s = (p_\eta + p_\pi)^2$

→ Scalar form factor:

$$f_0^{\eta\pi}(s) = f_+^{\eta\pi}(s) + \frac{s}{\Delta_{\eta\pi}} f_-^{\eta\pi}(s), \qquad \Delta_{PQ} = m_P^2 - m_Q^2$$

→ $f_0^{\eta\pi}(s)$ associated with divergence of current

$$\langle \eta(p_\eta)\pi(p_\pi) | i\partial^\mu j_\mu^{ud}(0) | 0 \rangle = \sqrt{2}\Delta_{\eta\pi}\, f_0^{\eta\pi}(s) \ ,$$

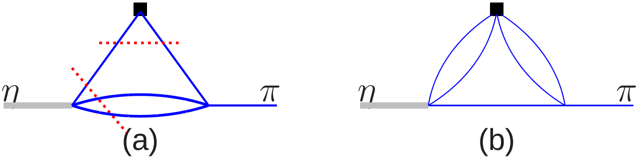
→ Current divergence obeys Ward identity

$$i\partial_\mu \bar{u}\gamma^\mu d = (m_d - m_u)\bar{u}d - eA_\mu \bar{u}\gamma^\mu d$$

→ Possibility of experimental determination of coupling of $a_0(980)$ to $\bar{u}d$ operator.

Analyticity of ηπ form factors: anomalous thresholds ?

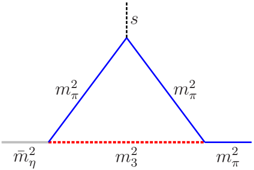
- η-meson is unstable. Impact on analyticity ?
- First investigate in toy model (w. local vertices)



- We consider only (a)-type diagrams

$$f^{\eta\pi}(s) = \frac{1}{16\pi^2}\int_{4m_\pi^2}^{\infty} dm_3^2 \sqrt{1-\frac{4m_\pi^2}{m_3^2}}\, K^{\eta\pi}(m_3^2, s) + \cdots$$

$K^{\eta\pi}(m_3^2, s)$ is a triangle diagram:

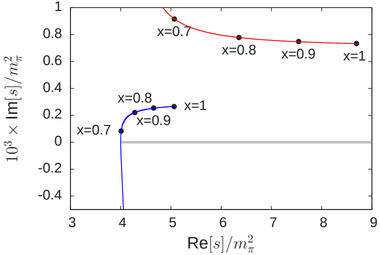


- Disp. representation of $K^{\eta\pi}$ (starting with $m_\eta^2 < 9m_\pi^2$)

$$K^{\eta\pi}(m_3^2, s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{L^{\eta\pi}(m_3^2, s')}{s' - s}$$

- Watch how singularities of the log. $L^{\eta\pi}(m_3^2, s')$ move when varying m_η^2 ($\equiv m_\eta^2 + i\epsilon$) [Mandelstam (1959)]

Figure:
$m_3^2 = 5m_\pi^2$
$x = m_\eta^2/(m_\eta^2)_{phys.}$



Crossing of real axis: $4m_\pi^2\left(1 - \frac{\epsilon^2}{4m_3^2(m_3^2 - 4m_\pi^2)}\right) < 4m_\pi^2$

Unitarity relations

- General expressions:

Vector: $$-2\sqrt{2}q_{\eta\pi}(s)\,\mathrm{Im}\,[f_+^{\eta\pi}(s)] = \frac{1}{2}\sum_n T^*_{\eta\pi^+\to n} \times \langle n|j_3^{ud}(0)|0\rangle$$

Scalar: $$-\frac{\sqrt{2}\Delta_{\eta\pi}}{\sqrt{s}}\,\mathrm{Im}\,[f_0^{\eta\pi}(s)] = \frac{1}{2}\sum_n T^*_{\eta\pi^+\to n} \times \langle n|j_0^{ud}(0)|0\rangle$$

- Leading contribution for vector FF: $n = \pi^+\pi^0$

$$\mathrm{Im}\,[f_+^{\eta\pi}(s)]_n = \frac{\theta(s-m_n^2)(4m_\pi^2-s)}{32\pi\sqrt{\lambda_{\eta\pi}(s)}} \times F_V^\pi(s) \times \int_{-1}^{1} dz\, z T^*_{\eta\pi^+\to\pi\pi}(s,t)$$

$\Downarrow$ Pion vector form factor

$\Downarrow$ $\eta \to 3\pi$ amplitude

Other contribs: $n = 4\pi, K^0K^+, \ldots$ unimportant below 1 GeV

- Contributions for scalar FF

  → $n = \pi^0\pi^+$: Double isospin suppressed

  → $n = \eta\pi^+$

$$\mathrm{Im}\,[f_0^{\eta\pi}(s)]_n = \frac{\theta(s-m_n^2)\sqrt{\lambda_{\eta\pi}(s)}}{32\pi s} f_0^{\eta\pi}(s) \times \int_{-1}^{1} dz T^*_{\eta\pi^+\to\eta\pi^+}(s,t)$$

[*Dominant contribution*]

  → $n = \bar{K}^0 K^+$

$$\mathrm{Im}\,[f_0^{\eta\pi}(s)]_n = \frac{\theta(s-m_n^2)\sqrt{s-4m_K^2}}{32\sqrt{2}\pi\ s} \frac{\Delta_{K^0K^+}}{\Delta_{\eta\pi}} f_0^{\bar{K}^0K^+}(s) \times \int_{-1}^{1} dz T^*_{\eta\pi^+\to\bar{K}^0K^+}(s,t)$$

[*Also relevant, in principle*]

At this level: analogous to $f_0^{K\pi}$

- Electromagnetic contributions ?

→ $n = \gamma\pi$: no contrib. to $f_0^{\eta\pi}$

→ $n = \gamma\pi\pi$

$$\mathrm{Im}\,[f_0^{\eta\pi}(s)] = -\frac{\theta(s-4m_\pi^2)}{2\Delta_{\eta\pi}} \times \sum_{\lambda=\pm 1} \int dLips_3\, e\, F_V^\pi(s_{\pi\pi}) e_\gamma(\lambda)(p_{\pi^0} - p_{\pi^+}) \times T^*_{\eta\pi^+\to\gamma(\lambda)\pi^0\pi^+}$$

Suppressed by phase-space below 1 GeV

Values near $s = 0$: ChPT calculations

- At LO: form factors are constant

$$f_0^{\eta\pi}(0) = \epsilon = \frac{\sqrt{3}(m_d - m_u)}{4\,(m_s - m_{ud})}$$

$$= \boxed{0.99 \times 10^{-2}} \quad \text{(PS masses +LO ChPT)}$$

- NLO calculations: [Scora, Maltman (1995)] [Neufeld, Rupertsberger (1995)] (also include $O(e^2 m_q)$)
  - → At $s = 0$: remarkable relation with K_{l3}^+, K_{l3}^0 decays

$$f_0^{\eta\pi}(0)|_{LO+NLO} = \frac{1}{\sqrt{3}}\left[\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^+}(0)} - 1\right]$$

  - → Exp. inputs from K-factories [NA48, ISTRA+, KLOE, BNL-E865] quite precise, yielding

$$\boxed{f_0^{\eta\pi}(0)|_{LO+NLO} = 1.49 \pm 0.23\, 10^{-2}}$$

→ Derivatives at $s = 0$

$$\dot{f}_+^{\eta\pi}(0) = \frac{\epsilon}{12F_\pi^2}\left[24\,L_9 - L_K - 2L_\pi - \tfrac{3}{16\pi^2}\right]$$

$$\dot{f}_0^{\eta\pi}(0) = \frac{\epsilon}{12F_\pi^2}\left[48\,L_5 - 9L_K - \tfrac{11}{16\pi^2} + 4m_\pi^2\dot{J}_{\eta\pi}(0)\right] + \frac{\sqrt{3}\,e^2}{18\Delta_{\eta\pi}}\left[-2(2S_2 + 3S_3) + \tfrac{11}{16\pi^2}Z\right]$$

L_5, L_9: Gasser-Leutwyler couplings: known

S_2, S_3: must be estimated from resonance models

Dispersion relation for $f_0^{\eta\pi}$

- Use phase dispersive representation

$$f_0^{\eta\pi}(s) = f_0^{\eta\pi}(0) \times \exp\left[\zeta s + \frac{s^2}{\pi}\int_{(m_\eta+m_\pi)^2}^{\infty} \frac{\phi^{\eta\pi}(s')}{(s')^2(s'-s)} ds'\right]$$

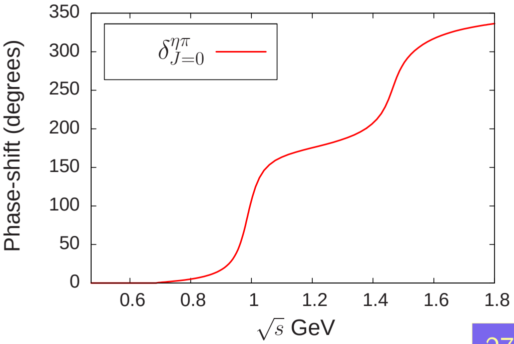
→ Watson: below $K\overline{K}$ threshold: $\phi^{\eta\pi}(s) = \delta_0^{\eta\pi}(s)$

→ $\zeta = \dot{f}_0^{\eta\pi}(0)/f_0^{\eta\pi}(0)$ (from ChPT)

→ $s \to \infty$: no exponential behaviour

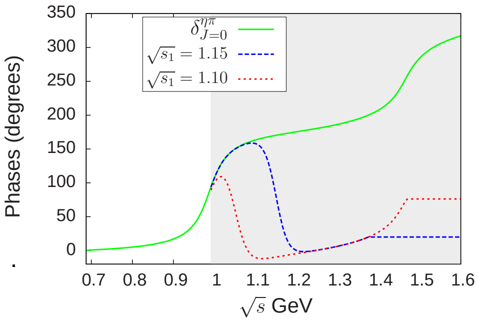
⇒ Sum rule: $$\zeta = \frac{1}{\pi}\int_{(m_\pi+m_\eta)^2}^{\infty} \frac{\phi^{\eta\pi}(s')}{(s')^2} ds'$$

- $\eta\pi$ scattering: scattering length $a_0 = (-0.02 \pm 0.77) \times 10^{-2}$ (NLO LET [Kubis, Schneider EPJ C62 (2009) 511])
- Phase-shift: use model [Black, Fariborz, Schechter, PR D61 (2000) 074030]
  - → LO ChPT+ scalar resonance exchanges, tested for $\pi\pi$, πK, $\eta' \to \eta\pi\pi$
  - → Merits: Simple, crossing symm., approx. unitarity
  - → Drawback: $a_0 = 3.1 \times 10^{-2}$



- Form factor phase: dip model inspired by πK

$$
\begin{array}{ll}
\text{Let } s_1,\ s_2: & 4m_K^2 < s_1 < s_2 \\
s \leqslant s_2: & \phi^{\eta\pi}(s) = \delta_0^{\eta\pi}(s) - \pi\theta(s - s_1) \\
s > s_2: & \phi^{\eta\pi}(s) = \text{constant}(= \phi^{\eta\pi}(s_2))
\end{array}
$$

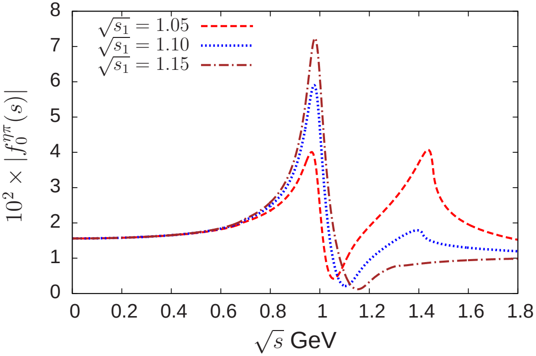


- s_1 given, s_2 determined from sum rule constraint

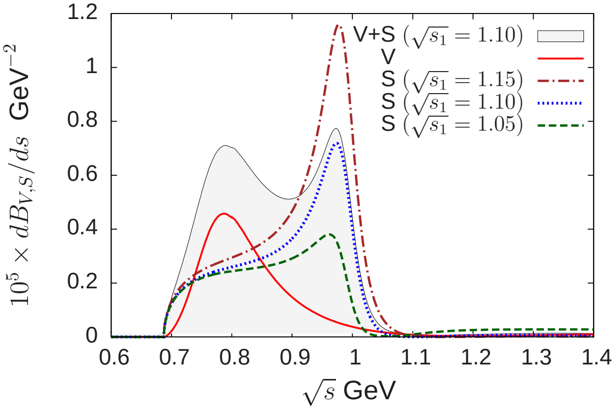
$$\zeta = \frac{1}{\pi}\int_{(m_\eta+m_\pi)^2}^{\infty} \frac{\phi^{\eta\pi}(s')}{(s')^2}\, ds' .$$

- $\sqrt{s_1}$ cannot exceed $\simeq 1.2$ GeV

- Exoticity of a_0 resonances:
  If dip is close to resonance mass $\Rightarrow$ resonance is exotic



- Results on τ decay energy distribution:



→ Vector form factor uses $\eta\pi \to \pi\pi$ from Khuri-Treiman eqs.

→ Significant scalar contribution *below* $a_0(980)$ peak

- Results on τ decay integrated branching fractions:

→ Experimental: $BF \leqslant 9.9 \times 10^{-5}$ [Babar (2011)]

→ Our estimate:
$$BF_{vect} \simeq 0.11 \times 10^{-5}$$
$$BF_{scal} \simeq 0.37^{+0.30}_{-0.20} \times 10^{-5}$$
(var. of s_{dip})

is on lower side of previous ones:

V	S	total (x10^5)	ref.
0.25	1.60	1.85	Tisserant, Truong (1982)
0.12	1.38	1.50	Pich (1987)
0.15	1.06	1.21	Neufeld, Rupertsberger(1995)
0.36	1.00	1.36	Nussinov, Soffer (2008)
[0.2-0.6]	[0.2-2.3]	[0.4-2.9]	Paver, Riazuddin (2010)
0.44	0.04	0.48	Volkov,Kostunin (2012) (NJL)

Summary

- Estimates of $\eta\pi$ isospin violating form factors: inputs from K^+_{l3}/K^0_{l3}, $\eta \to 3\pi$ (vector), analogy with $K\pi$ (scalar)
- Whether $f_0(980)$ or $f_0(1430)$ depends on position of phase cusp.
- Exp. observation of $\tau \to \eta\pi$? large number of τ's produced at B factories:

$$N_\tau = 5 \times 10^8 \text{ (Babar)}$$
$$N_\tau = 9 \times 10^8 \text{ (Belle)}$$

  but not optimized for τ physics (large backgrounds). Better situation at future facilities ? Super B, Super charm-tau
- Other sources of information on $\eta\pi$: $\gamma\gamma \to \eta\pi$ [Belle, PR D80 (2009) 032001], lattice QCD