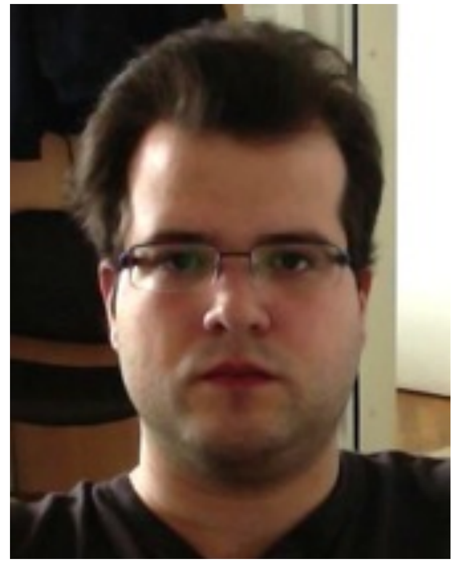
Hadron scattering from a lattice perspective

C. B. Lang
University of Graz, Austria



Sasa Prelovsek and Luka Leskovec
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(Fermilab, USA)

Valentina Verduci
(Univ. Graz, Austria)

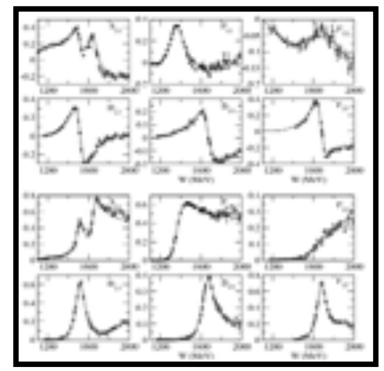
Thanks to my collaborators!

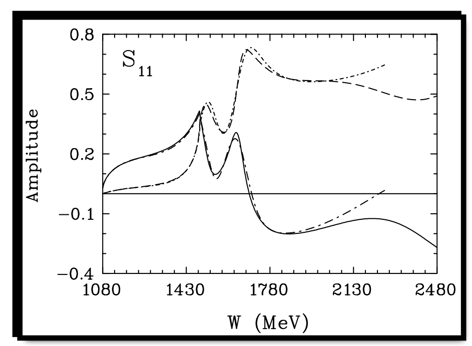
Richard Woloshyn
(TRIUMF, Canada)

Themes

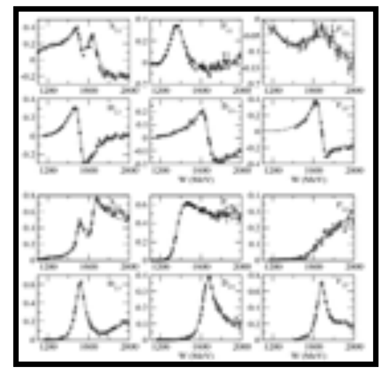
1. Lattice QCD
2. Bound states, resonances, phase shifts
3. Examples

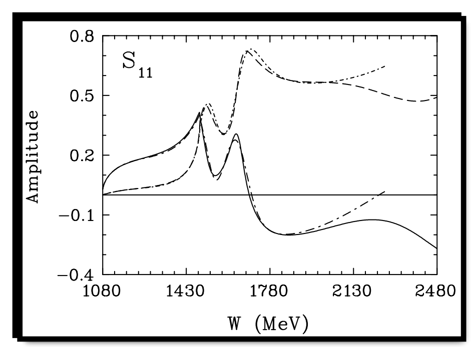
experiments





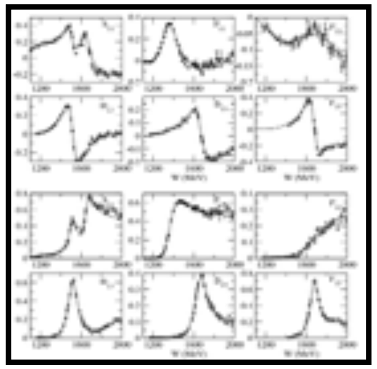
experiments

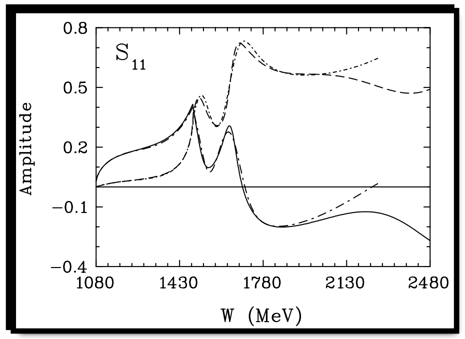




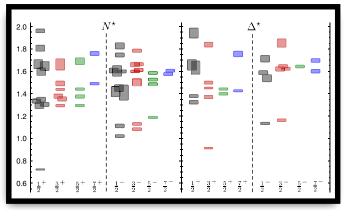
QCD

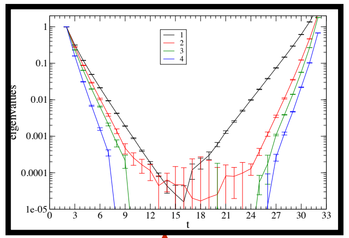
experiments

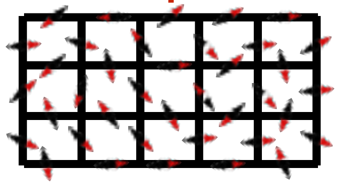




lattice QCD

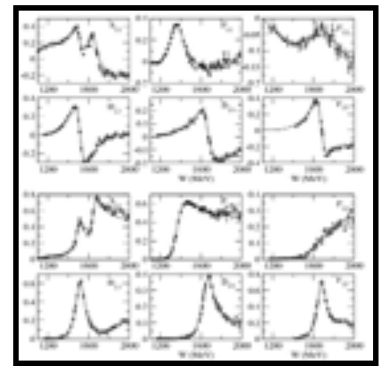


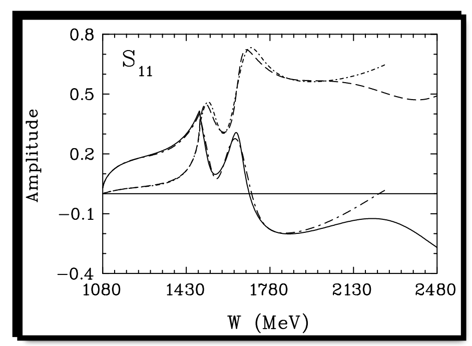




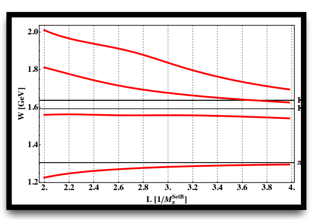
QCD

experiments





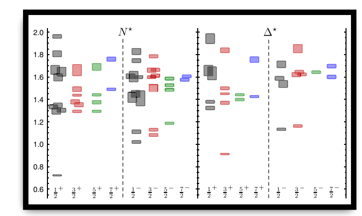
models

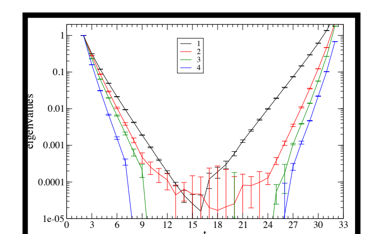


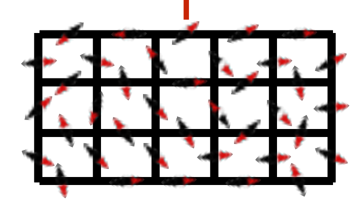
$$M_\pi^2 = M_{0\,\pi}^2\left[1+\mu_\pi-\frac{\mu_\eta}{3}+\frac{16M_{0\,K}^2}{f_0^2}\left(2L_6^r-L_4^r\right)\right.$$
$$\left.+\frac{8M_{0\,\pi}^2}{f_0^2}\left(2L_6^r+2L_8^r-L_4^r-L_5^r\right)\right], \quad (3)$$

$$M_K^2 = M_{0\,K}^2\left[1+\frac{2\mu_\eta}{3}+\frac{8M_{0\,\pi}^2}{f_0^2}\left(2L_6^r-L_4^r\right)\right.$$
$$\left.+\frac{8M_{0\,K}^2}{f_0^2}\left(4L_6^r+2L_8^r-2L_4^r-L_5^r\right)\right], \quad (4)$$

lattice QCD

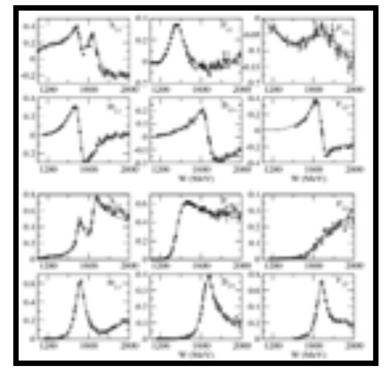


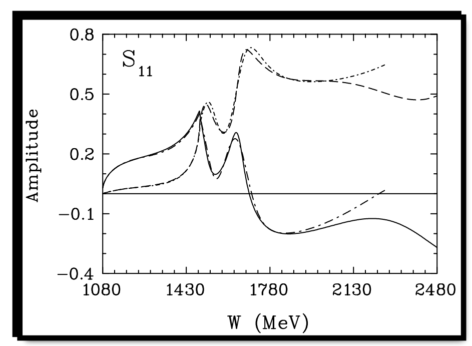




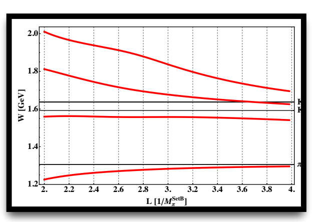
QCD

experiments





models



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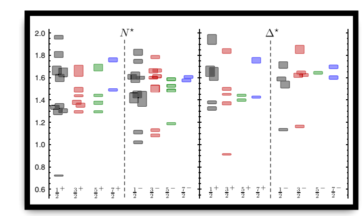
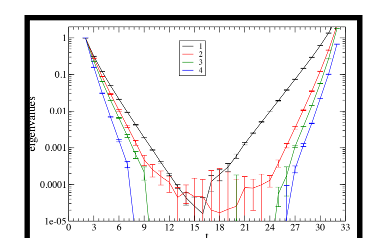
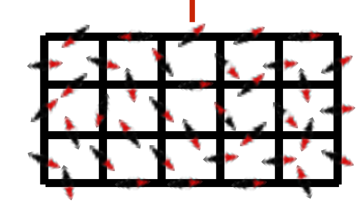
lattice QCD







QCD

What is QCD?

- We assume that QCD is *the* relativistic quantum field theory of quarks and gluon fields with SU(3) gauge symmetry, defined by the action

$$S = \int d^3x\, dt\, L$$

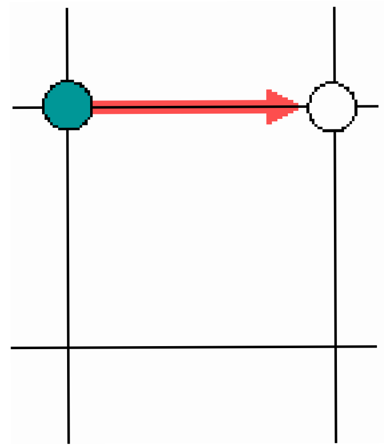
$$L = -\frac{1}{2g^2}\mathrm{tr} F_{\mu\nu}F^{\mu\nu} + \sum_f \overline{\psi}_f (i\not{D} - m_f)\psi_f$$

- This theory can be solved from first principles with minimal number of input parameters (quark masses and scale fixing parameter)

- Hadron properties should be computable from QCD

Regularization: Lattice QCD *(1974)*

Ken Wilson

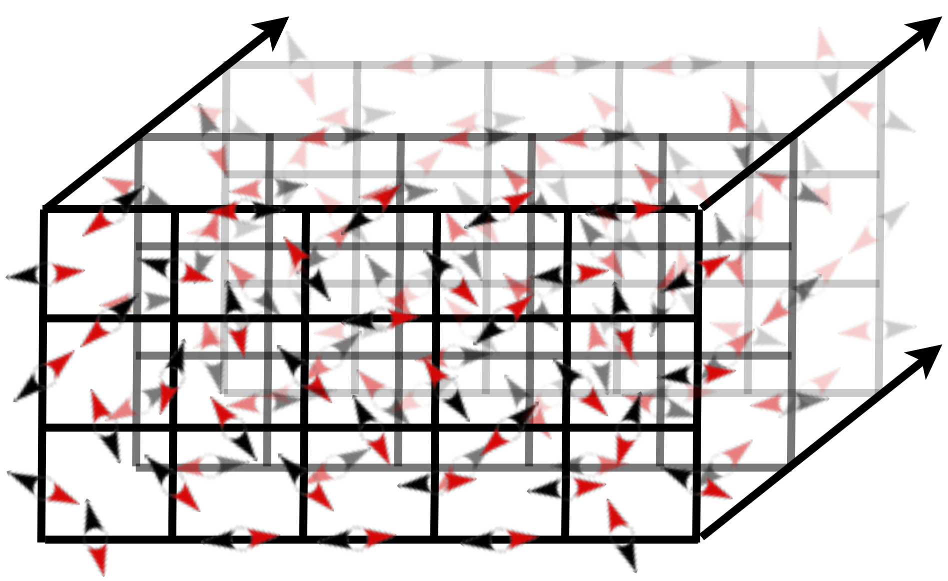


lattice spacing a

Gluon $U_\mu = e^{i\,A_\mu} \in \mathrm{SU}(3)$

Quark ψ

Antiquark $\overline{\psi}$



$U_\mu(x, y, z, \tau)$

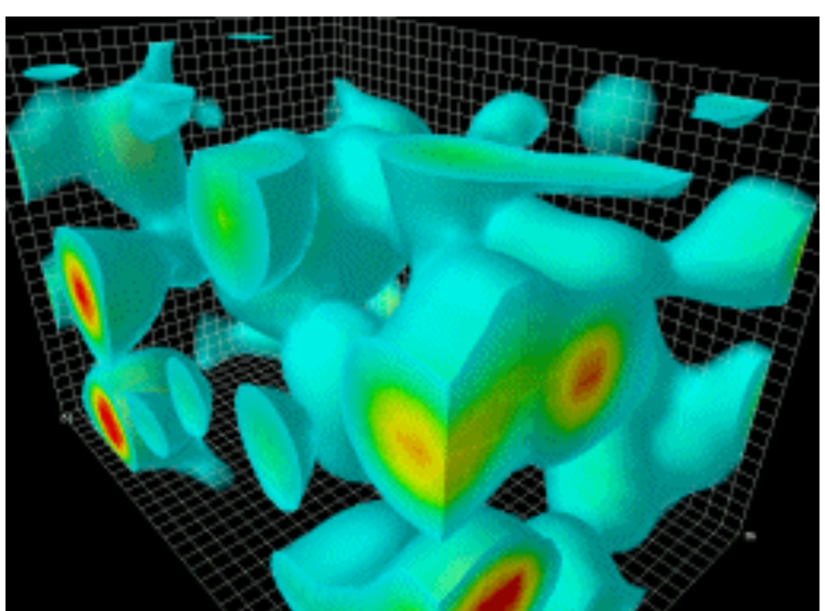
Quantization:

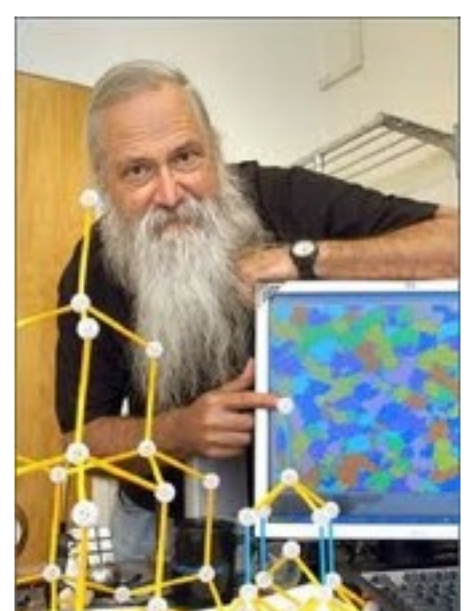
$$\int [dU][d\psi][d\bar{\psi}] \to \sum_{\{U.\psi,\bar{\psi}\}}$$

The path integral becomes a well-defined (very large) sum over field configurations

Let's compute *(1980)*

…functional integral is approximated by computer (Monte Carlo) sampling of field configurations……..





Let’s compute *(1980)*

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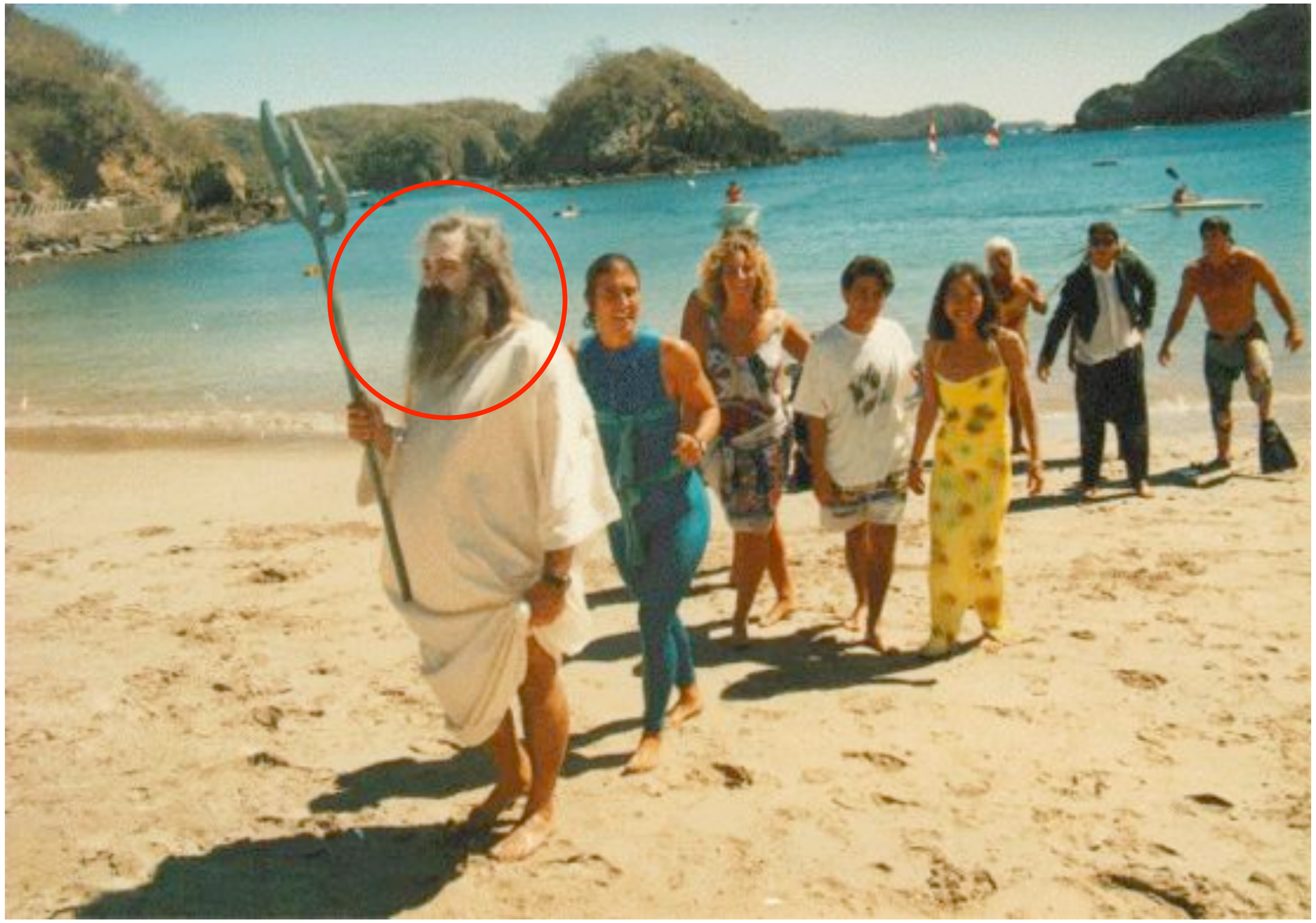
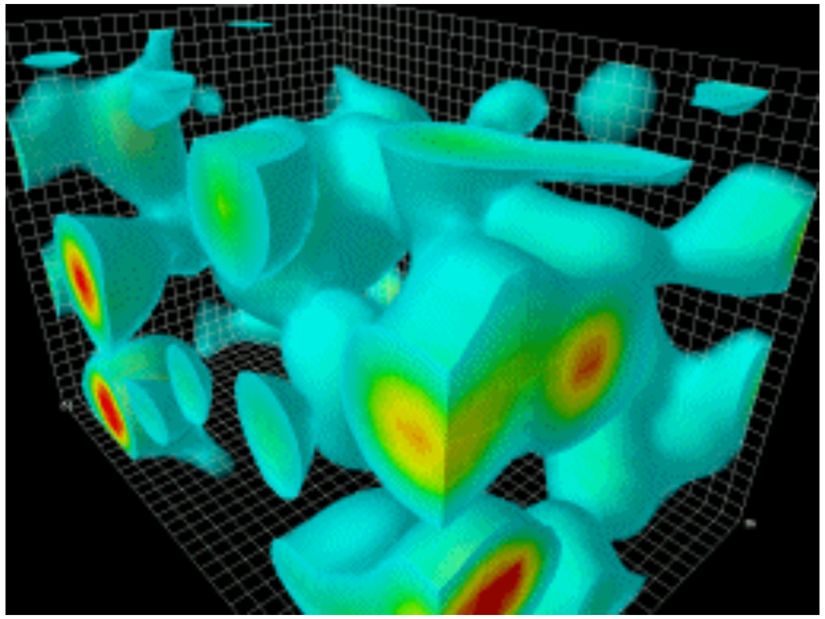
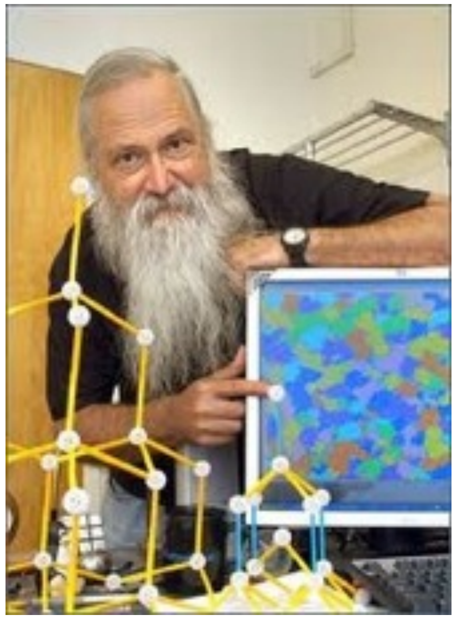
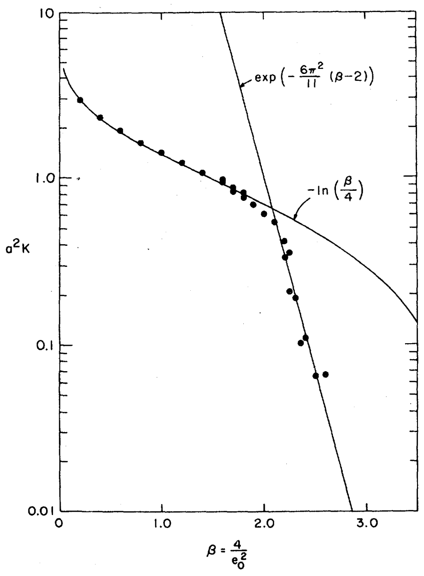






Mike Creutz

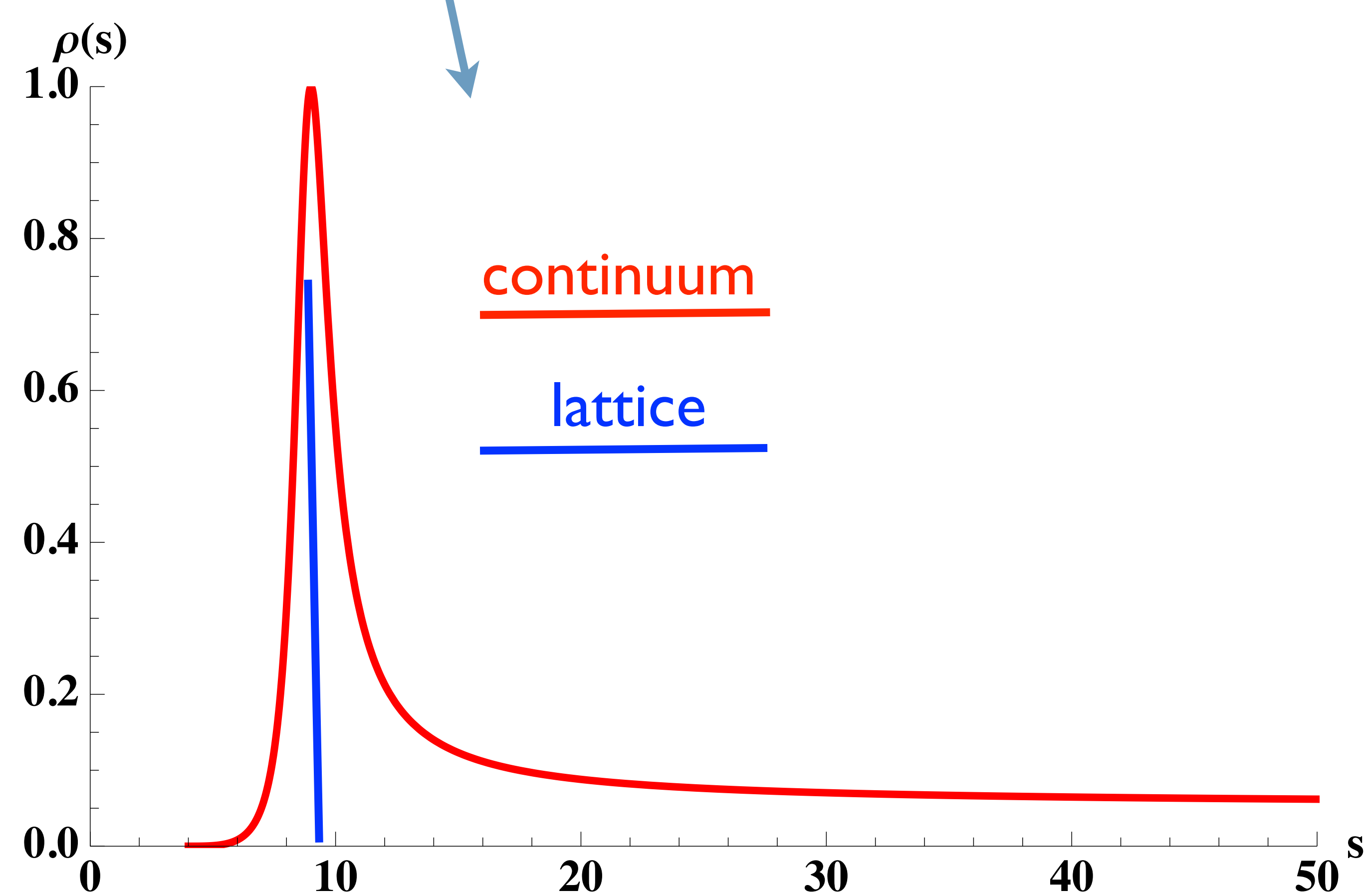


Continuum vs. lattice

Correlation functions:

$$\langle X(t) X^\dagger(0) \rangle \equiv C(t) = \int_{\omega_0}^{\infty} d\omega \; \rho(\omega) \, e^{-\omega t}$$

Finite volume:
Discrete energy levels!



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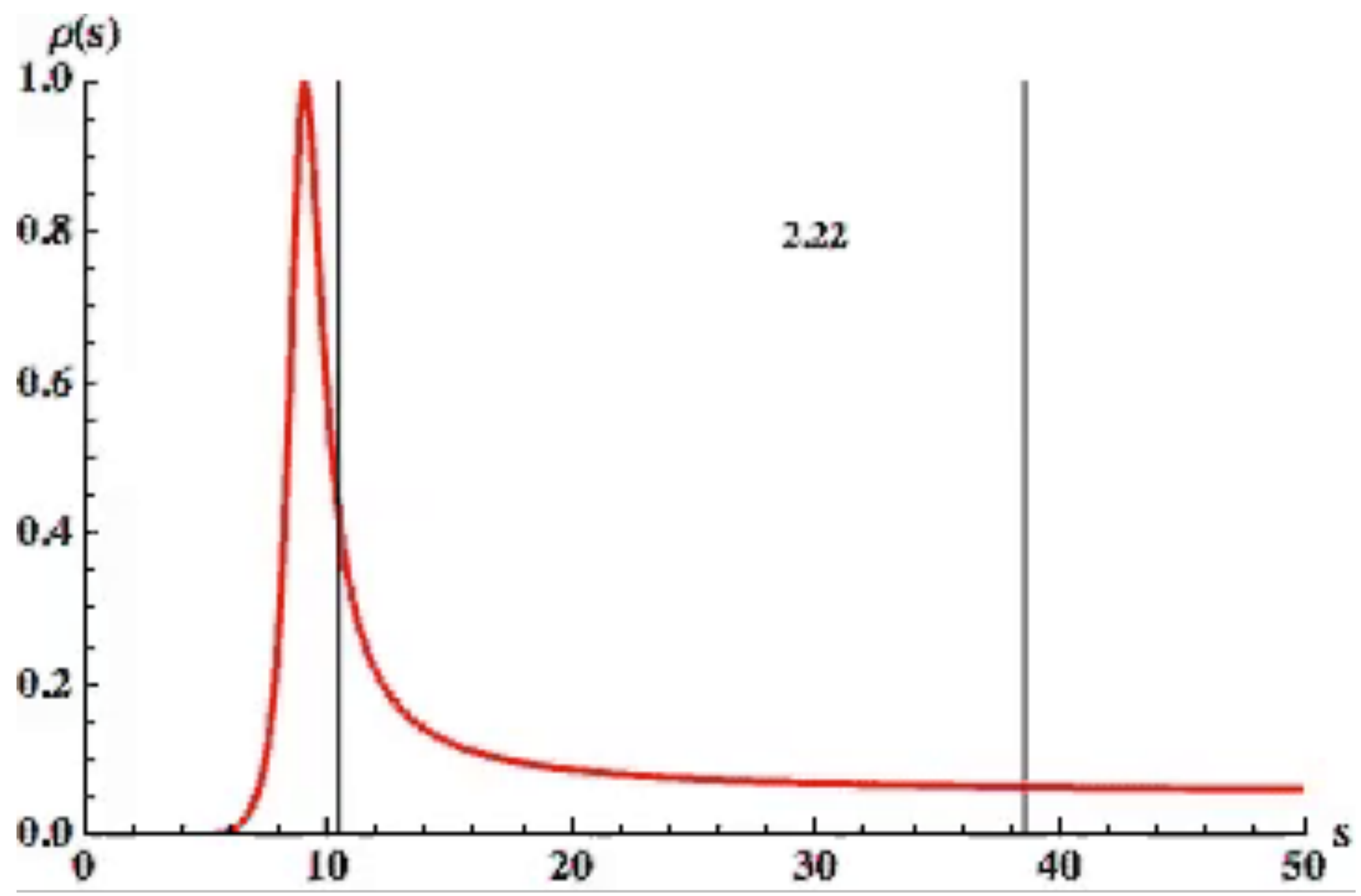

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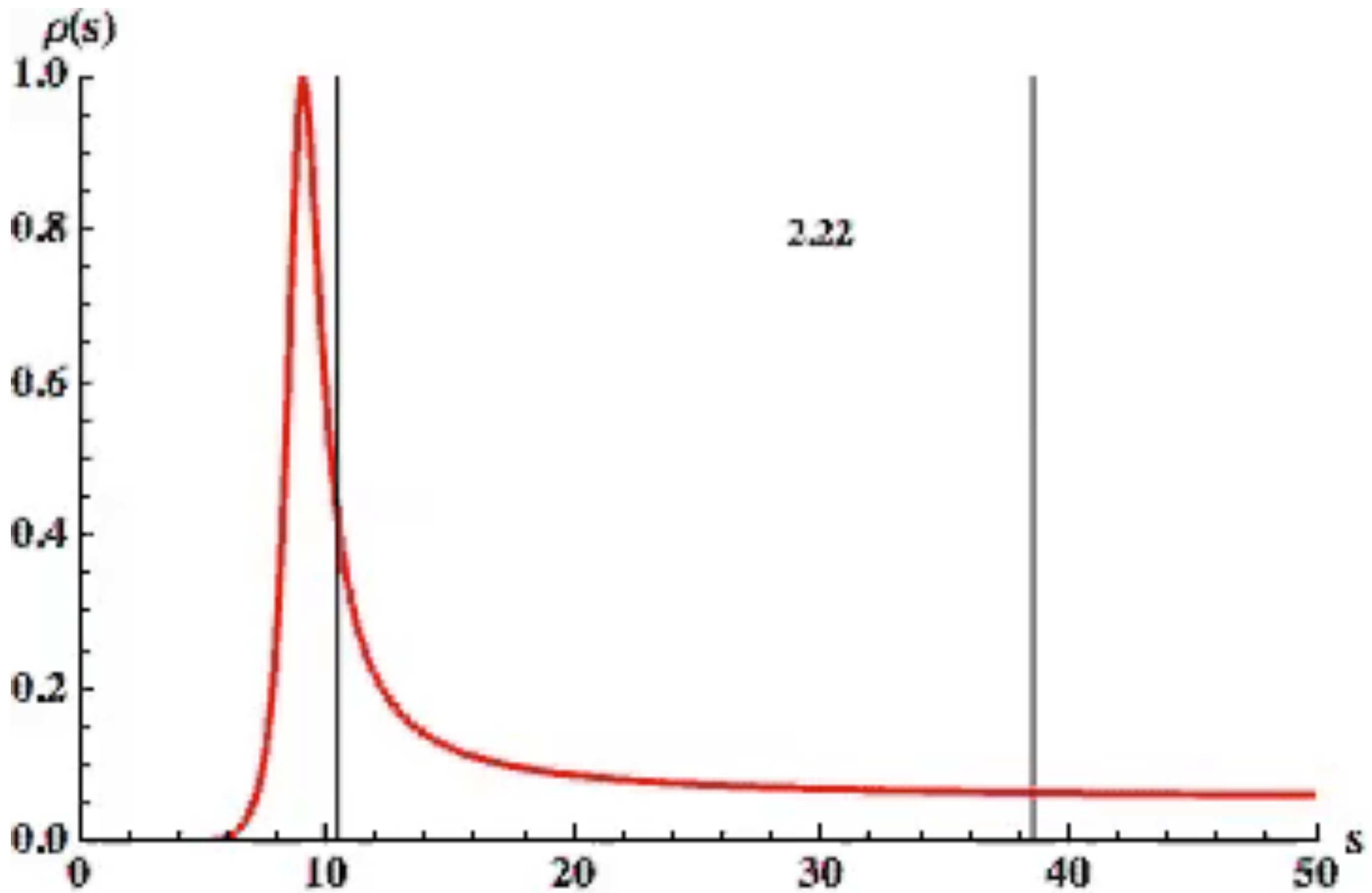
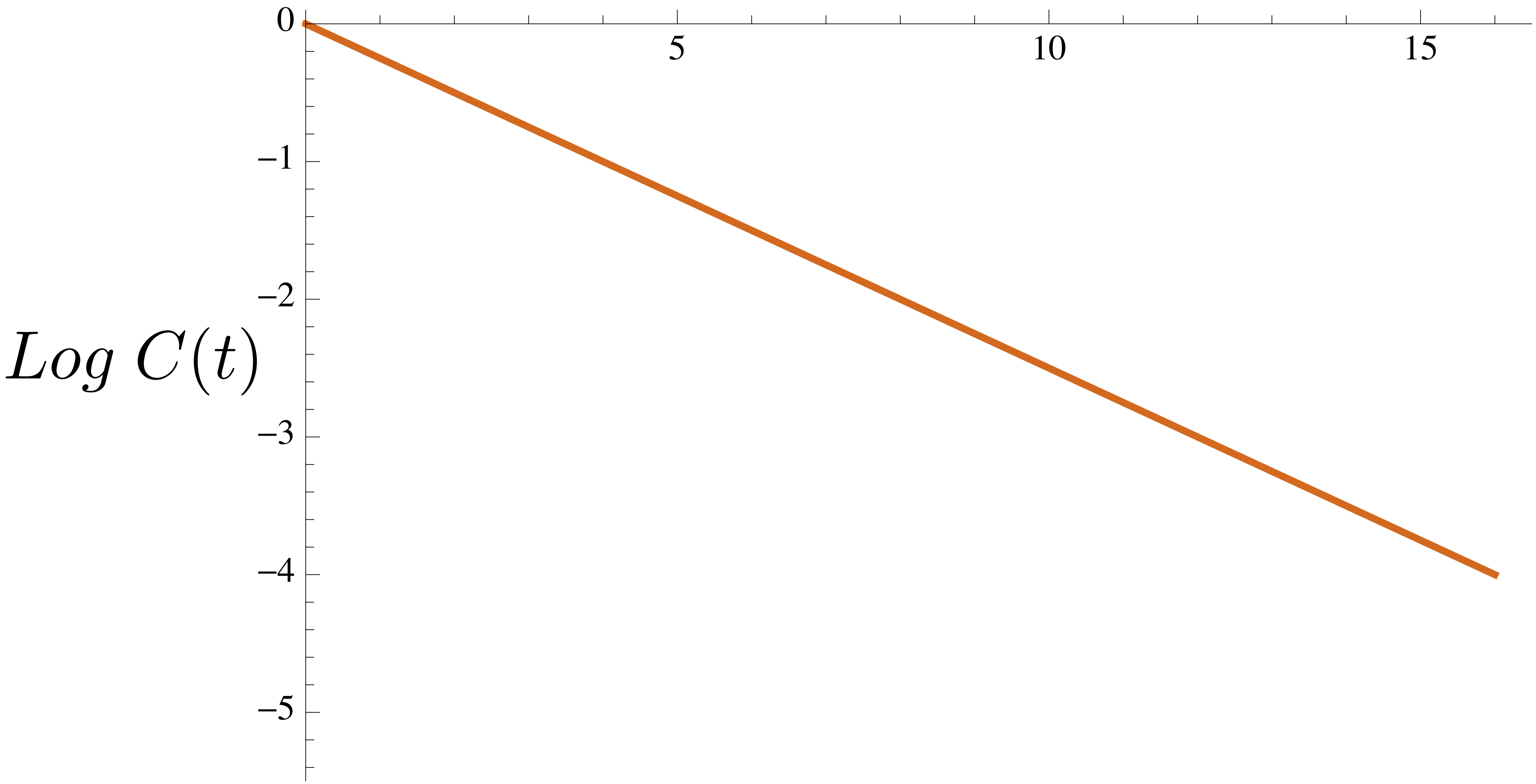


Hadron correlation function - propagator

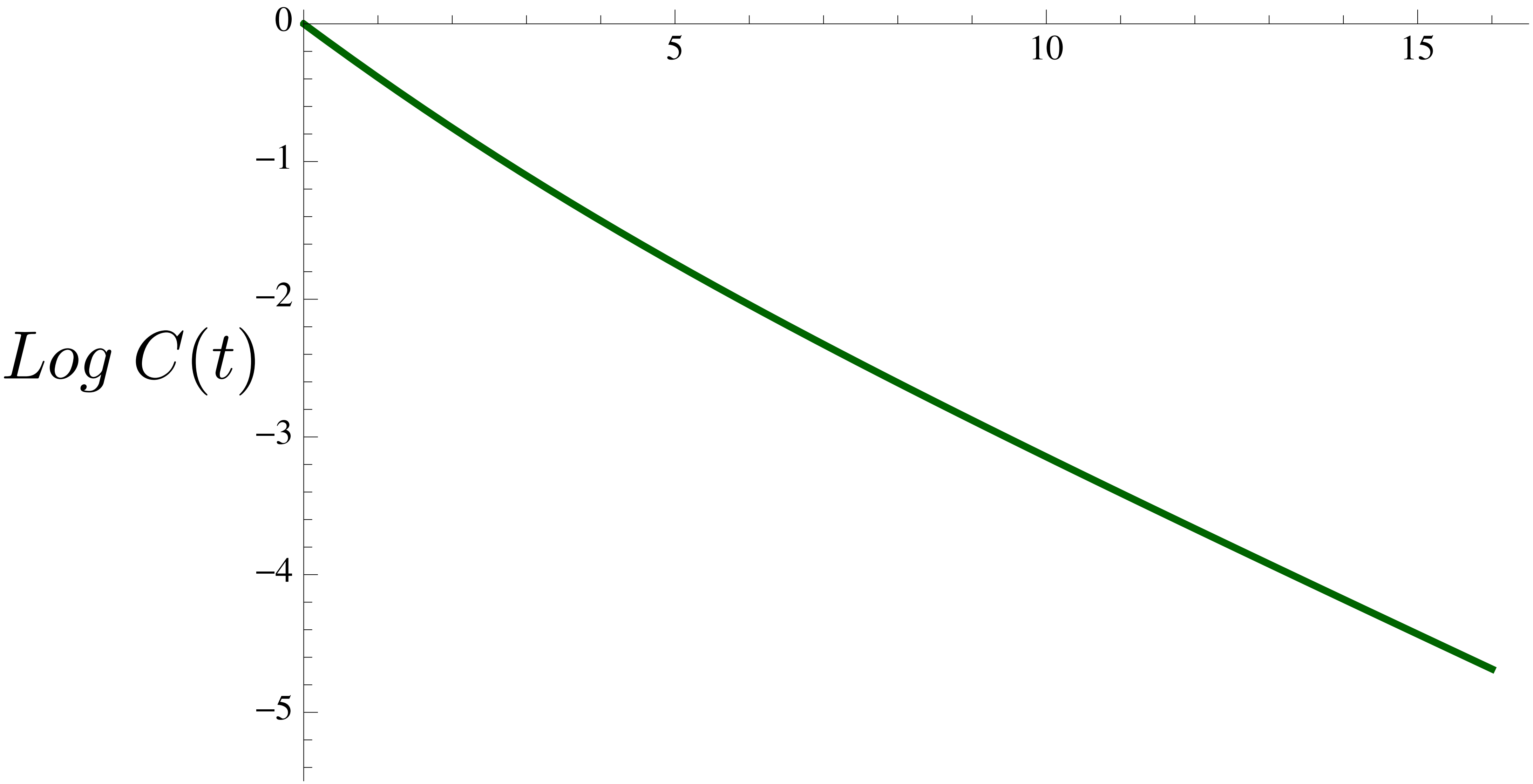
$$C(t) = e^{-0.25\ t}$$



signal/noise ~ $\exp\left(-(m_M - m_\pi)t\right)$
$\exp\left(-(m_B - 1.5 m_\pi)t\right)$

Hadron correlation function - propagator

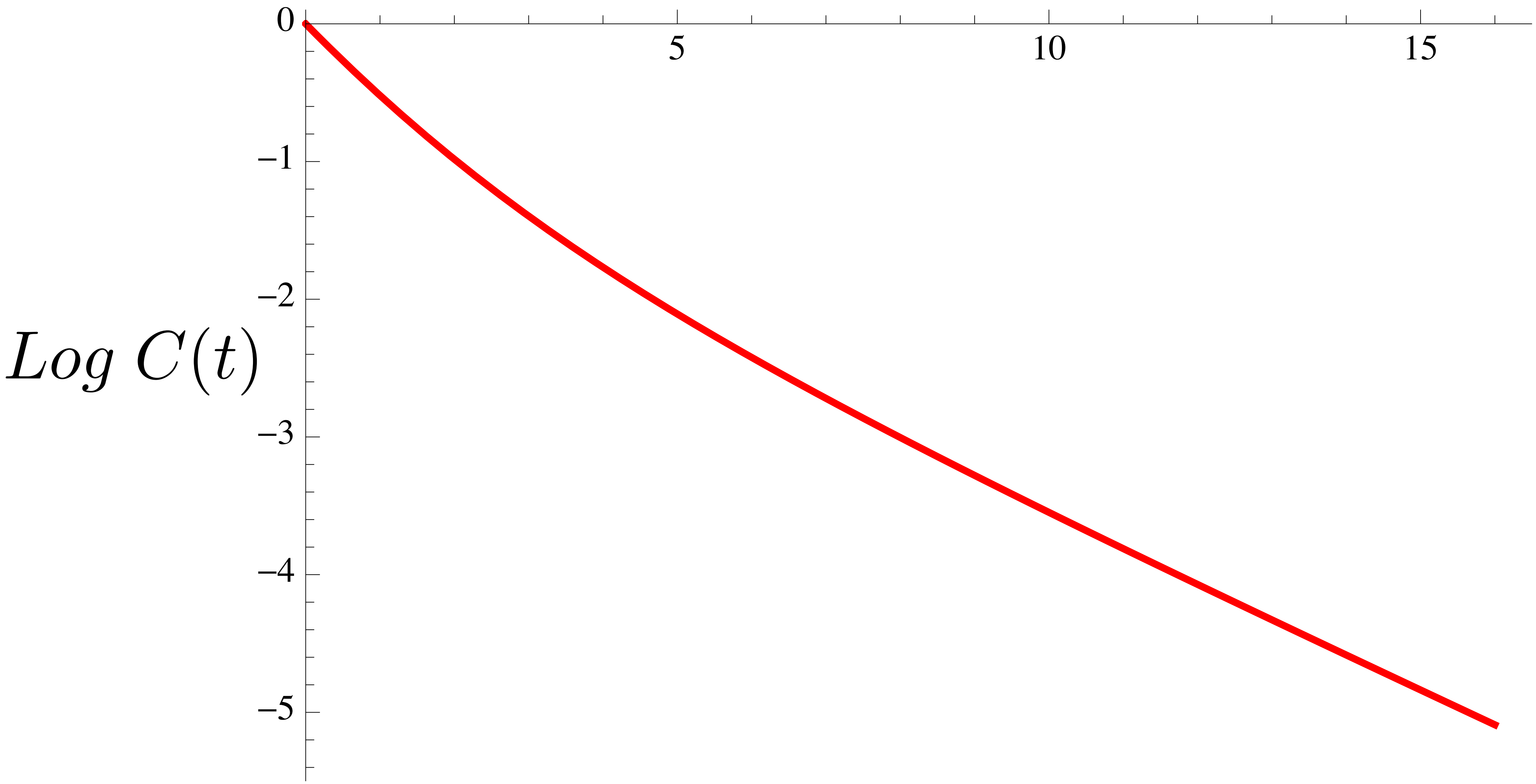
$$C(t) = e^{-0.25\ t} + e^{-0.55\ t}$$



signal/noise ~ $\dfrac{\exp\left(-(m_M - m_\pi)t\right)}{\exp\left(-(m_B - 1.5 m_\pi)t\right)}$

Hadron correlation function - propagator

$$C(t) = e^{-0.25\ t} + e^{-0.55\ t} + e^{-0.85\ t}$$



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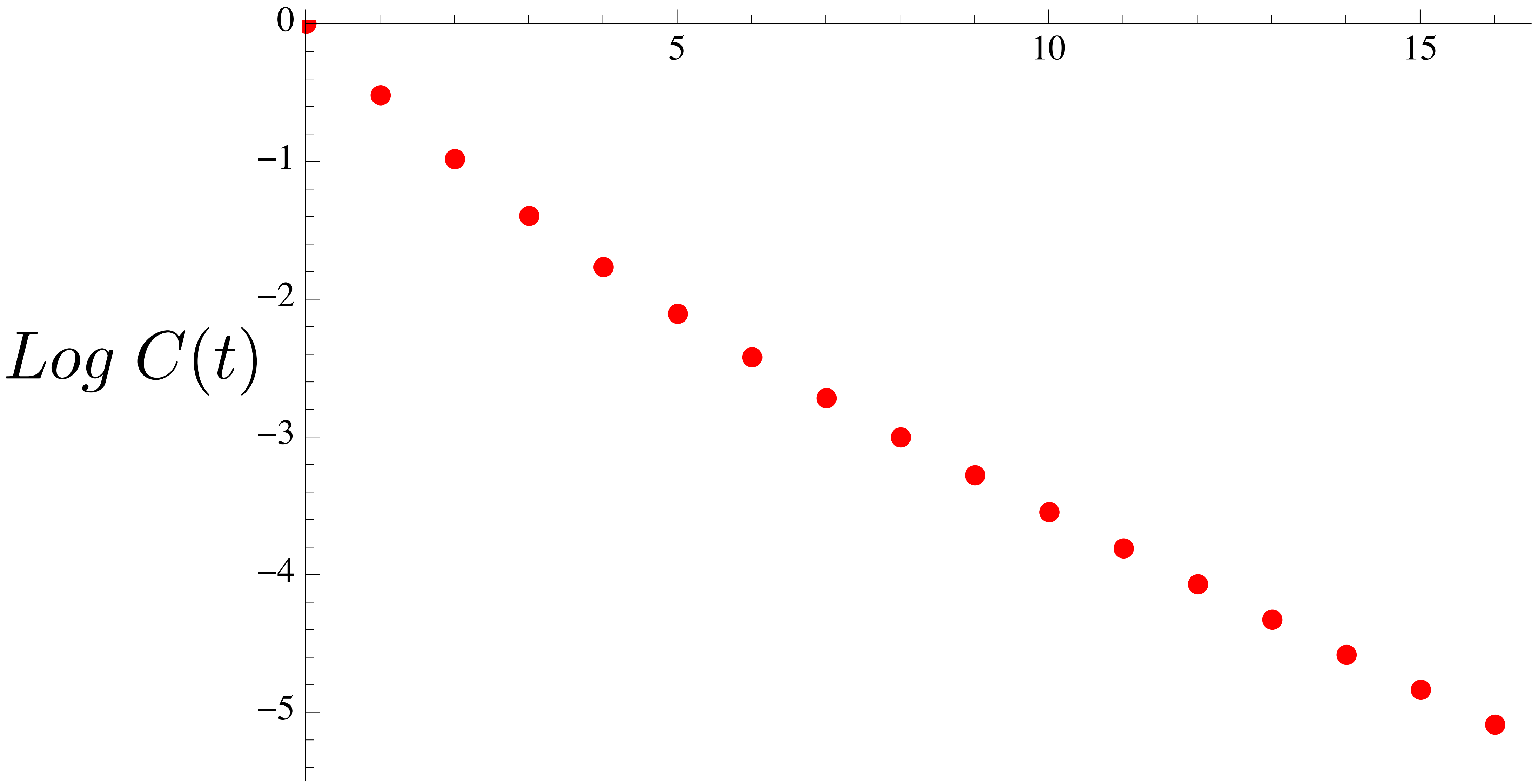
Hadron correlation function - propagator

$$C(t_n) = e^{-0.25\,t} + e^{-0.55\,t} + e^{-0.95\,t}$$



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Hadron correlation function - propagator

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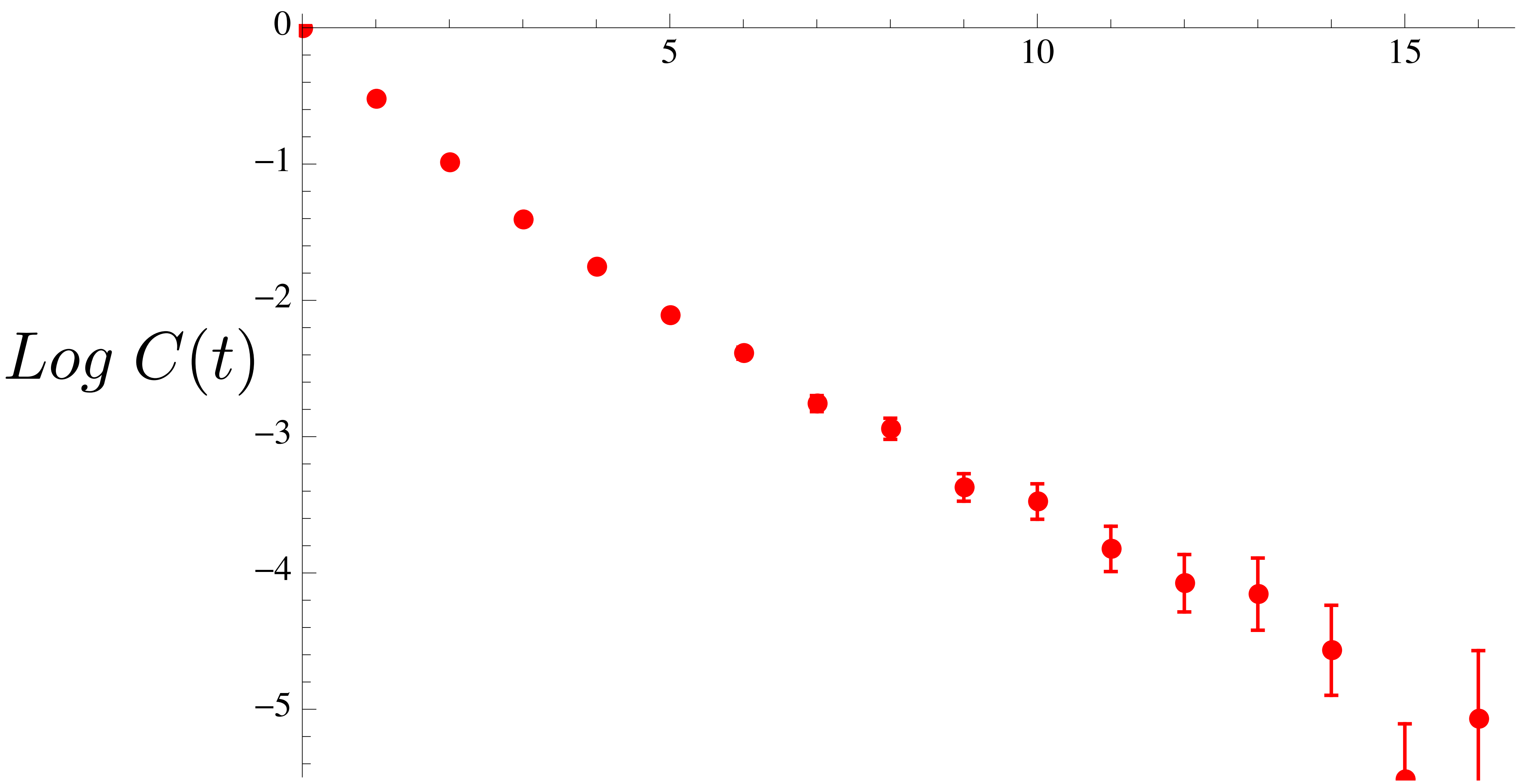

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1. Lattice QCD
2. Bound states, resonances, phase shifts
3. Examples

Lattice spectroscopy

How to obtain and understand
the energy spectrum?

Ground state spectroscopy

Is correct only for stable particles -
most hadrons decay -
when can we believe mass results ?

Resonances and bound states

require inclusion of hadron-hadron
channels in the calculation

Finite volume

leads to discretised momenta-
smallest $k=2\pi/L$
(for L=3 fm k=400 MeV)

Variational analysis

Lüscher, Wolff: NPB339(90)222
Michael, NPB259(85)58
See also Blossier et al., JHEP0904(09)094

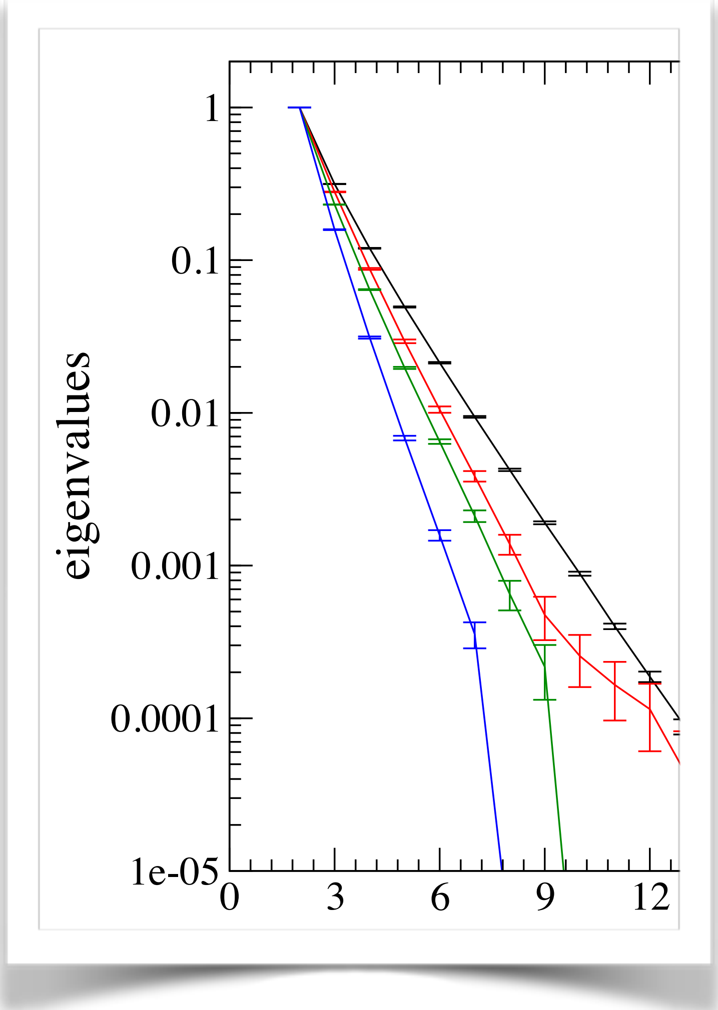
- Compute all cross-correlations for several interpolators

$$C_{ij}(t) = \langle X_i(t) X_j^\dagger(0) \rangle$$
$$= \sum_n \langle X_i(t)|n\rangle e^{-t\,E_n} \langle n|X_j^\dagger(0)\rangle$$

- Solve the eigenvalue problem. The eigenvalues give the energy levels (masses):

$$\lambda^{(n)}(t) \propto e^{-t\,E_n}\left(1 + \mathcal{O}(e^{-t\Delta E_n})\right)$$

- The eigenvectors are "fingerprints" of the state and allow to identify the "composition" of the state



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Z_i^n "overlap"

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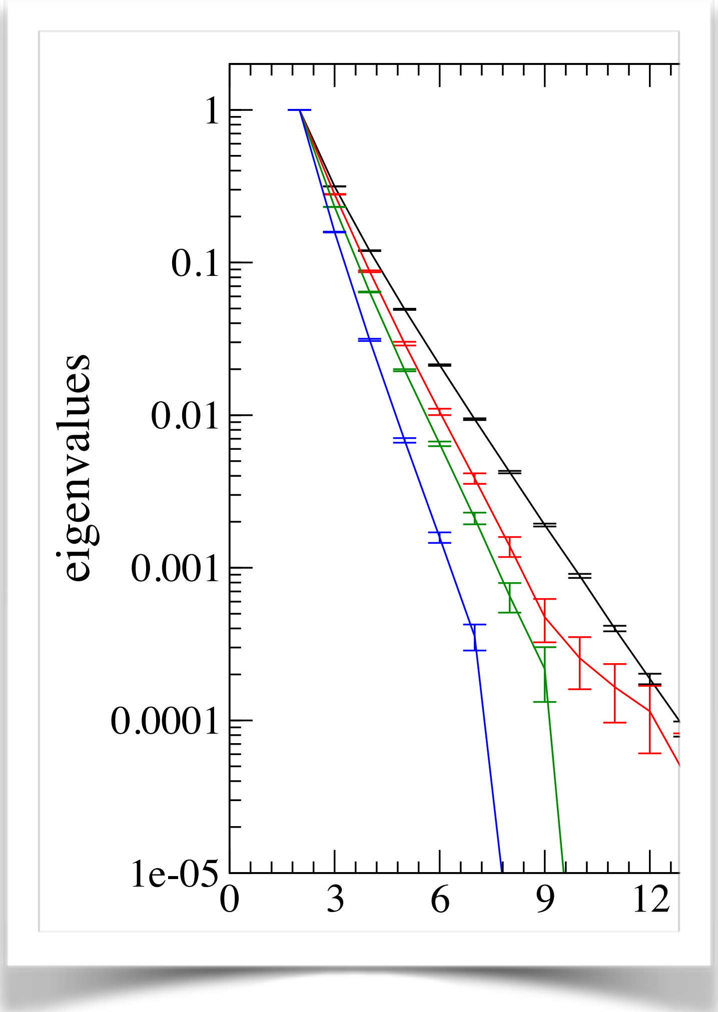


Interpolators and propagators $\langle X_i(t) X_j^\dagger(0) \rangle$

$\boxed{X_i(t)}$

- should have the correct quantum numbers (but: lattice symmetry mixes diff. angular momenta)

- should build a complete set of operators in the quantum channel

- “single hadron approximation”: use only interpolators of type $\boxed{\bar{q}\,q}$ and $\boxed{q\,q\,q}$
- 2-hadron interpolators of type $\boxed{\bar{q}\,q\,\bar{q}\,q}$ and $\boxed{\bar{q}\,q\,q\,q\,q}$
- coupled channels

Example:

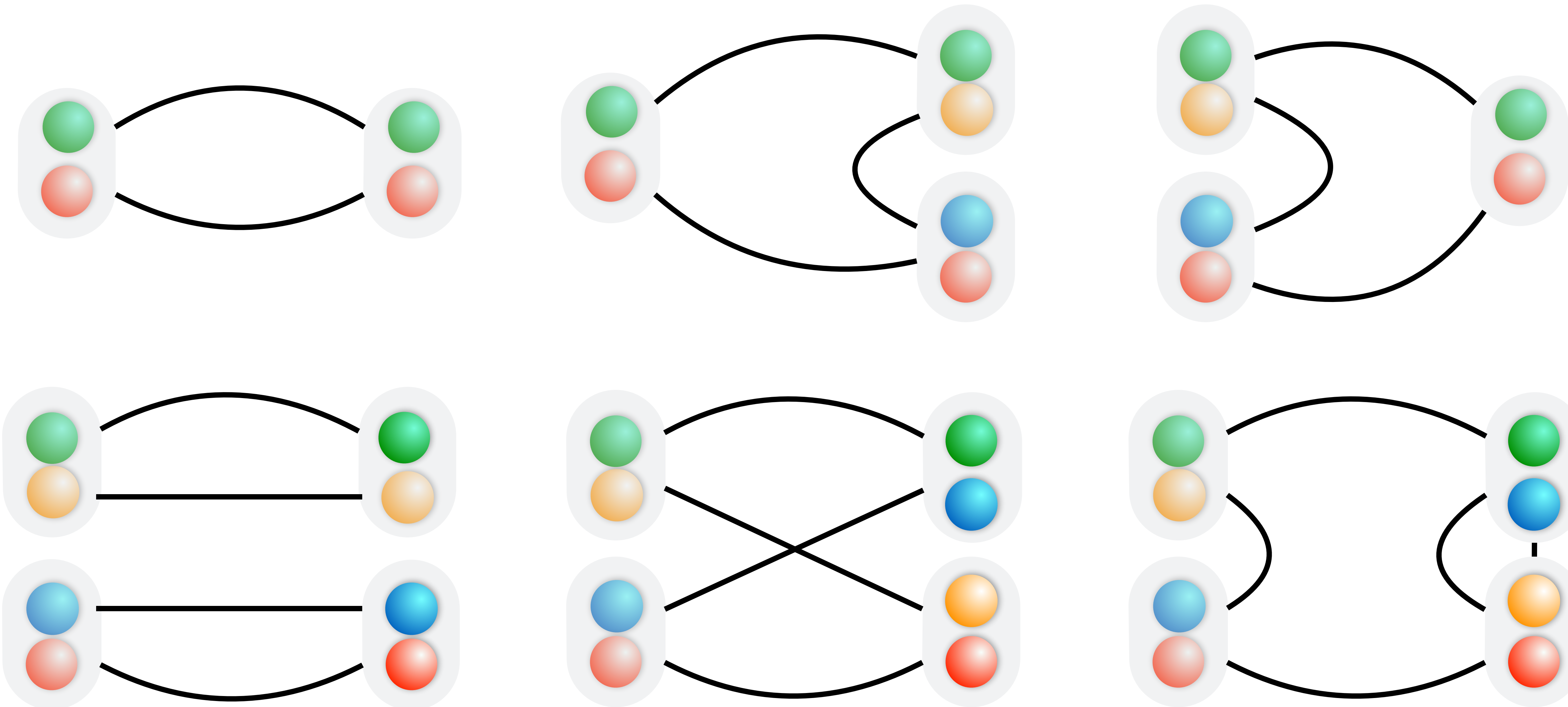
$\boxed{N}$

$\boxed{N,\ N\pi}$

$\boxed{N,\ N\pi,\ N\sigma,\ N\pi\pi}$

Wick contractions

for $(M, M_1 M_2) \longleftrightarrow (M, M_1 M_2)$



Energy levels and phase shift

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237

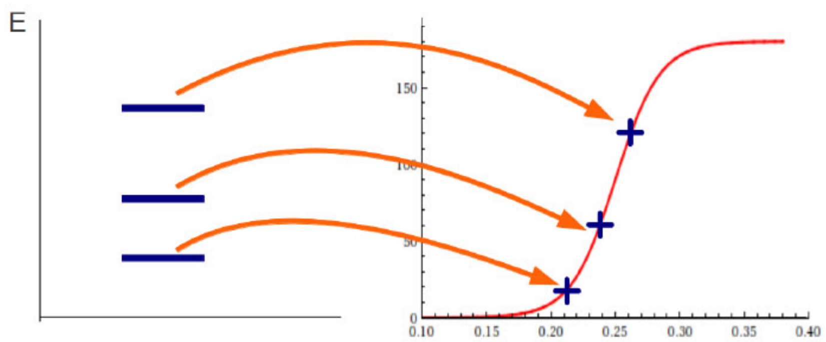
2d resonance example:
Gattringer & cbl, NPB391 (93) 463

Example:
One (periodic) dimension

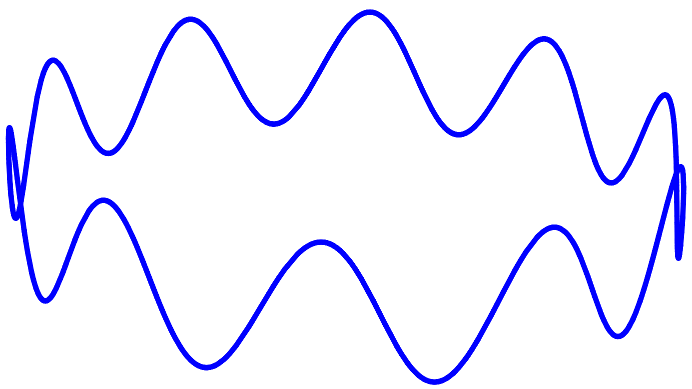
$$e^{ikL+2i\delta(k)} = 1$$

$$k_n L + 2\delta(k_n) = 2n\pi$$

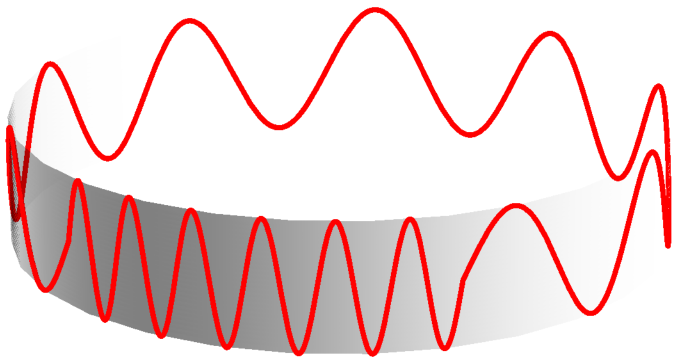
Discrete momentum → discrete energy
→ phase shift value at that energy



$$k_n \rightarrow \delta(k_n)$$



$$V = const.$$
$$\delta = 0$$



$$V = localized$$
$$\delta \neq 0$$

Waves and Scattering in a Finite Volume

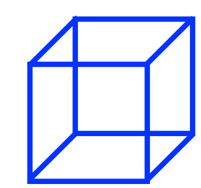
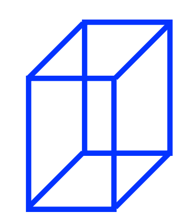
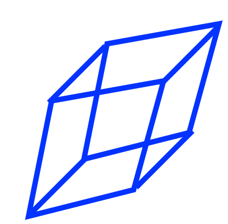
Elastic regime, rest frame:

$$\cot\delta(q) = \frac{\mathcal{Z}_{00}(1;q^2)}{\pi^{3/2}q}$$

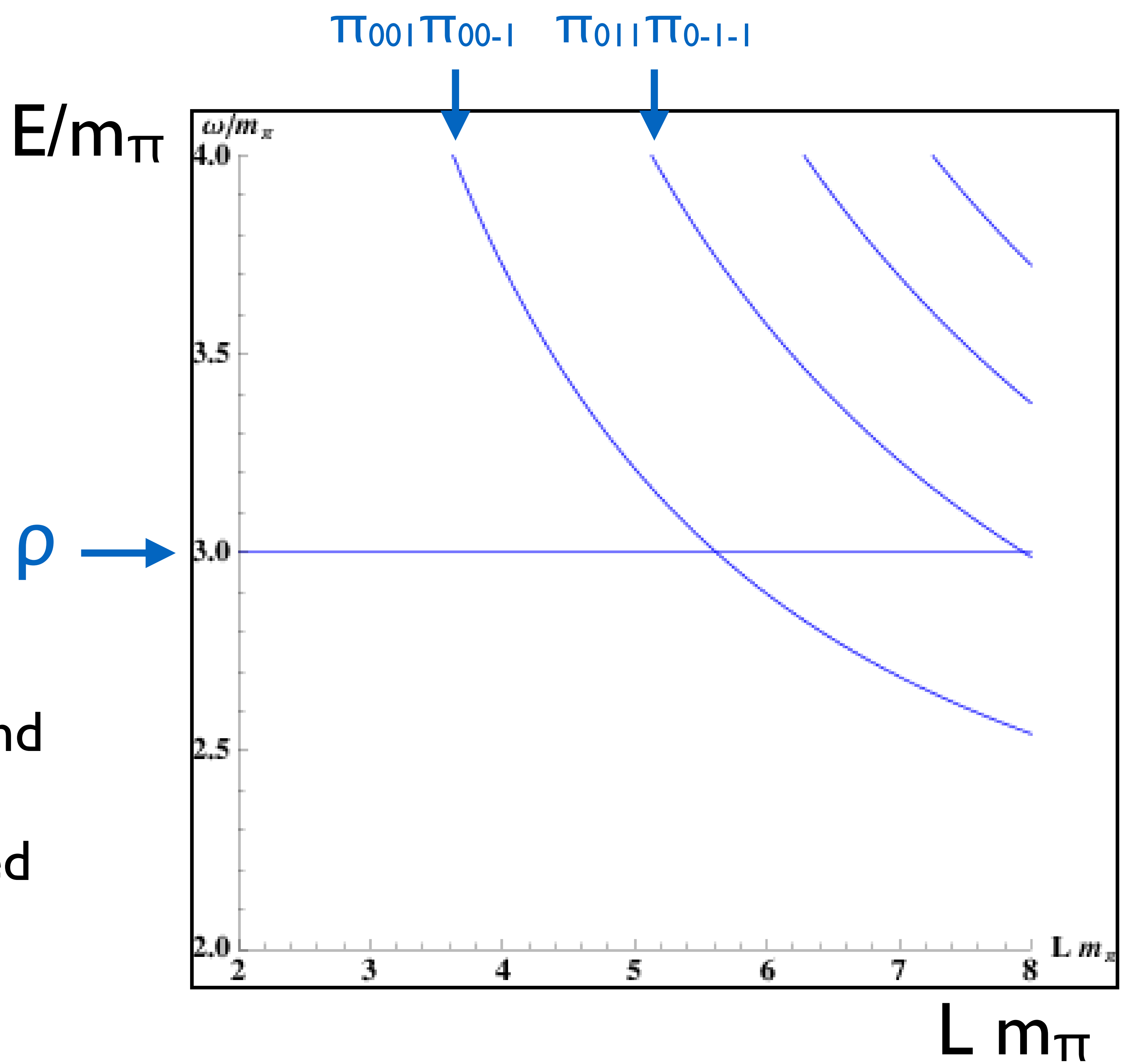
Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237

Moving frames:

Rummukainen, Gottlieb: NP B 450(1995) 397
Kim, Sharpe: NP B 727 (2005) 218
Feng, Jansen, Renner: PoS LAT10 (2010) 104
Fu, PR D85 (2012) 014506
Leskovec, Prelovsek, PR D85 (2012) 114507
Göckeler et al., PR D 86, 094513 (2012)
Döring et al., Eur.Phys.J.A48 (2012)114

	Relativistic distortion	Symmetry group
$\vec{p}=(0,0,0)$		O_h
$\vec{p}=(0,0,1)$		D_{4d}
$\vec{p}=(1,1,0)$		D_{2d}

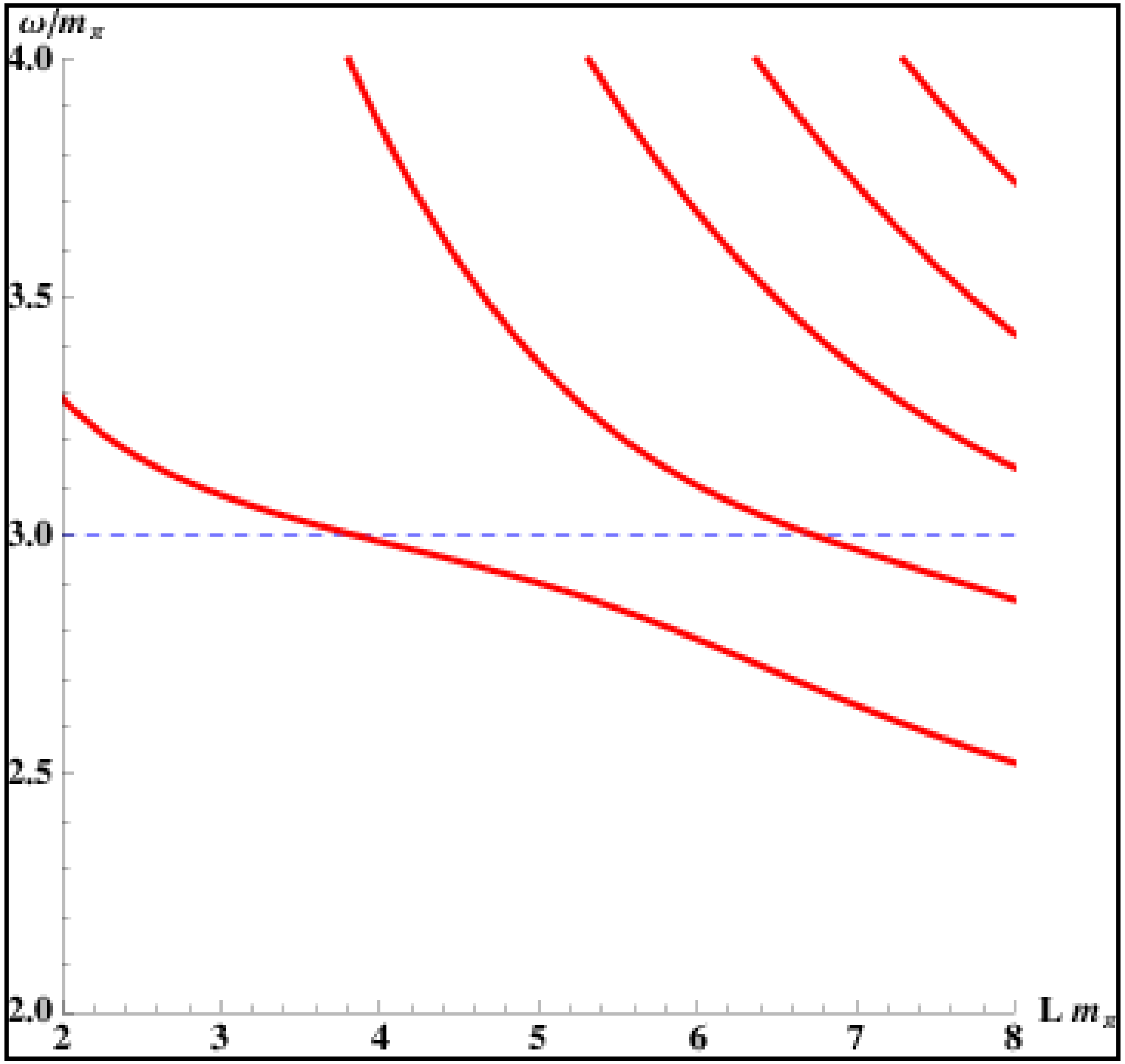
Energy Levels and Phase Shifts



The energy levels depend on the spatial volume.
Resonance region: avoided level crossing

Energy Levels and Phase Shifts

E/m_π



L m_π

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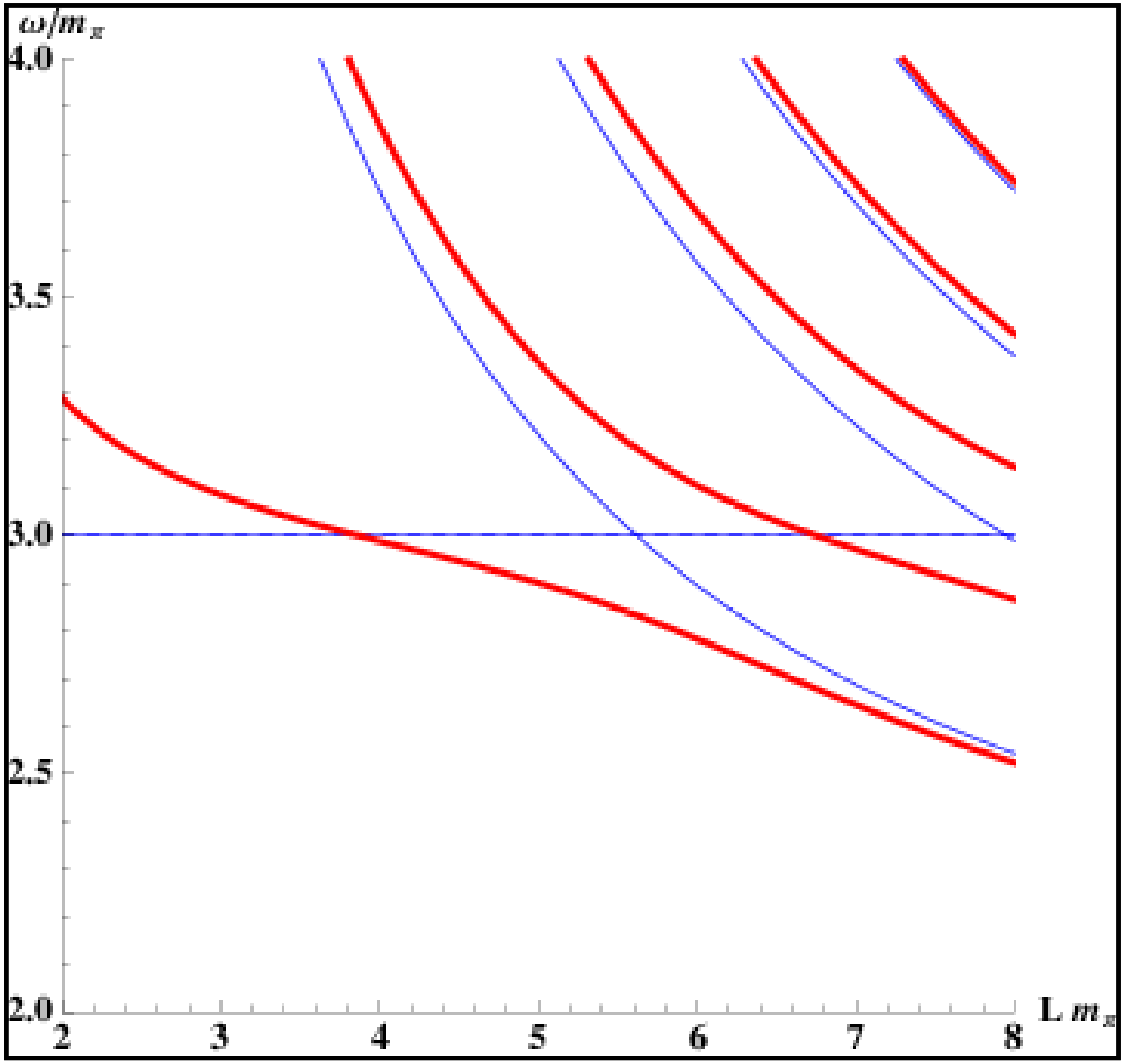
Energy Levels and Phase Shifts

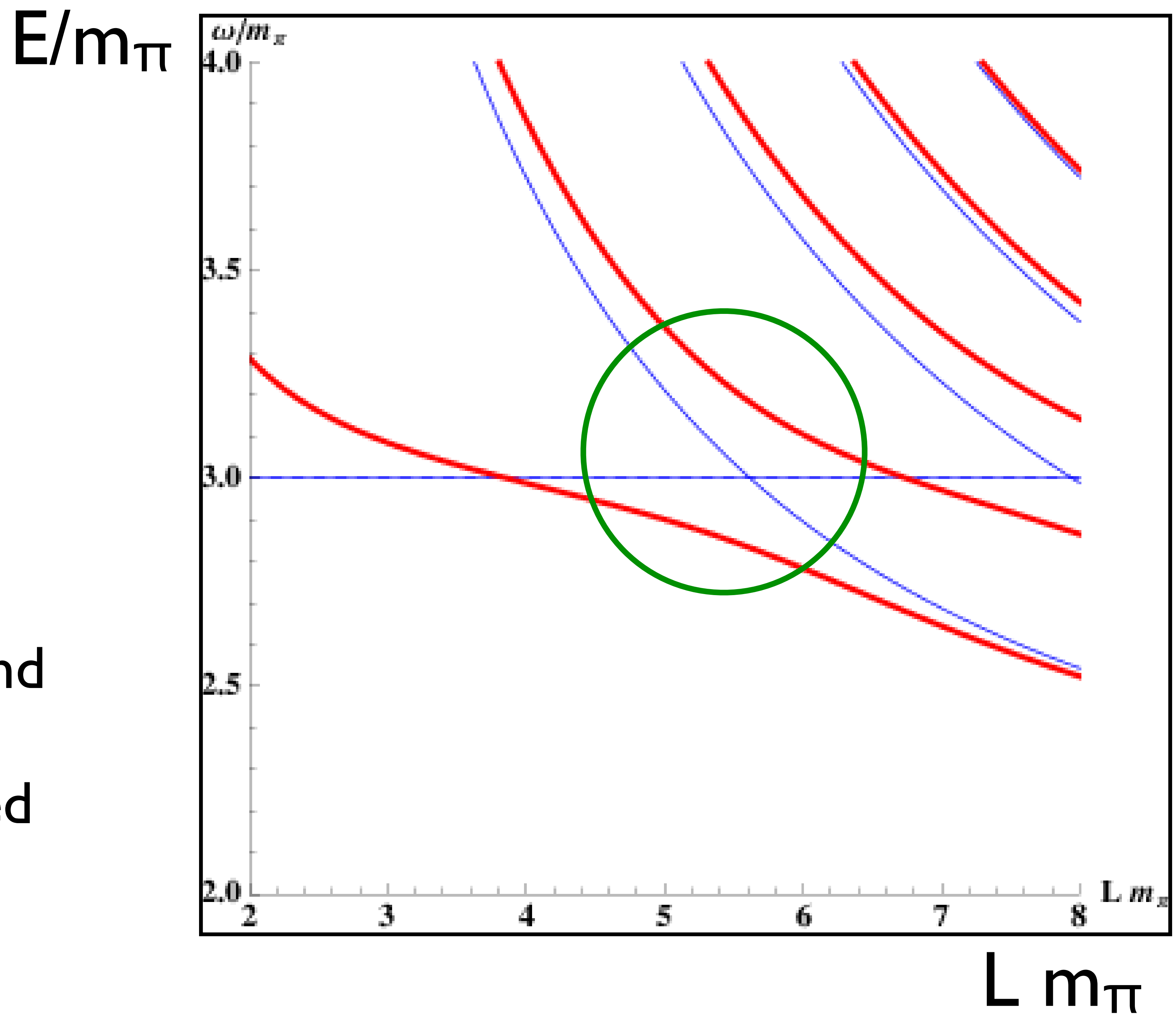
E/m$_\pi$



L m$_\pi$

The energy levels depend on the spatial volume.
Resonance region: avoided level crossing

Energy Levels and Phase Shifts



The energy levels depend on the spatial volume.
Resonance region: avoided level crossing

Near threshold

$$T^{-1} = \begin{cases} K^{-1} - i\,p & \text{for } p^2 > 0 \\ K^{-1} + |p| & \text{for } p^2 < 0 \end{cases}$$

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237

$$K^{-1} = \frac{2\mathcal{Z}_{00}(1; (\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

$$\approx \frac{1}{a_0} + \frac{1}{2} r_0\, p^2$$

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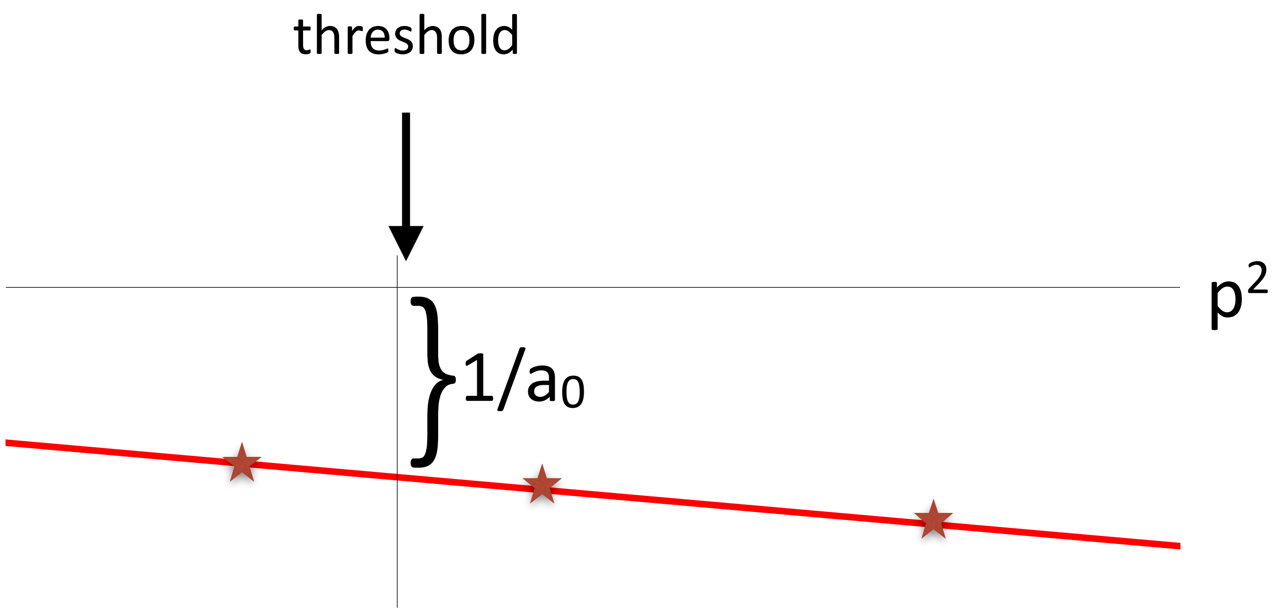

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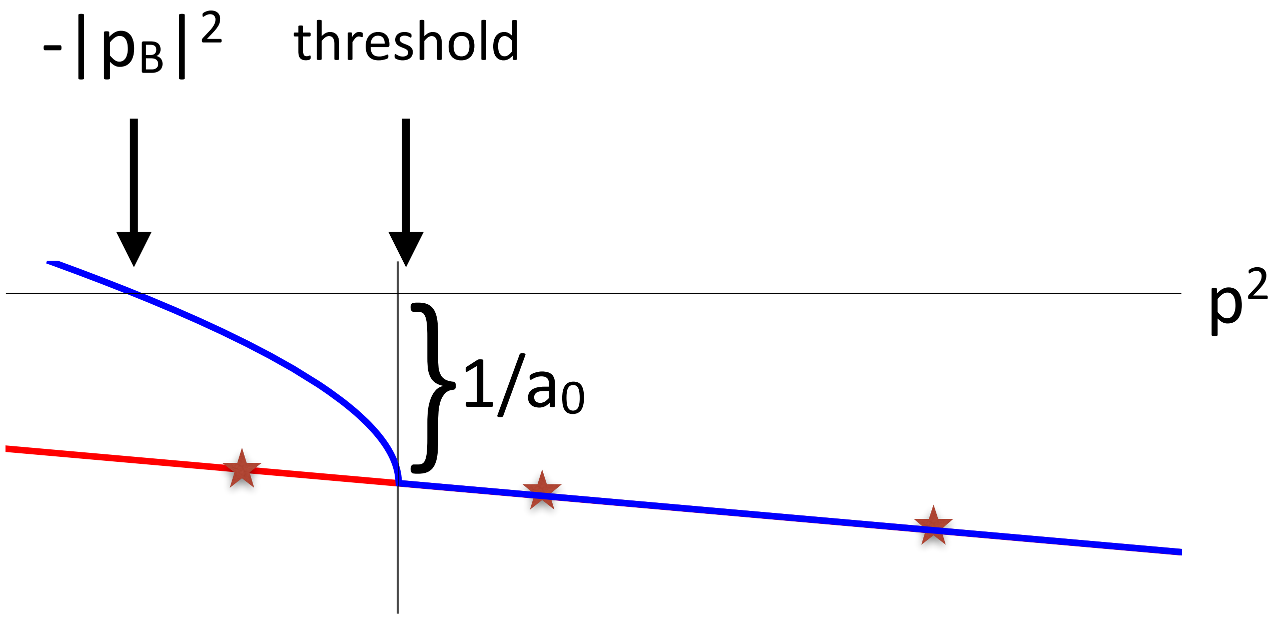

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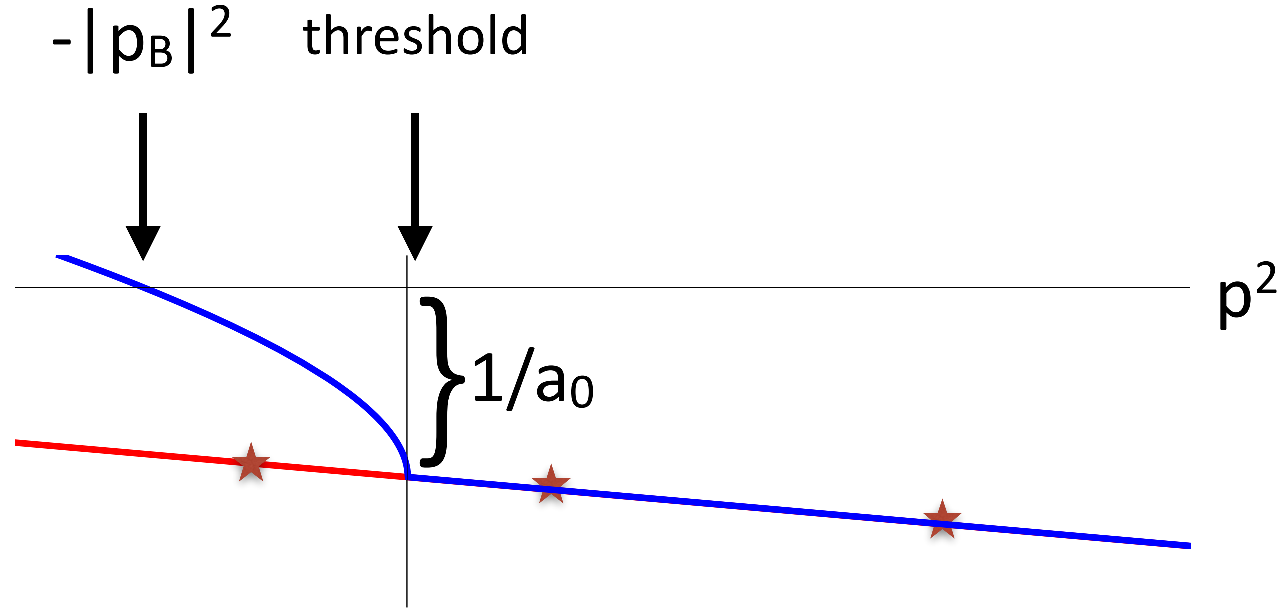
bound state



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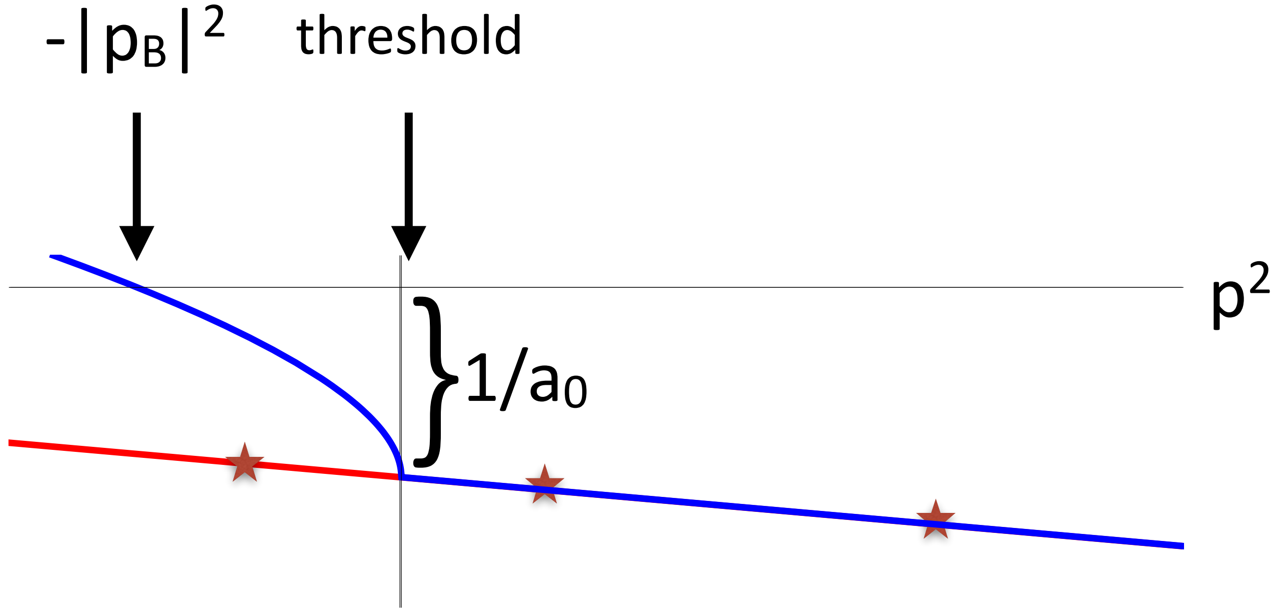
$$K^{-1} = p\,\cot\delta(p) \quad \text{for } p^2 > 0$$

bound state



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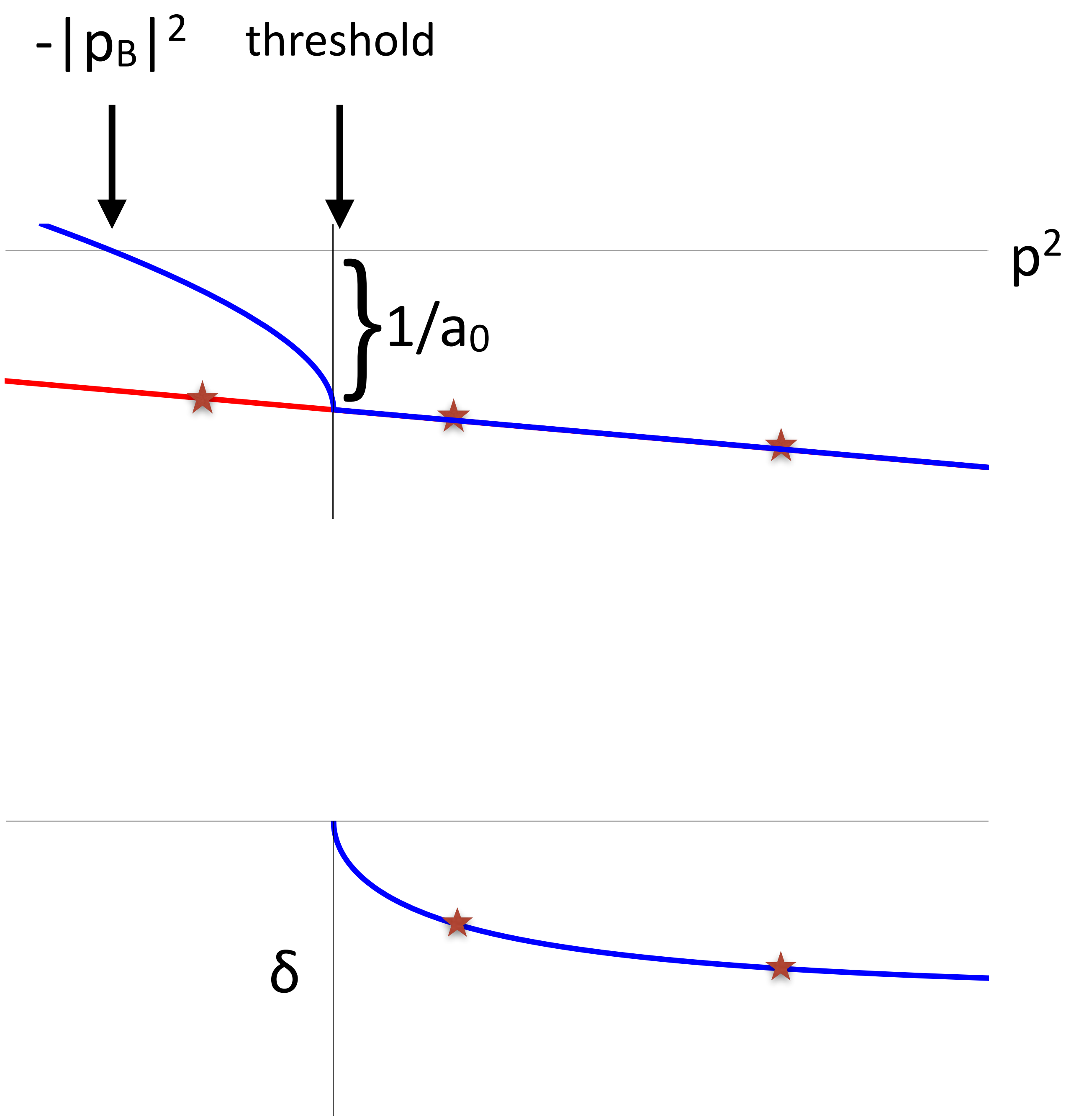

1. Lattice QCD
2. Bound states, resonances, phase shifts
3. Examples

Example for equal masses: ππ p-wave (ρ)

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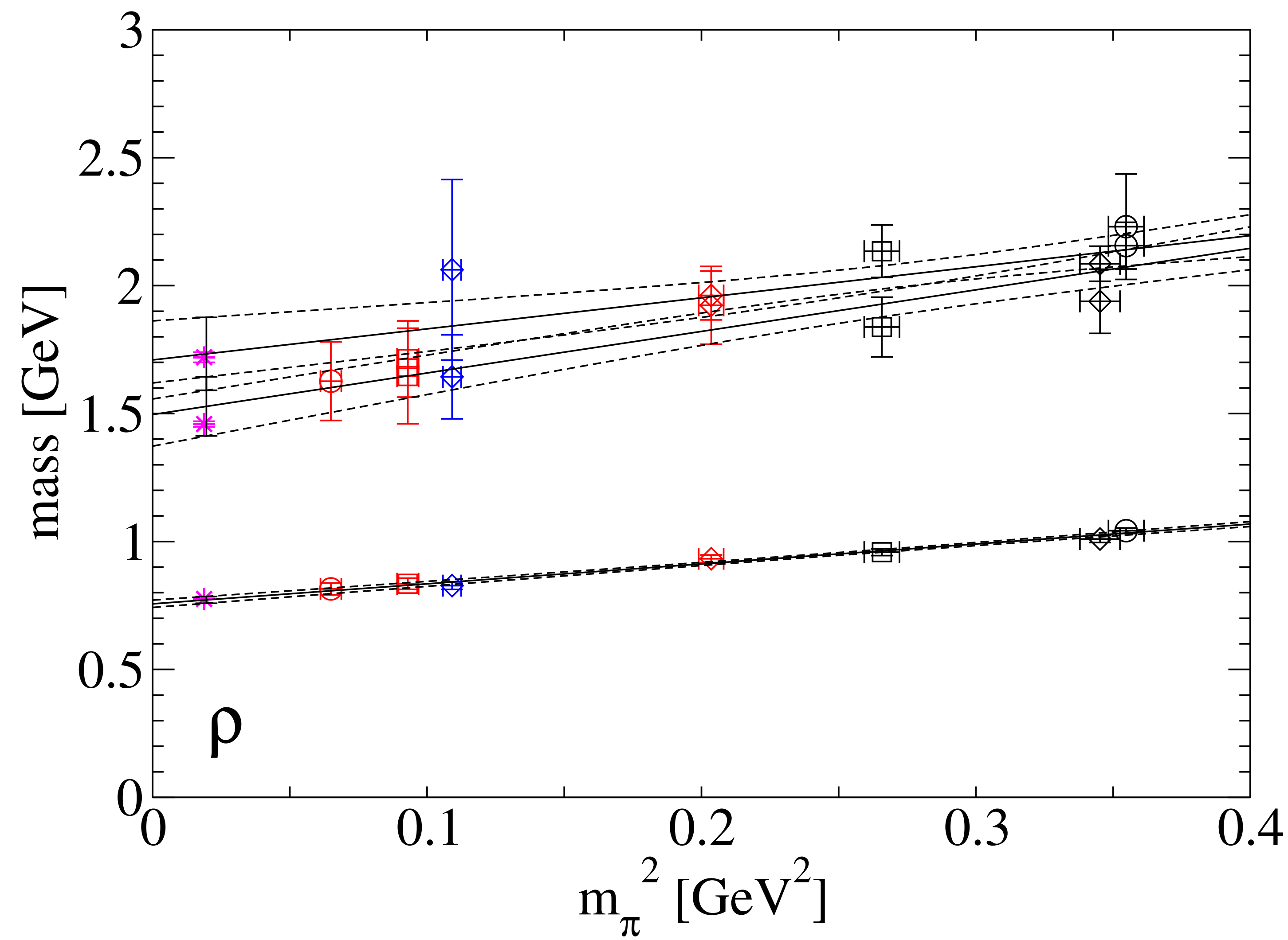
Moving frames and
the prototype resonance: ***ρ***
elastic and elegant

Example for equal masses: ππ p-wave (ρ)

In the traditional single hadron approach the ρ seems to be stable: no ππ signals are found!

Traditional approach: single hadron operators (qq)

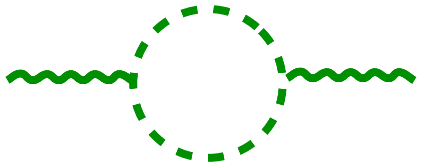


Engel et al. PRD 85 (2012) 034508

Example for equal masses: ππ p-wave (ρ)

In the traditional single hadron approach the ρ seems to be stable: no ππ signals are found!

Where is the ππ state?



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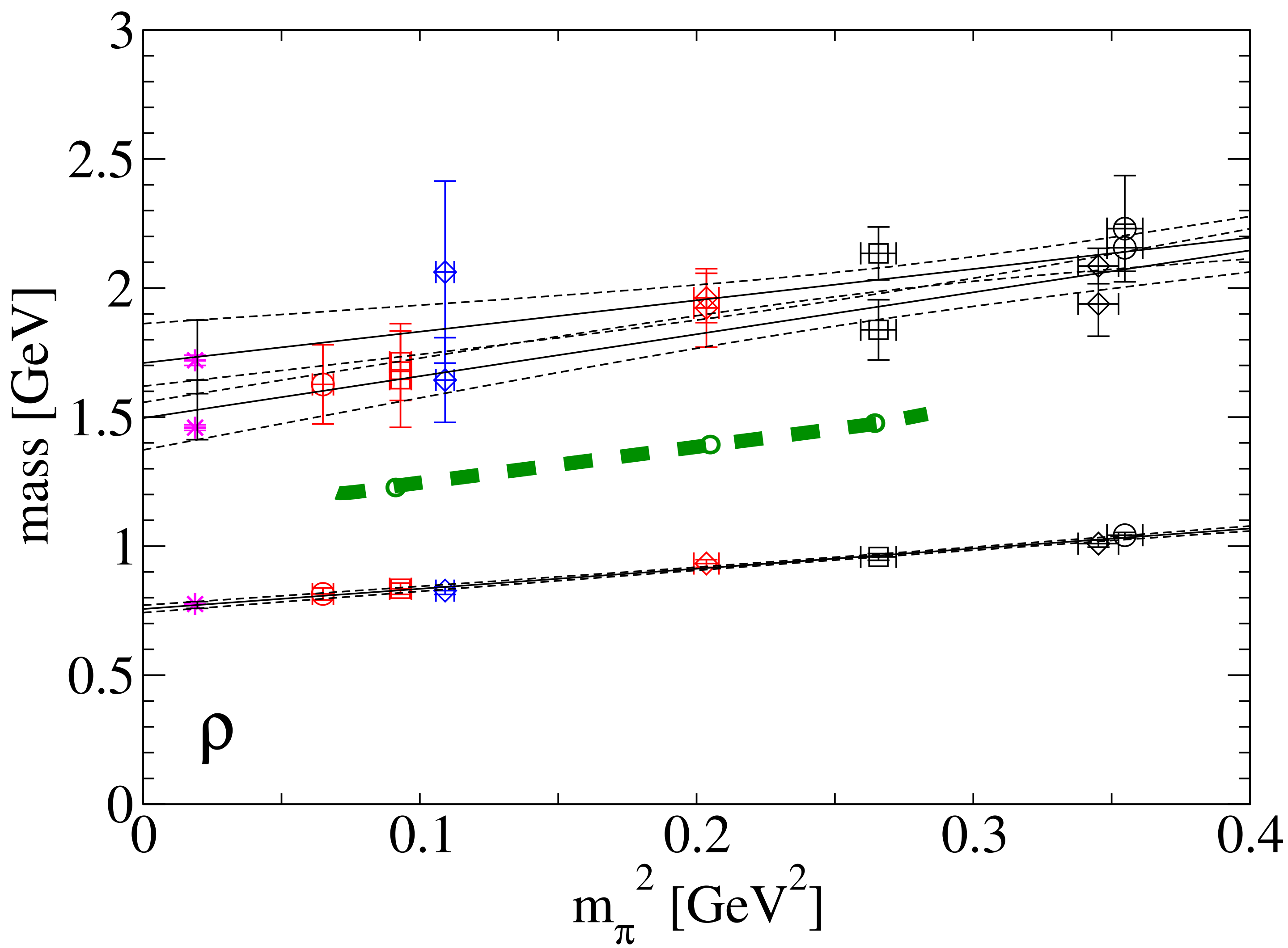


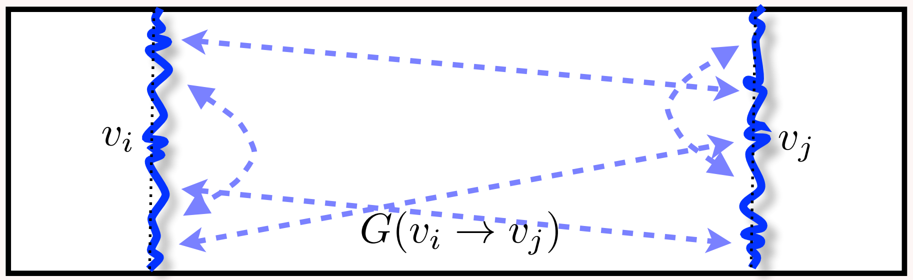
Engel et al. PRD 85 (2012) 034508

Configurations

- Ensemble 1:
  - Hasenfratz et al., PRD 78, 014515 & 054511 (2008)
  - n_f=2 Wilson improved, 4 nHYP
  - 16^3x32, L_x=2 fm, 279 configs.
  - m_π=266 MeV, m_K=552 MeV

Propagators

- Distillation
  - HSC, Peardon et al., PRD80, 054506 (2009)
  - n_v=96
  - perambulators



$$\tau_{ij}^{\overline{\alpha}\overline{\beta}}(t', t) = v_i^*(t')\, G^{\overline{\alpha}\overline{\beta}}(t'; t)\, v_j(t)$$

$$\langle M(t')\, M^\dagger(t)\rangle = -\mathrm{tr}\,[\phi(t')\tau(t', t)\phi(t)\tau(t, t')]$$

Interpolators

Up to 18 ρ and ππ operators
P=(000), (001), (011)

$$\mathcal{O}_1^s(t) = \sum_{\mathbf{x},i} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\ A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ u_s(x)\ -\{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_2^s(t) = \sum_{\mathbf{x},i} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\ \gamma_t A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ u_s(x)\ -\{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_3^s(t) = \sum_{\mathbf{x},i,j} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\overleftarrow{\nabla}_j\ A_i\gamma_i\ \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ \overrightarrow{\nabla}_j u_s(x)\ -\{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_4^s(t) = \sum_{\mathbf{x},i} \tfrac{1}{\sqrt{2}}\ \bar{u}_s(x)\ A_i\ \tfrac{1}{2}[\mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\ \overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}]u_s(x)\ -\{u_s \leftrightarrow d_s\} \qquad (s=n,m,w)\ ,$$

$$\mathcal{O}_5^s(t) = \sum_{\mathbf{x},i,j,k} \tfrac{1}{\sqrt{2}}\ \epsilon_{ijl}\ \bar{u}_s(x)\ A_i\gamma_j\gamma_5\ \tfrac{1}{2}[\mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}\overrightarrow{\nabla}_l - \overleftarrow{\nabla}_l \mathrm{e}^{\mathrm{i}\mathbf{P}\mathbf{x}}]u_s(x) - \{u_s \leftrightarrow d_s\} \quad (s=n,m,w)\ ,$$

$$\mathcal{O}_6^{s=n}(t) = \tfrac{1}{\sqrt{2}}[\pi^+(\mathbf{p_1})\pi^-(\mathbf{p_2}) - \pi^-(\mathbf{p_1})\pi^+(\mathbf{p_2})]\ , \qquad \pi^\pm(\mathbf{p_i}) = \sum_{\mathbf{x}} \bar{q}_n(x)\gamma_5\tau^\pm \mathrm{e}^{\mathrm{i}\mathbf{p_i}\mathbf{x}} q_n(x)\ .$$

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... include ππ operator

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P=(000), (001), (011)

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... include ππ operator

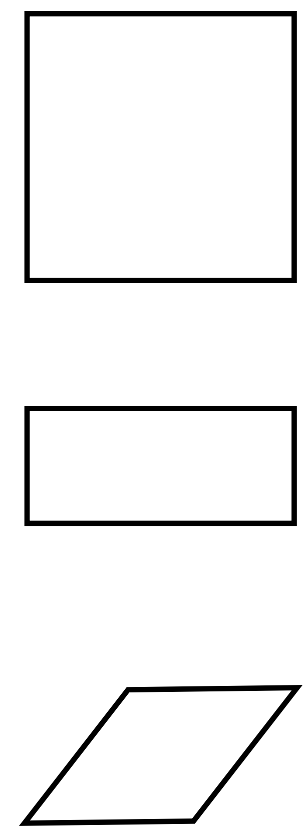
... and three quark widths (s, m, w)

Moving frames: more energy values

$\vec{p} = (0,0,0) \quad (\text{units} 2\pi/L)$

$\vec{p} = (0,0,1)$

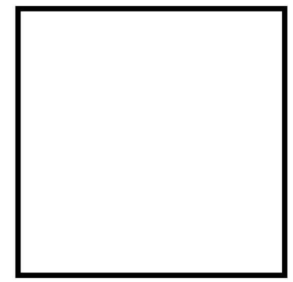
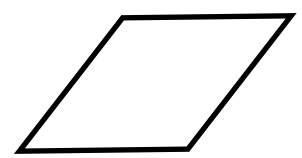
$\vec{p} = (1,1,0)$



Relativistic distortion

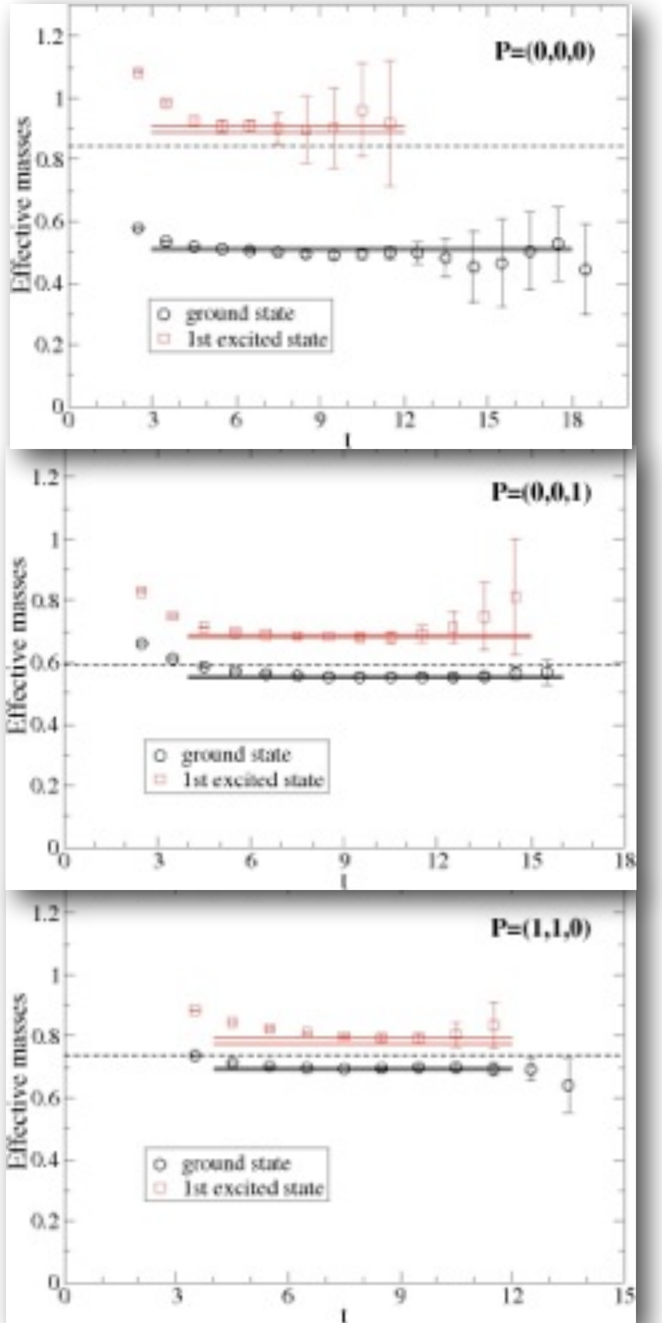
Moving frames: more energy values

and less symmetry…

	Relativistic distortion	Symmetry group	Irrep for ρ
$\vec{p} = (0,0,0) \quad (\text{units} 2\pi/L)$		O_h	T_1^-
$\vec{p} = (0,0,1)$		D_{4d}	A_2^-
$\vec{p} = (1,1,0)$		D_{2d}	B_1^-

ρ meson

CBL, Mohler, Prelovsek, Vidmar;
Phys. Rev. D **84**, 054503 (2011)
Erratum Phys. Rev. D 89 (2014) 059903(E)



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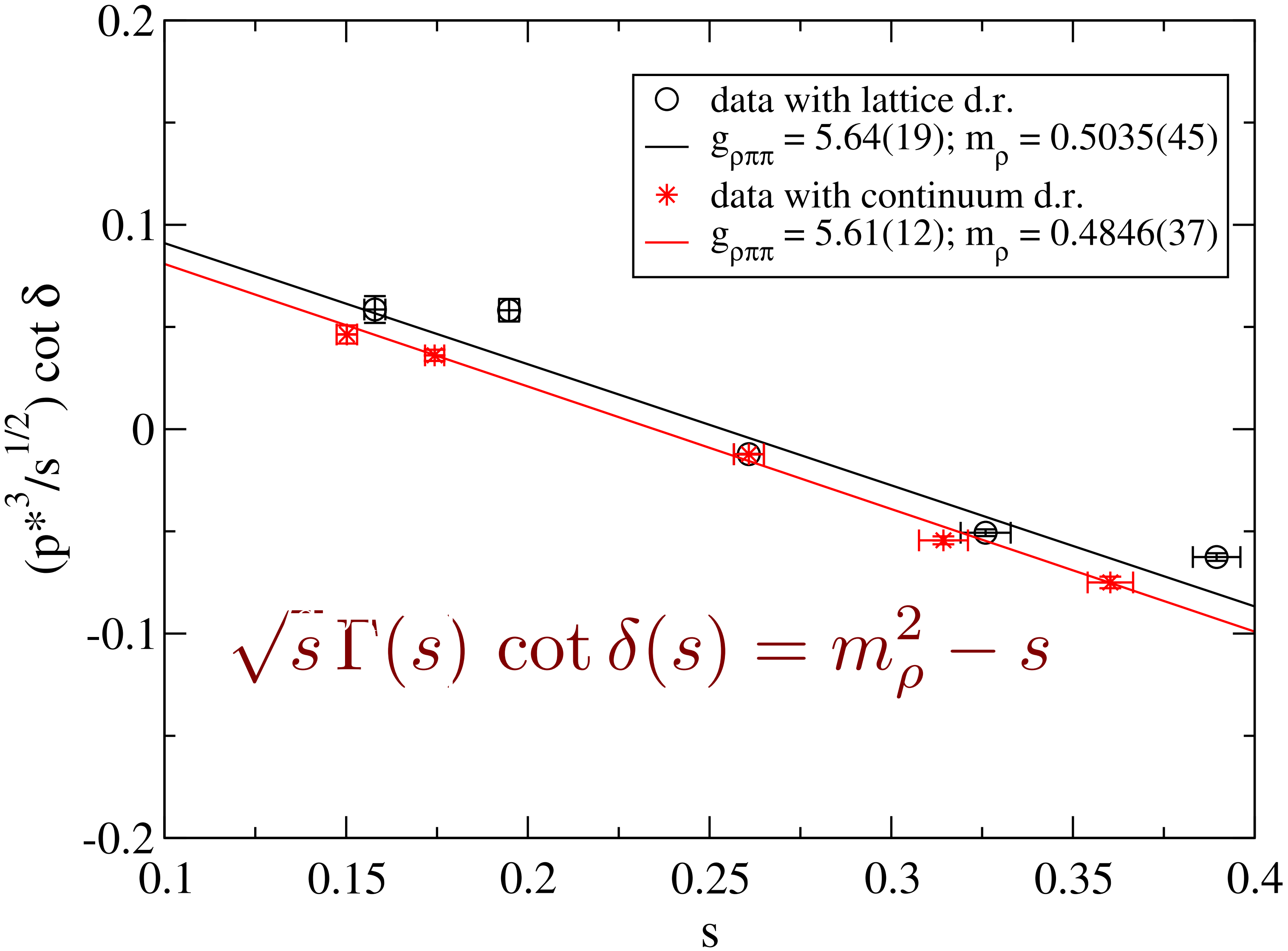
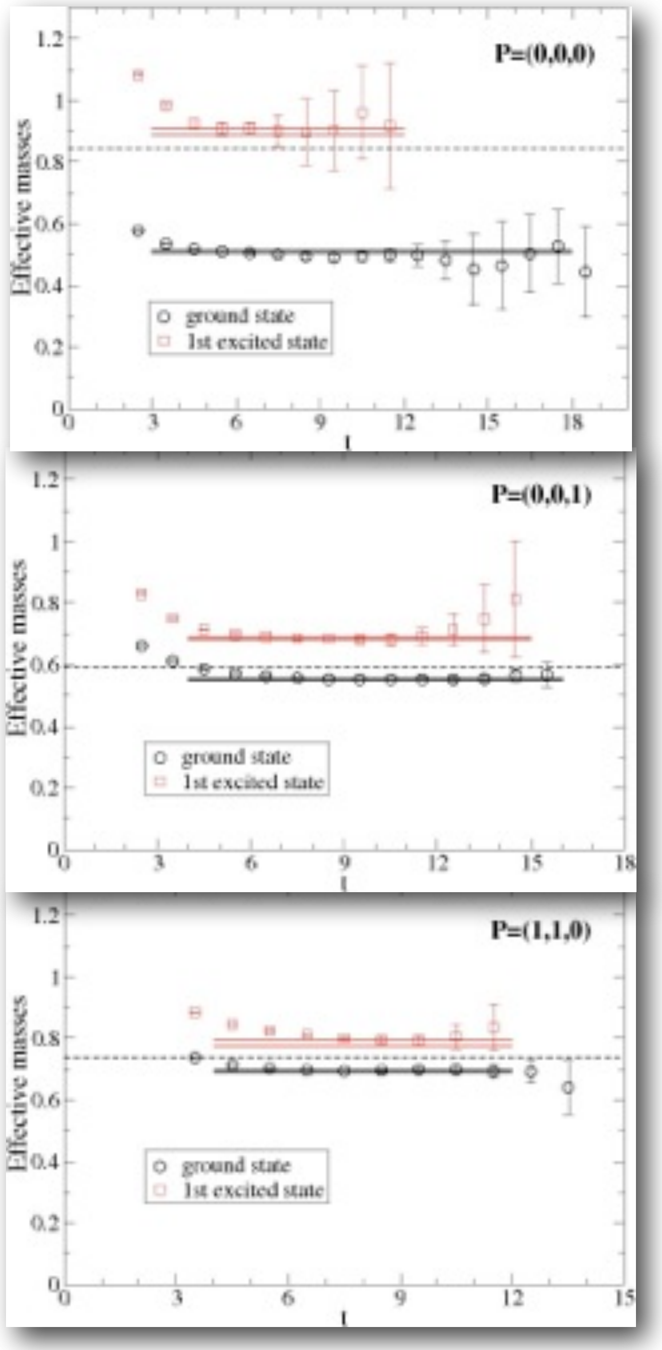
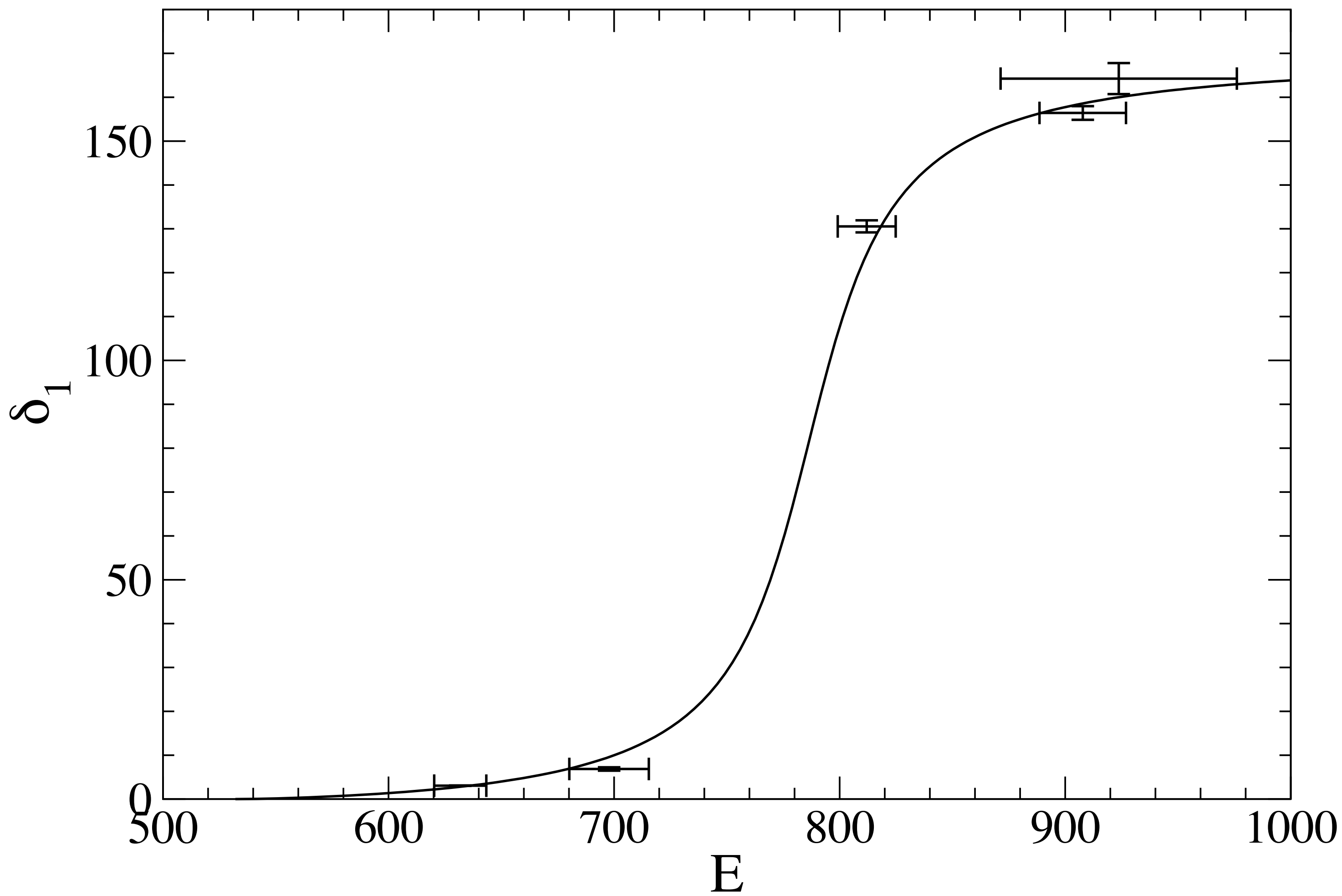


$$\sqrt{s}\,\Gamma(s)\,\cot\delta(s) = m_\rho^2 - s$$



Phase shift: ρ meson

CBL, Mohler, Prelovsek, Vidmar;
Phys. Rev. D **84**, 054503 (2011)
Erratum Phys. Rev. D 89 (2014) 059903(E)



$$g_{\rho\pi\pi} = 5.61(12)$$

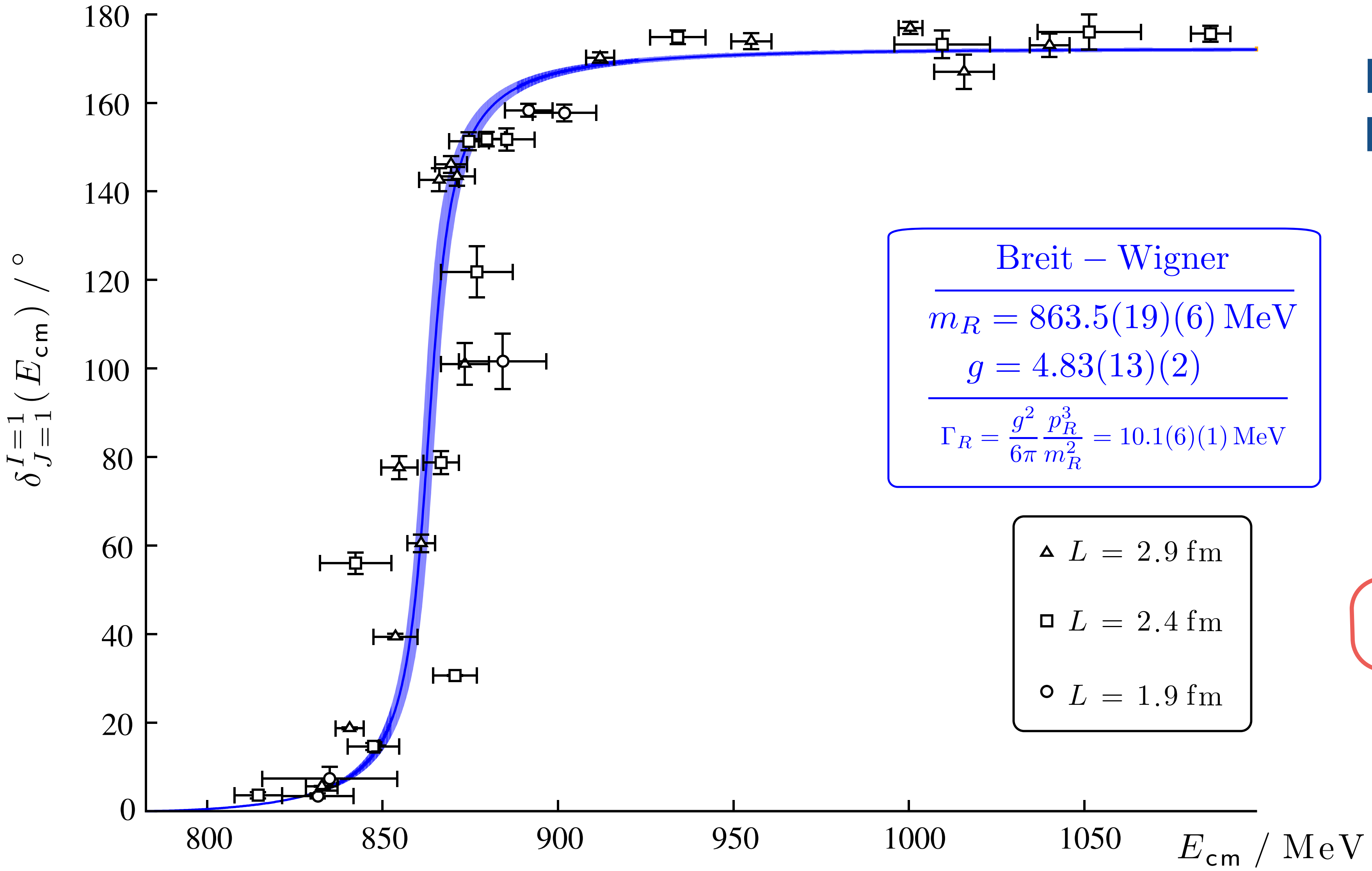
$$\boxed{m_\pi = 266(3)(3)\,\mathrm{MeV}}$$

$$m_\rho = 772(6)(8)\,\mathrm{MeV}$$

$$g_{\rho\pi\pi,exp} = 5.96$$

See also:
Aoki et al. (PACS-CS), PoS LAT10(10)108
Feng et al. (ETMC) PoS LAT10(10)104
Frison et al. (BMW, PoS LAT10(10)139
Dudek, Edwards, Thomas, Phys.Rev. D 87, 034505 (2013)

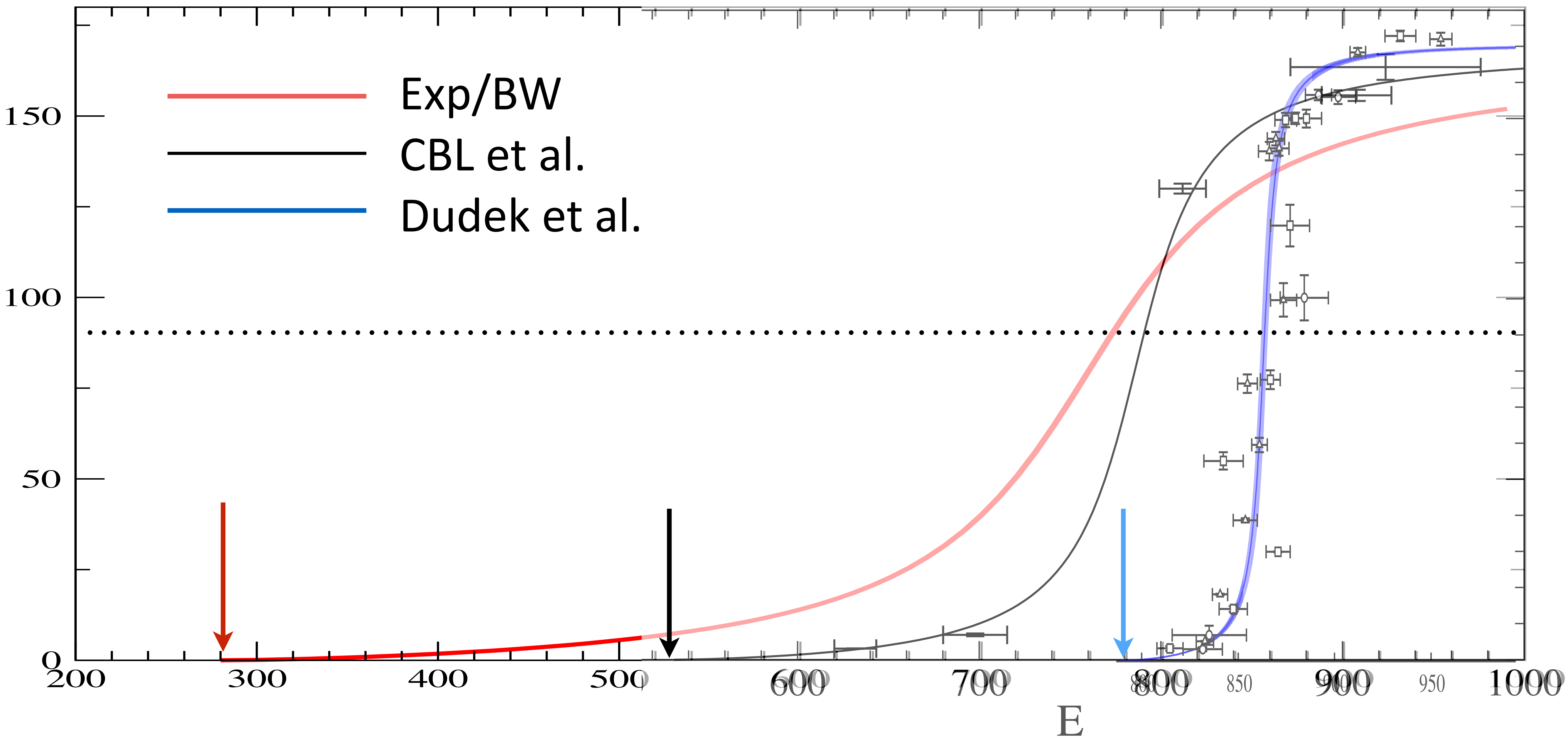
Further progress



Dudek, Edwards, Thomas, PR D87, 034505 (2013).

Different volumes
several momenta
m_π=391 MeV

Comparison with BW-fit to experiment



Unequal masses: Kπ

s-wave and p-wave
total momentum P=(000), (001), (011)
(m_π=266 MeV, m_K=552 MeV)

Coupled system of 5 $\bar{q}q$ and 3 Kπ operators:

$$\mathcal{O}^{\bar{q}q}_{I=1/2} = \sum_x e^{iPx}\bar{s}(x)\hat{\Gamma}u(x) \ ,$$

5 Operators
(n_v=96)

$$\mathcal{O}^{K\pi}_{I=1/2} = \sum_j f_j[\sqrt{\tfrac{1}{3}}K^+(p_{Kj})\pi^0(p_{\pi j}) \ ,$$
$$+ \sqrt{\tfrac{2}{3}}K^0(p_{Kj})\pi^+(p_{\pi j})] \ ,$$

3 Operators,
various momenta
(units 2π/L), (n_v=64)

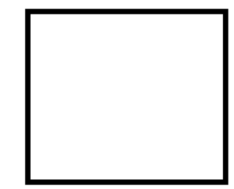
$$\mathcal{O}^{K\pi}_{I=3/2} = \sum_j f_j \ K^+(p_{Kj})\pi^+(p_{\pi j}), \ \ p_{Kj} + p_{\pi j} = P$$

3 Operators,
various momenta
(units 2π/L), (n_v=64)

Moving frames

$P = 0:$ $\quad O_h\,,\ \text{irrep } T_1^-\,,$ $\quad l = 1$

$$\tan\delta(q) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}$$

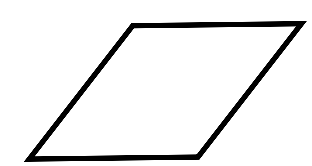


$P = \frac{2\pi}{L} e_z:$ $\quad C_{4v}\,,\ \text{irreps } E(e_{x,y}),\ E(e_x \pm e_y),$ $\quad l = 1, 2$

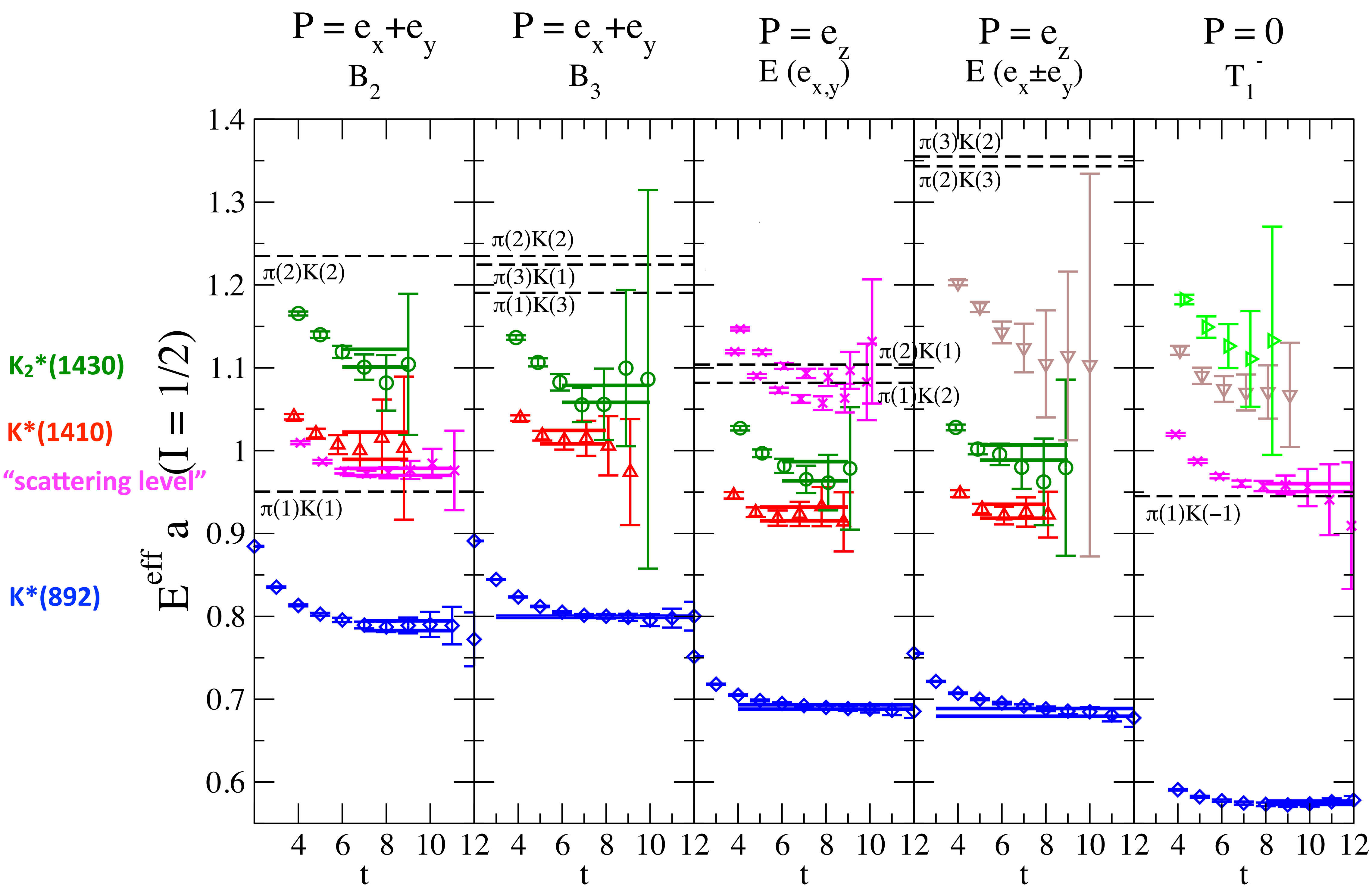
(irrep A_1 mixes $l = 0, 1 \ldots$)

Fu & Fu, PRD86 (2012) 094507

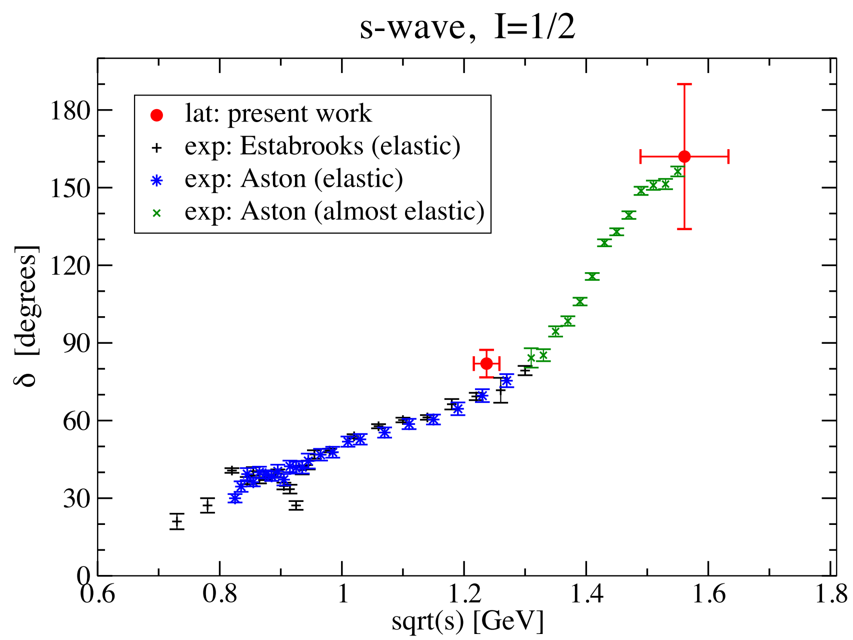
$P = \frac{2\pi}{L}(e_x + e_y):$ $\quad C_{2v}\,,\ \text{irreps } B_2,\ B_3,$ $\quad l = 1, 2$

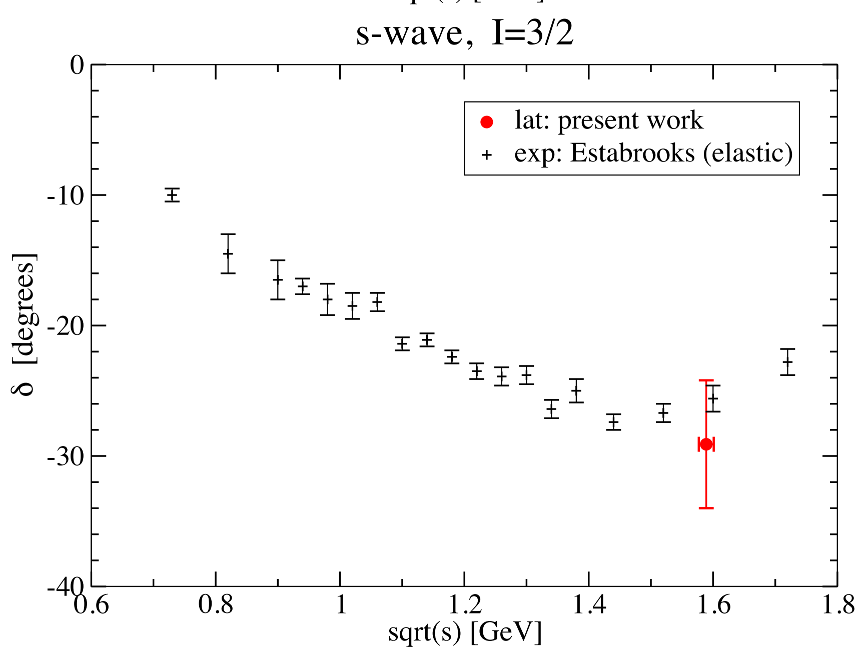


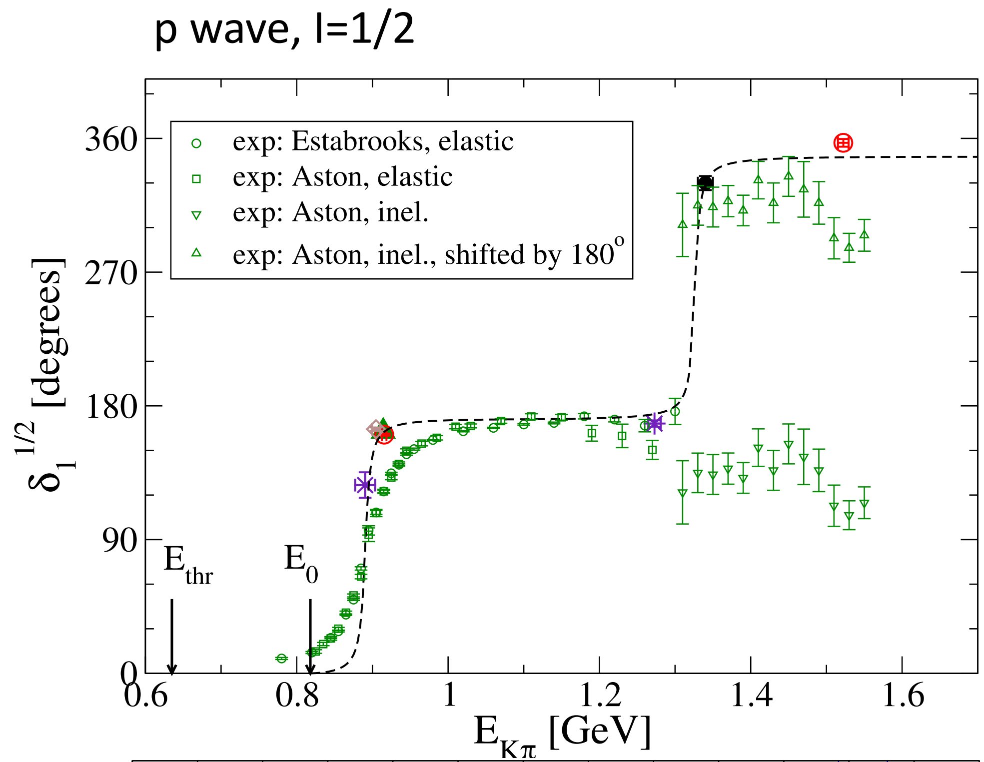
Effective energies (I=1/2)

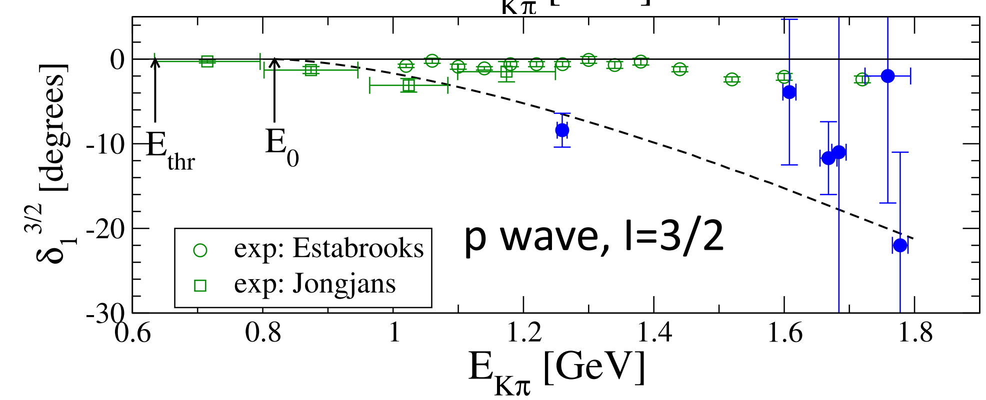


Kπ scattering and the K* width









	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]	$m_{K^*(1410)}$ [GeV]	$g_{K^*(1410)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6	1.33 ± 0.02	input
exp	891.66 ± 0.26	5.72 ± 0.06	1.414 ± 0.0015	1.59 ± 0.03

Lang, Leskovec, Prelovsek, Mohler,
Phys. Rev. D86 (2012) 054508; arXiv:1207.3204
Phys. Rev. D88 (2013) 054508, arXiv: 1307.0736
and PoS Lattice2013 (2013) 260; arXiv:1310.4958

see also the recent coupled πK, ηK study:
Dudek et al., arXiv 1406.5148

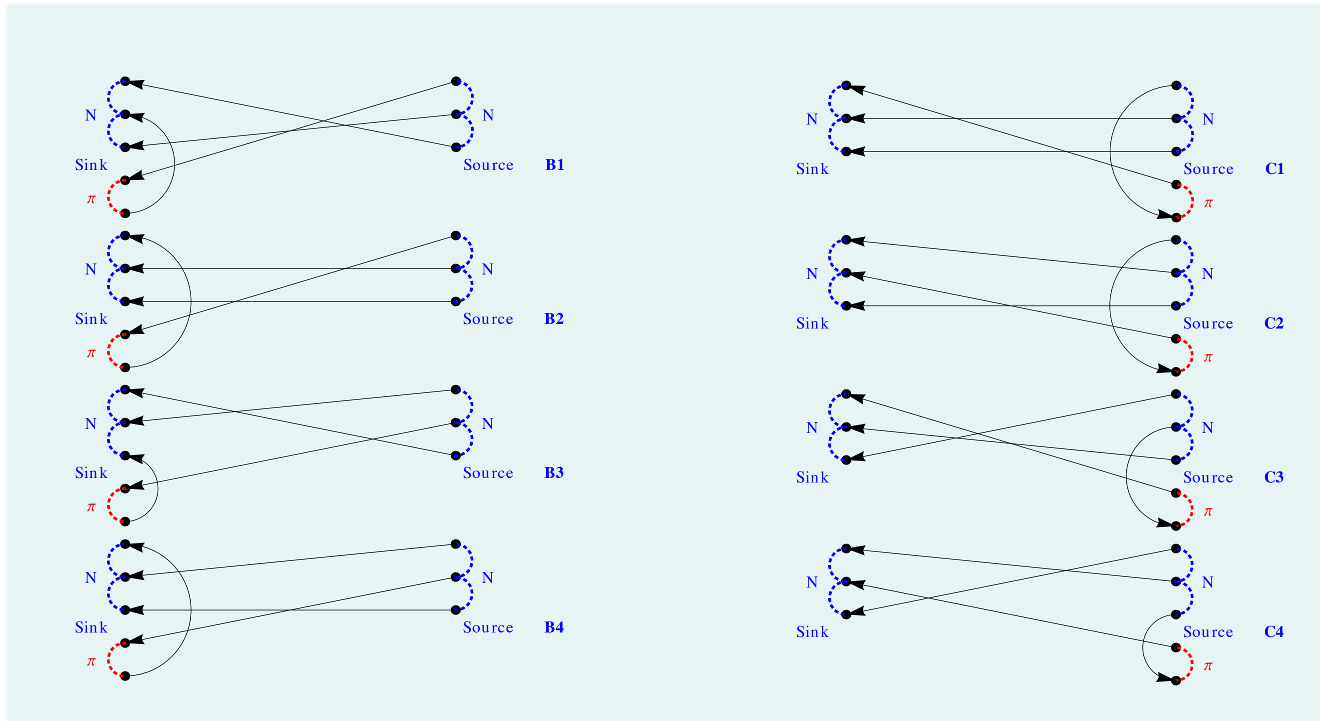
Baryons: Nπ

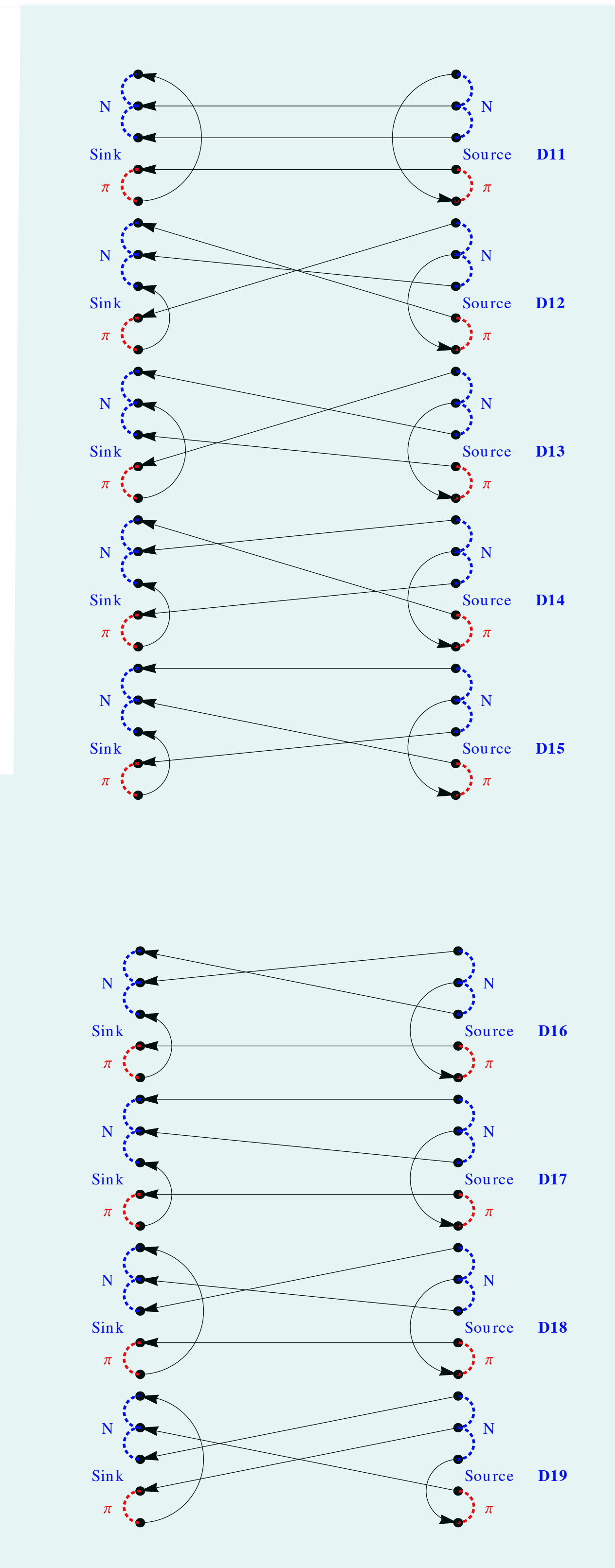
Baryons: Nπ

this the real stuff:
5 quark operators
and 29 contraction terms

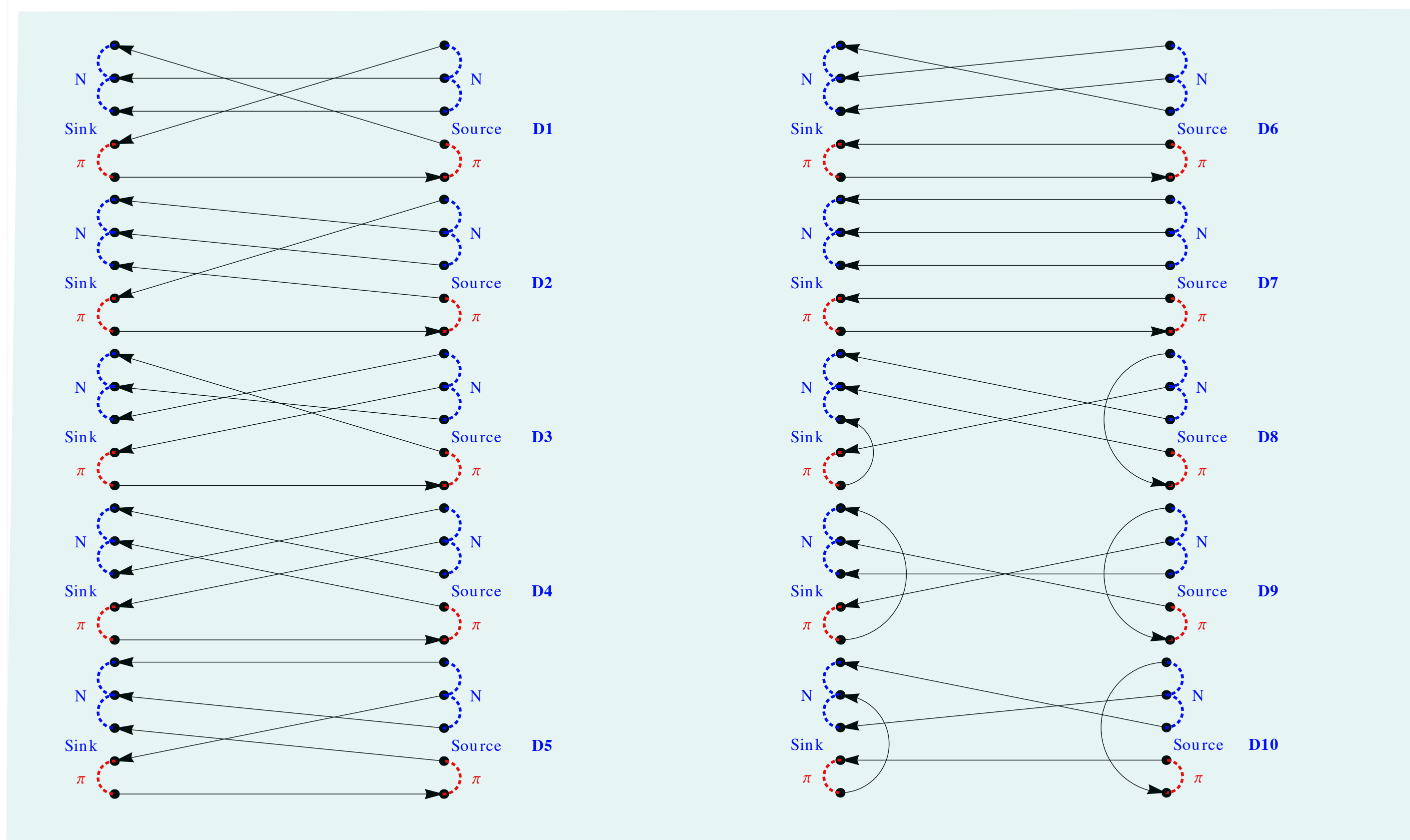
Baryons: Nπ

$$N \to N\pi, N\pi \to N \qquad N\pi \to N\pi$$





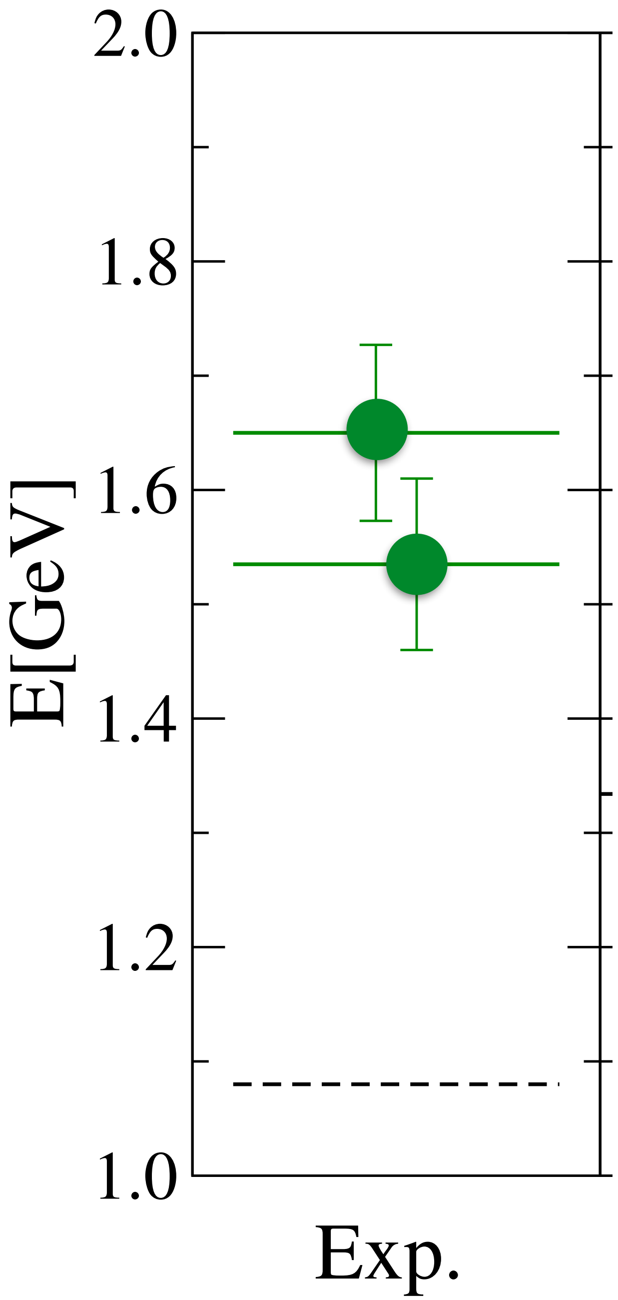
The background is fully dynamical quarks and gluons



Baryons: Nπ

$N(\frac{1}{2}^-)$

N*(1650)
N*(1535)



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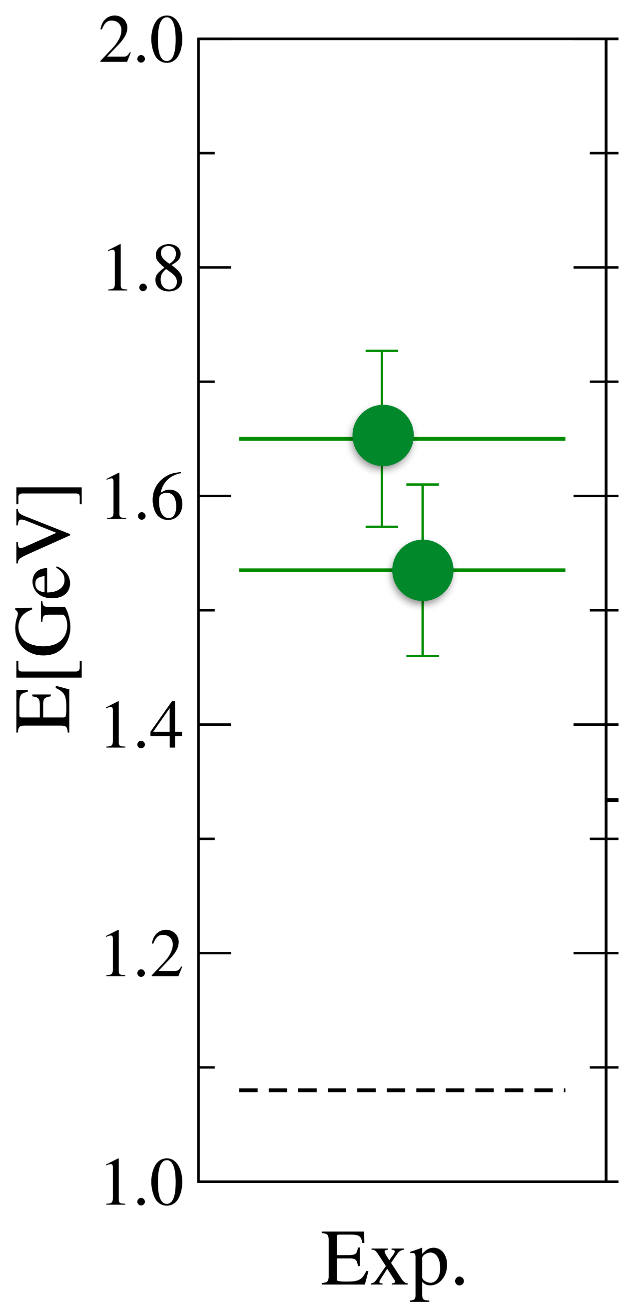

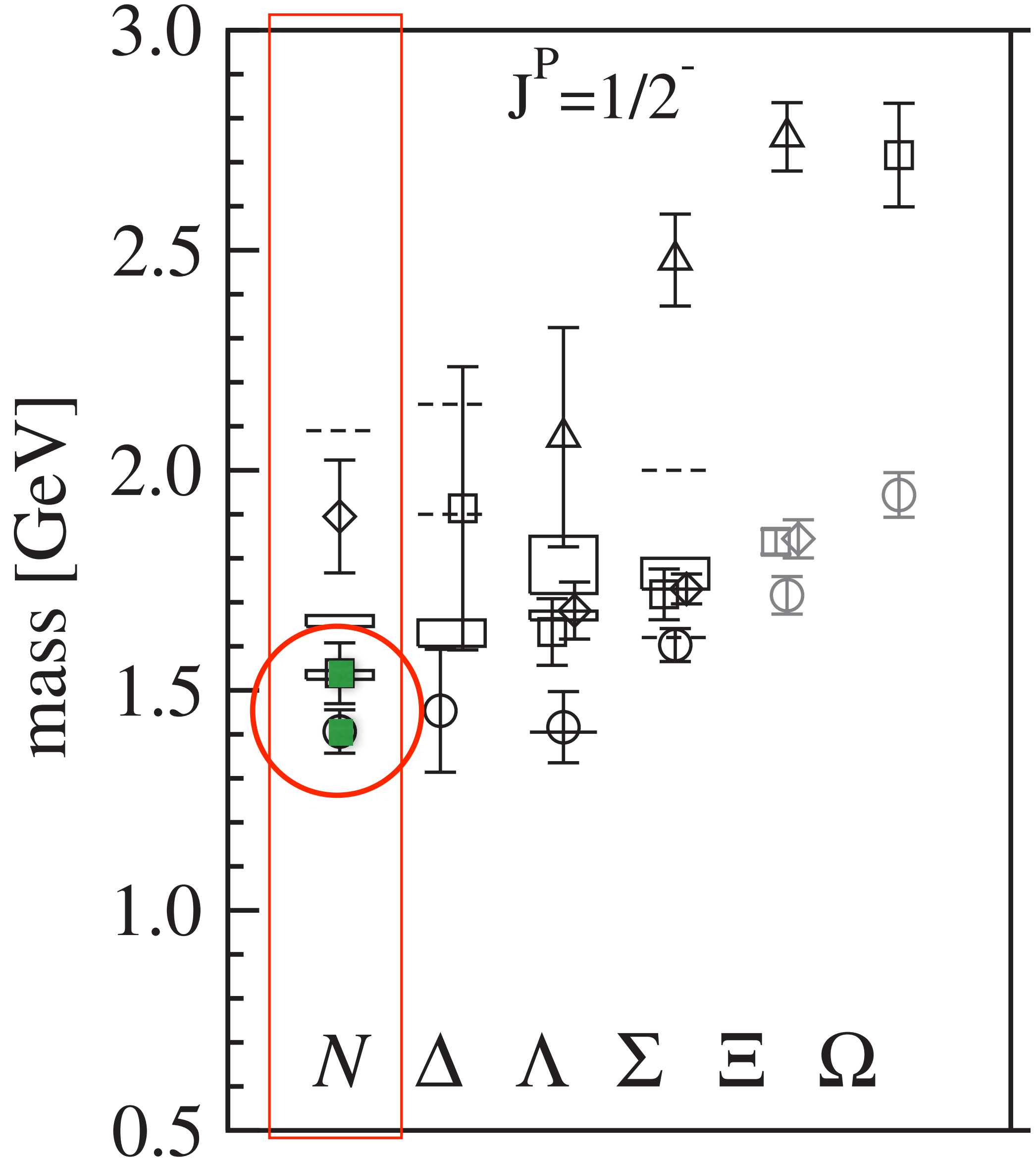

Engel et al., PRD87, 074504 (2013).

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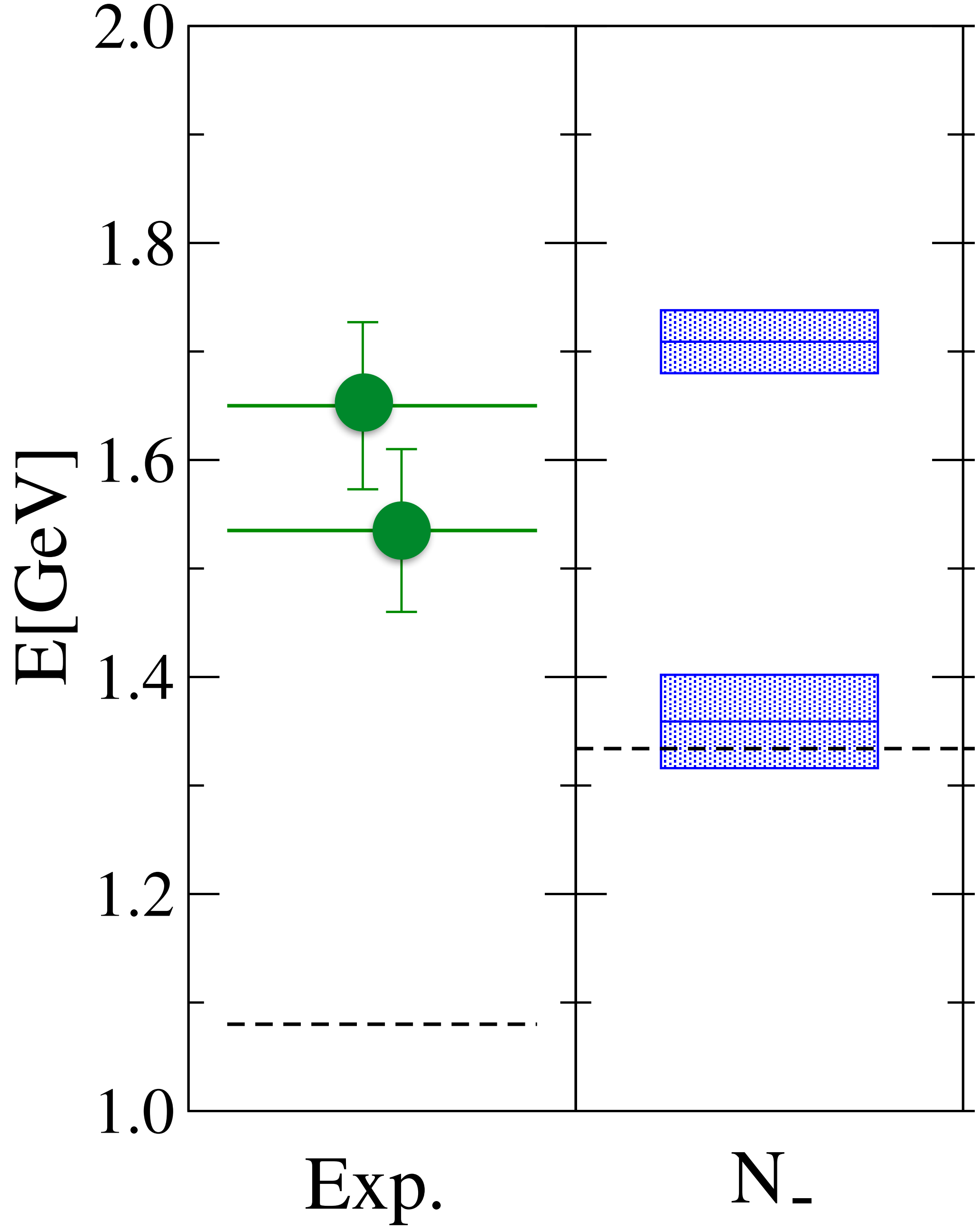

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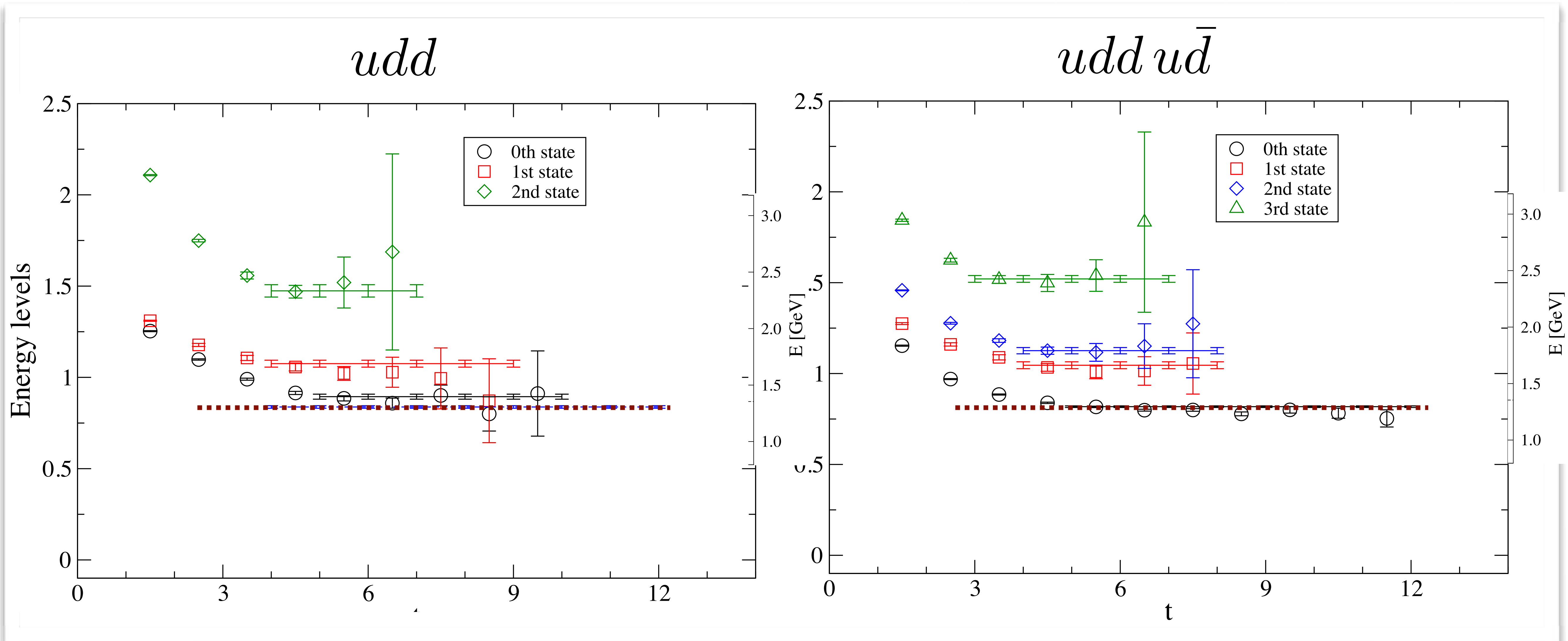
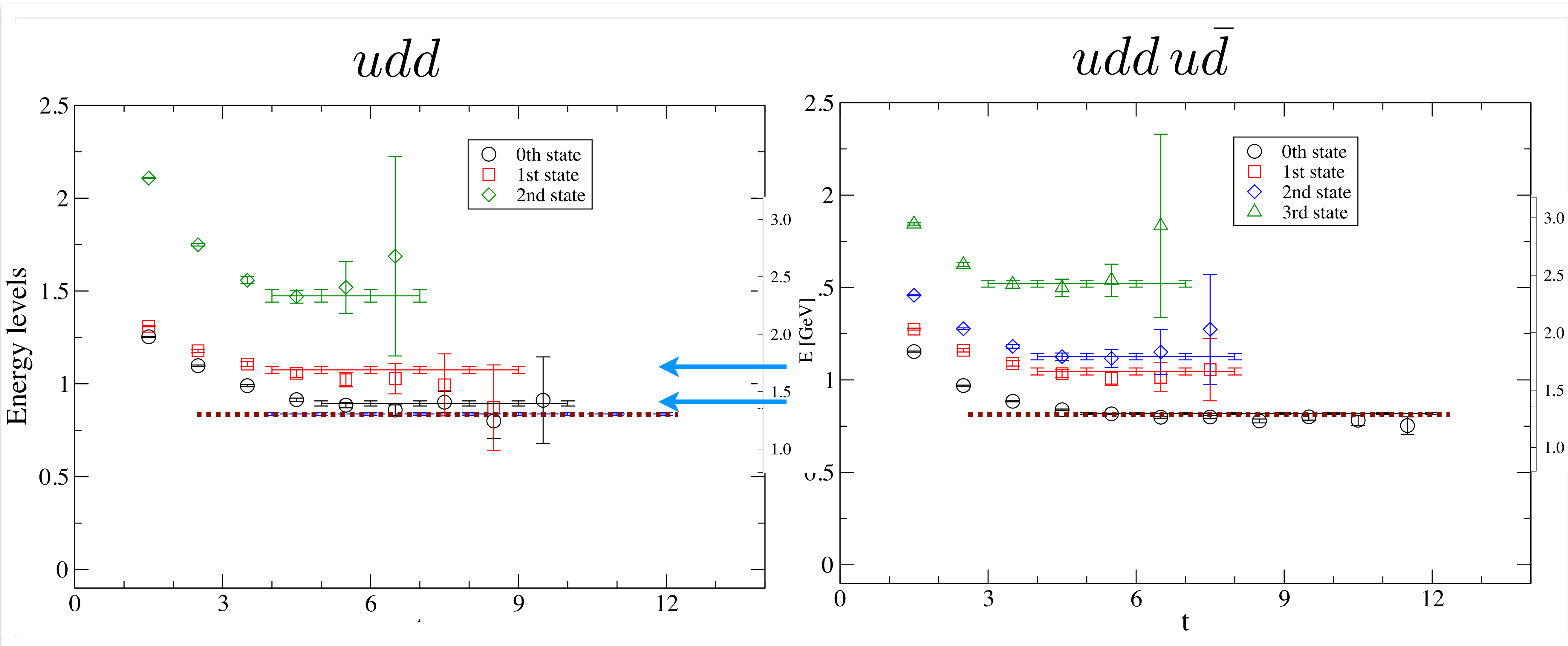


udd
$udd\,u\bar{d}$
0th state
1st state
2nd state
3rd state
Energy levels
E [GeV]
t

Baryons: Nπ



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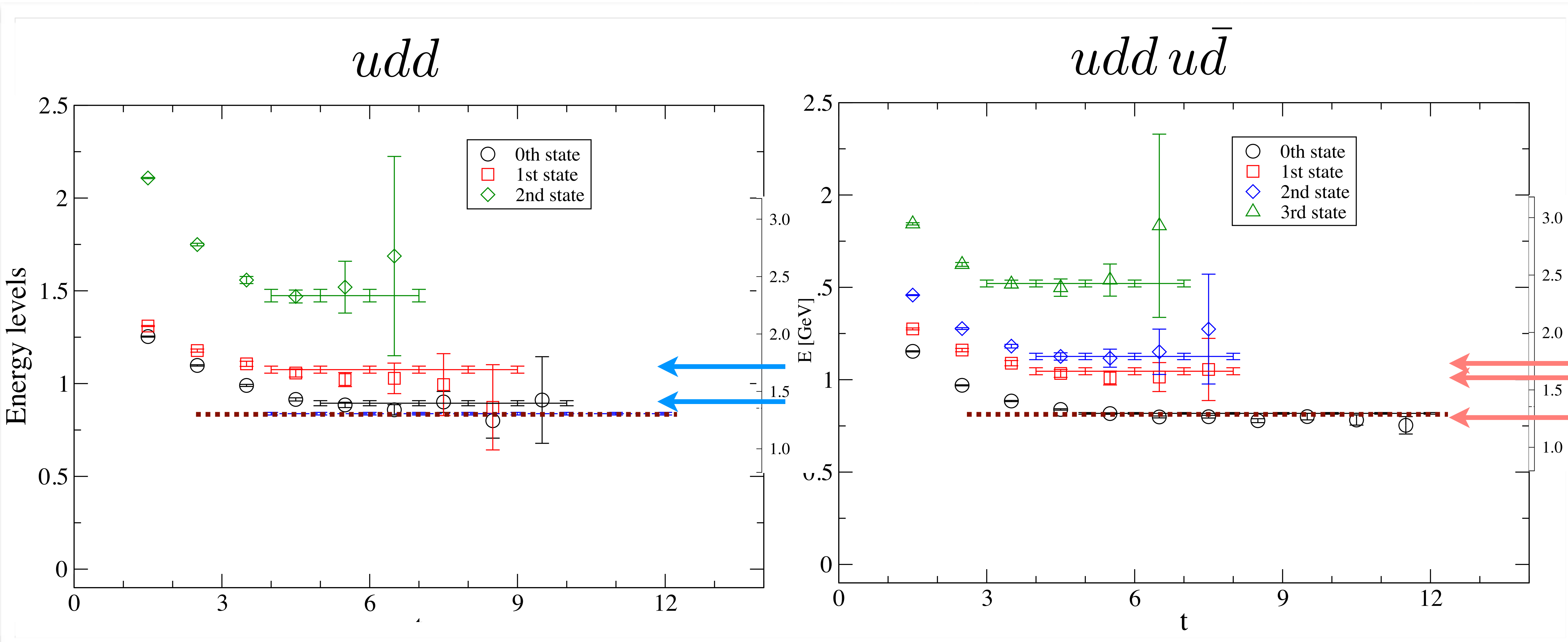

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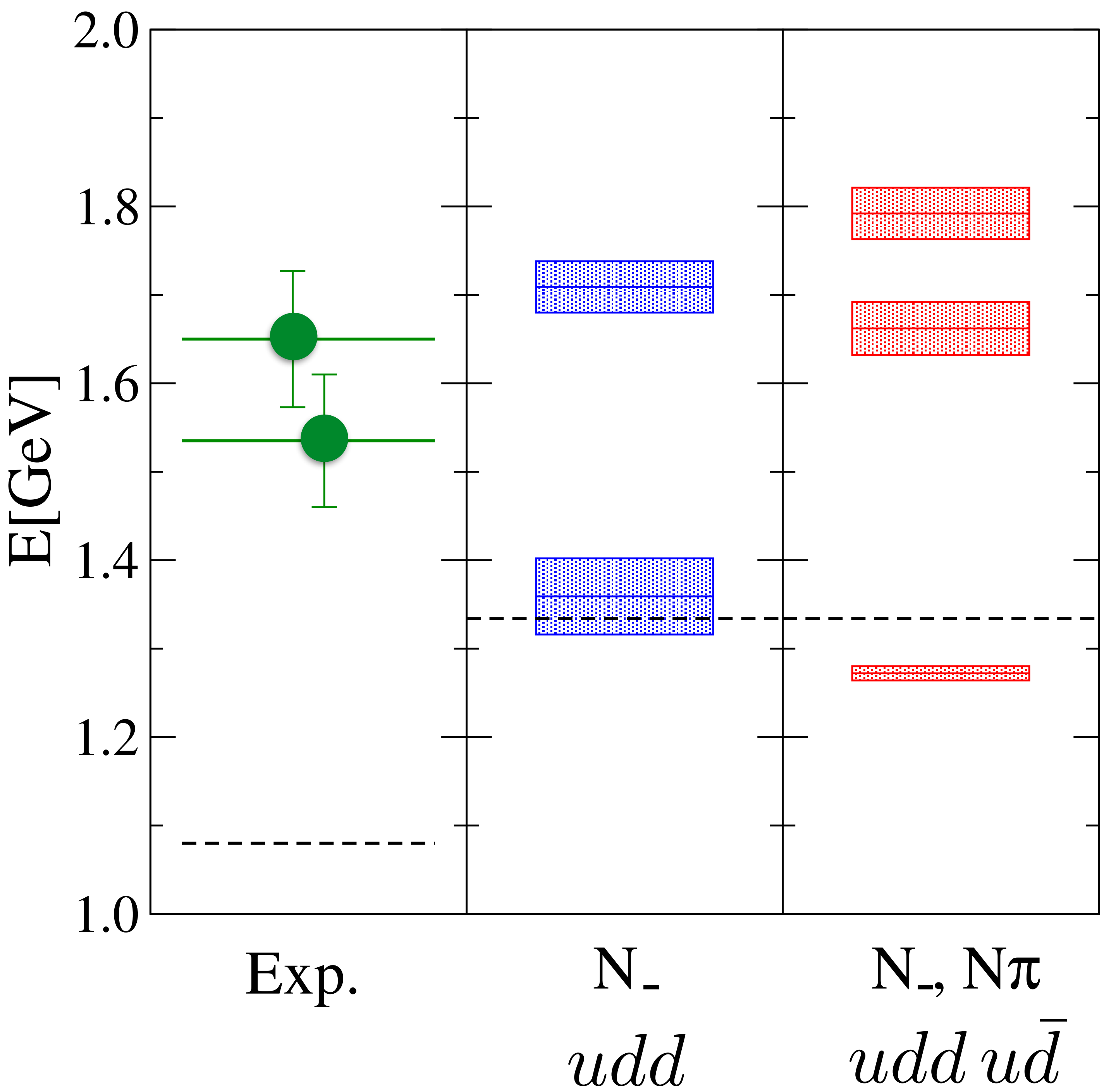
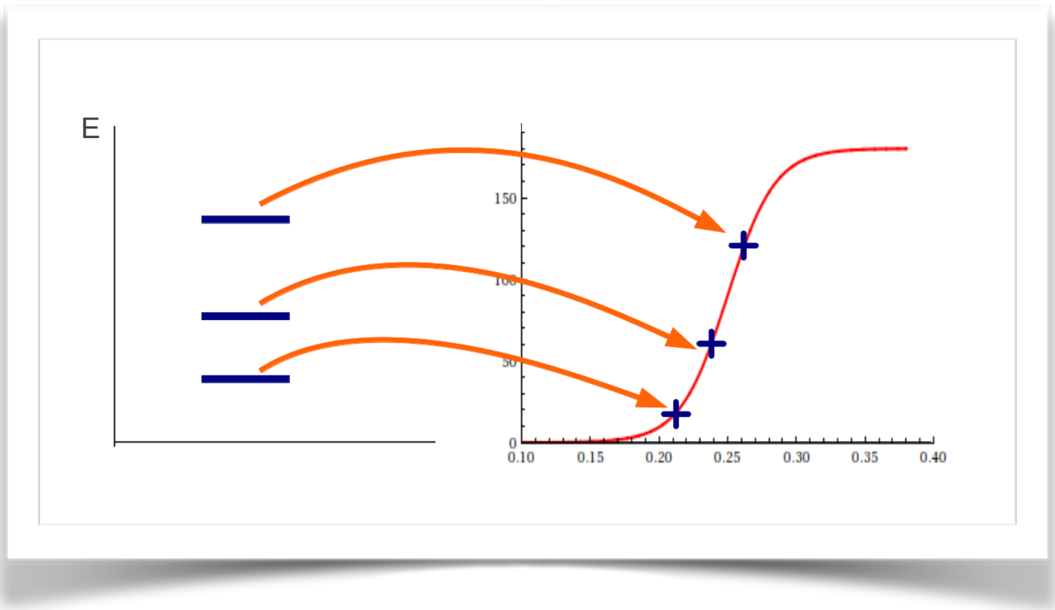


cbl & Verduci, PRD86 ***(2013)*** 054502

Lüscher (p=0):

$$\cot\delta(q) = \frac{\mathcal{Z}_{00}(1;q^2)}{\pi^{3/2}q}$$



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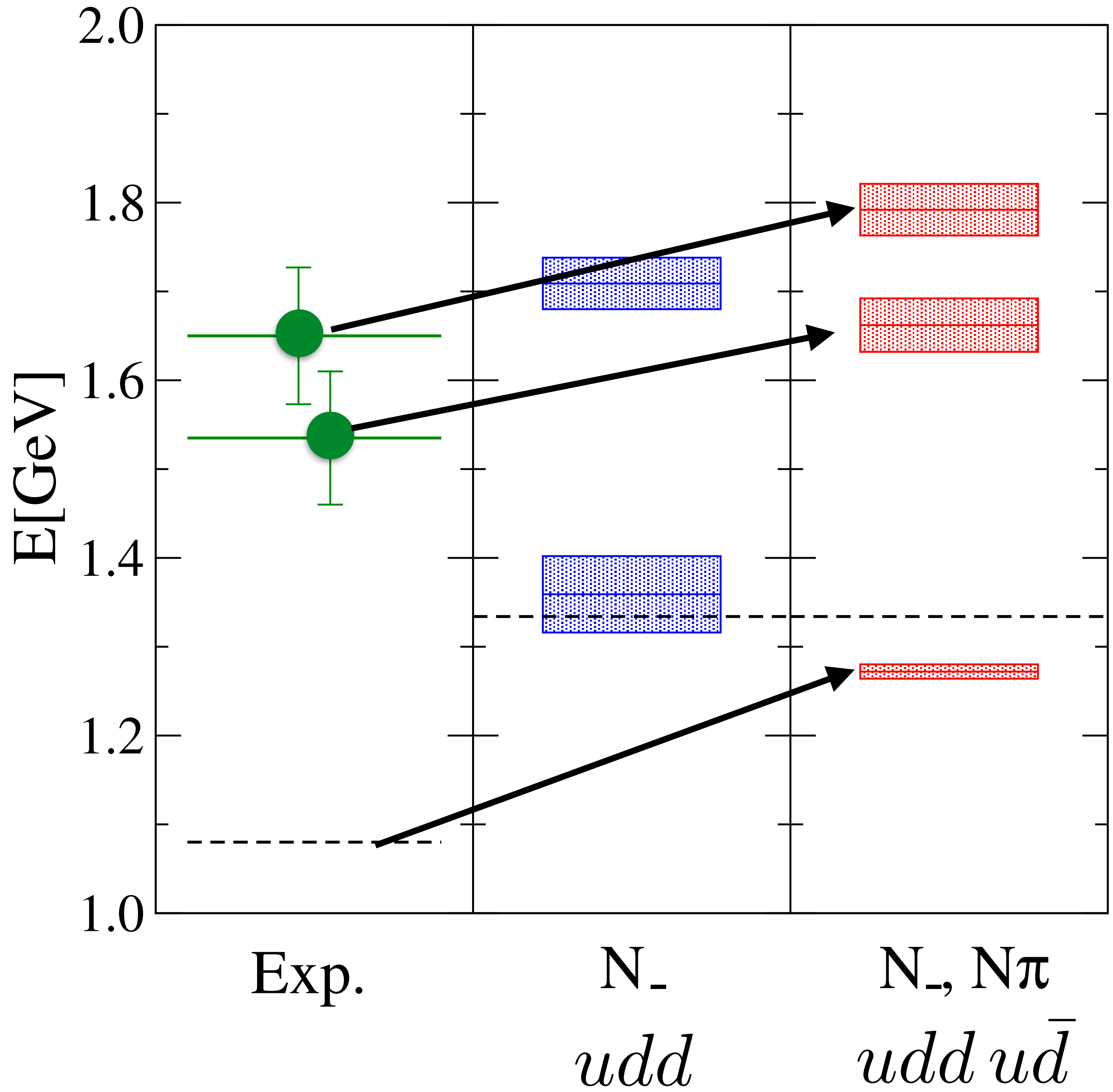

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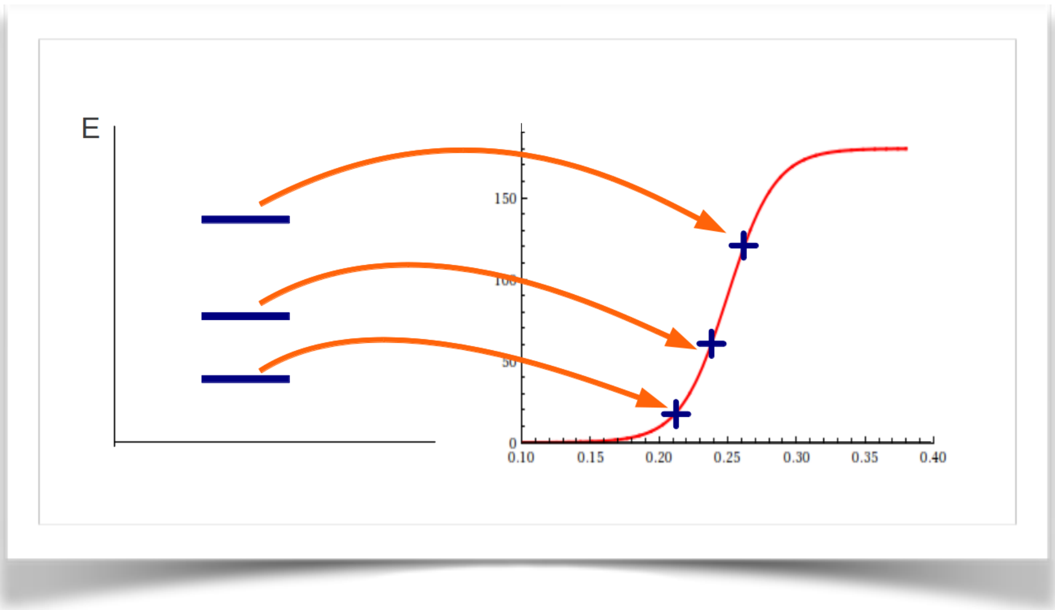

Charmed mesons: D_s

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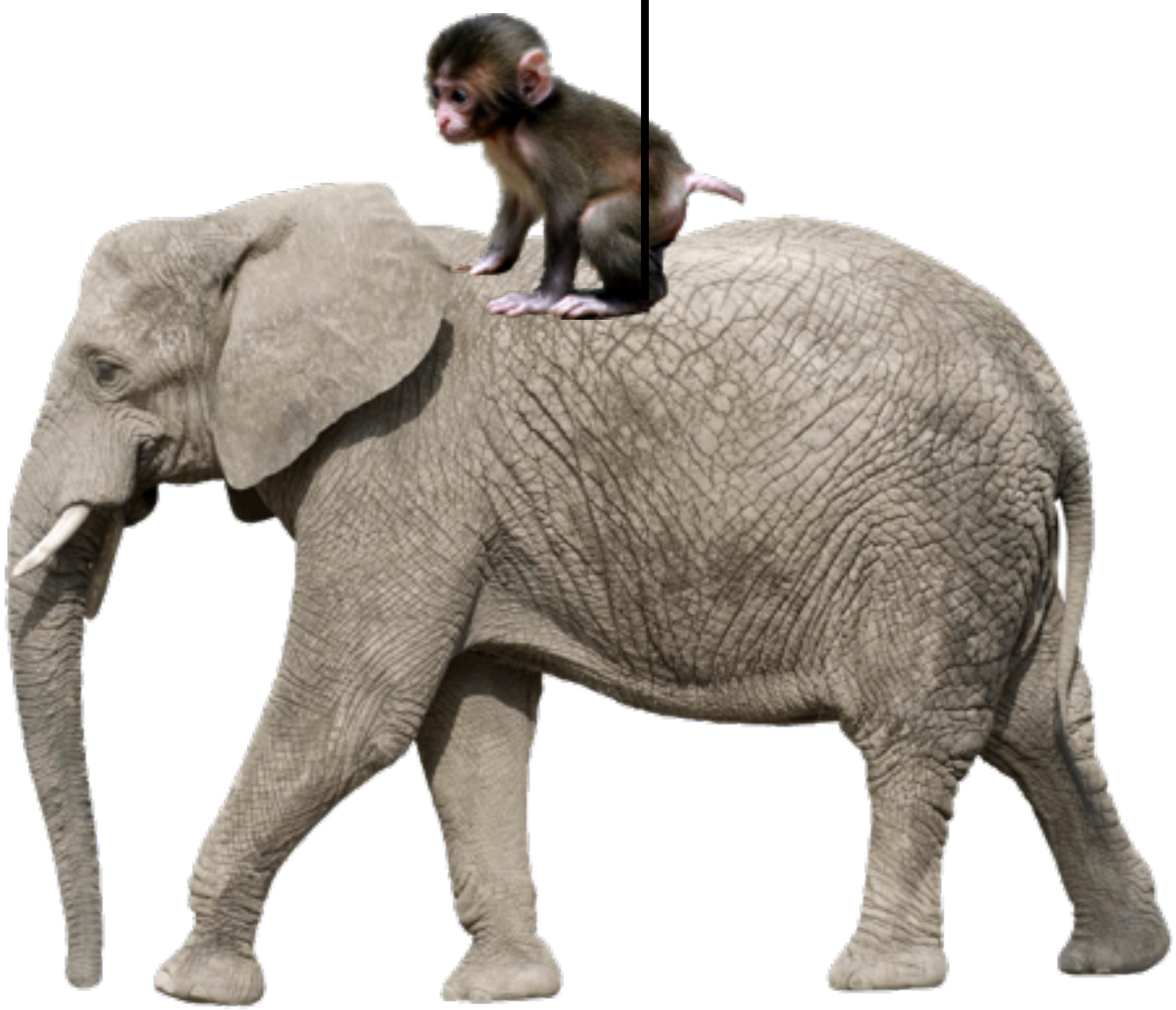


the heavy light quark experience:
how to disperse?

Charmed mesons: D_s

c s

CHARMED, STRANGE MESONS

Particles

$D_s^{\pm}$

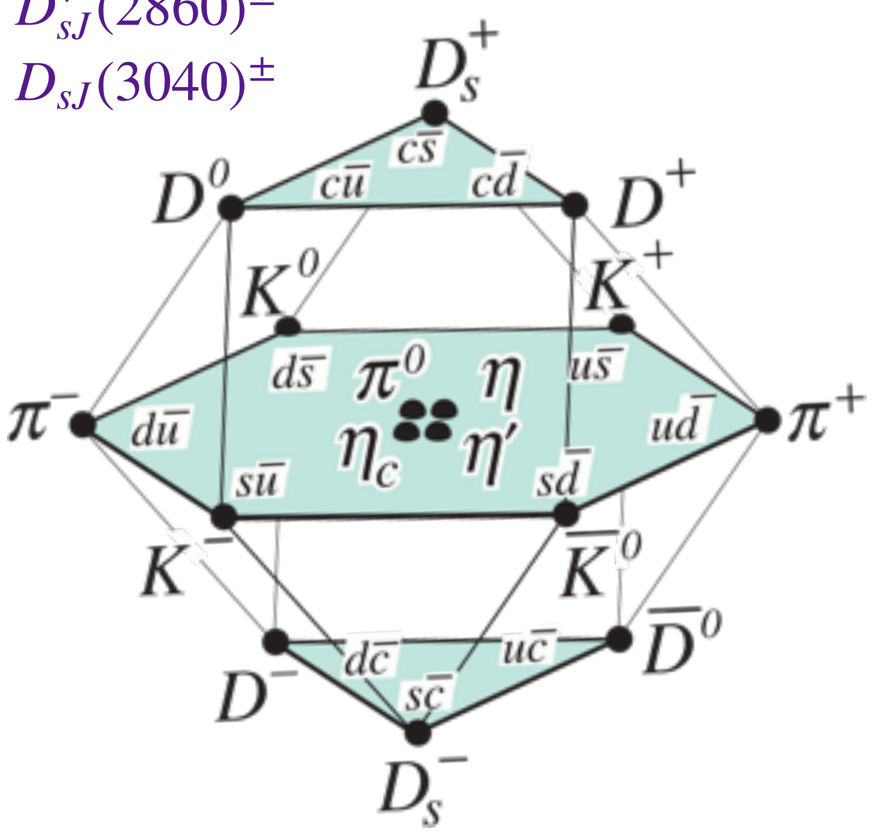
$D_s^{*\pm}$

	HQL:
$D_{s0}^*(2317)^{\pm}$	s-wave
$D_{s1}(2460)^{\pm}$	s-wave
$D_{s1}(2536)^{\pm}$	d-wave
$D_{s2}^*(2573)$	d-wave

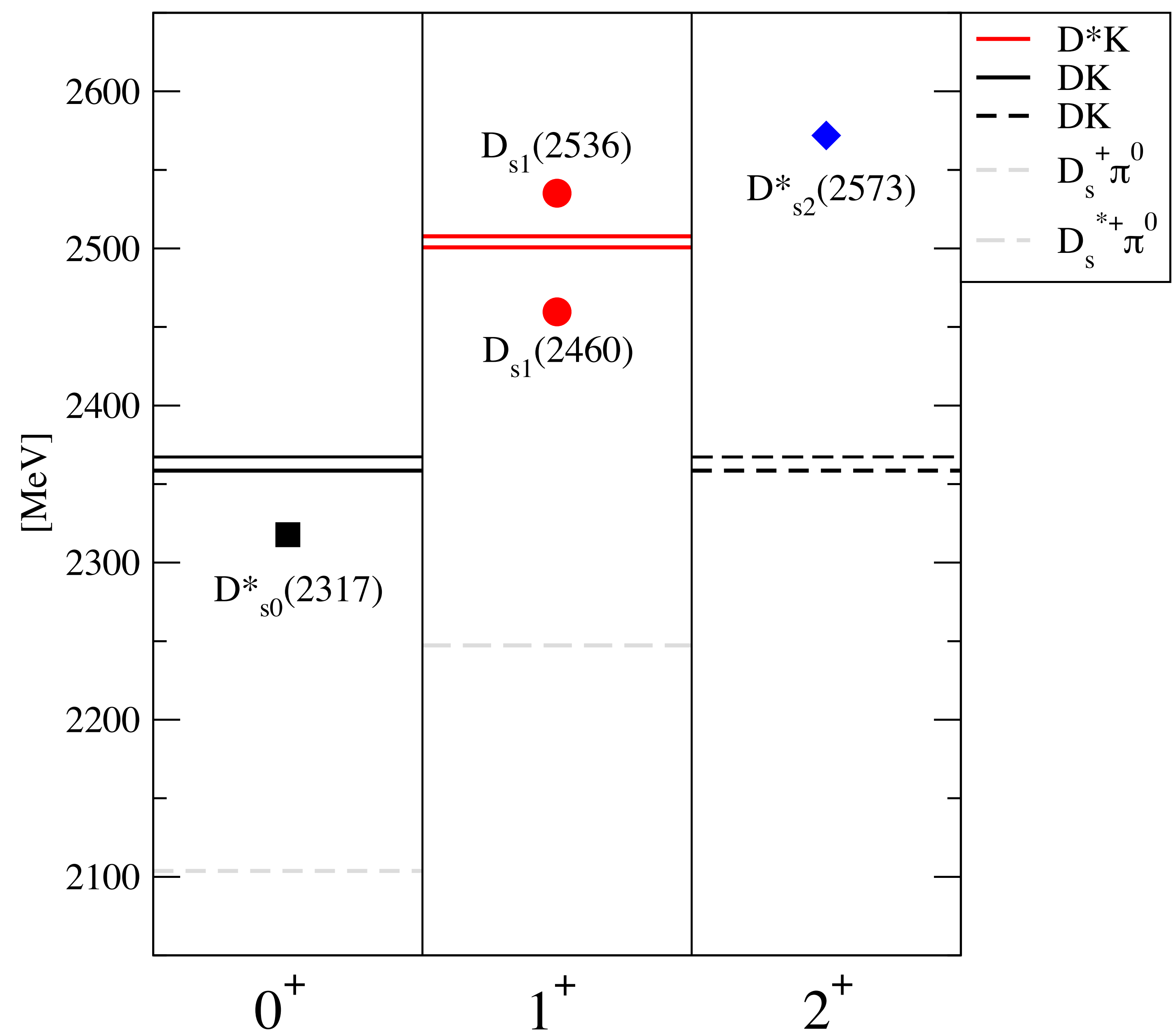
$D_{s1}^*(2700)^{\pm}$

$D_{sJ}^*(2860)^{\pm}$

$D_{sJ}(3040)^{\pm}$



Experiment



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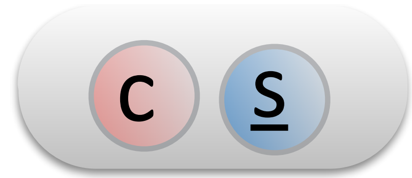

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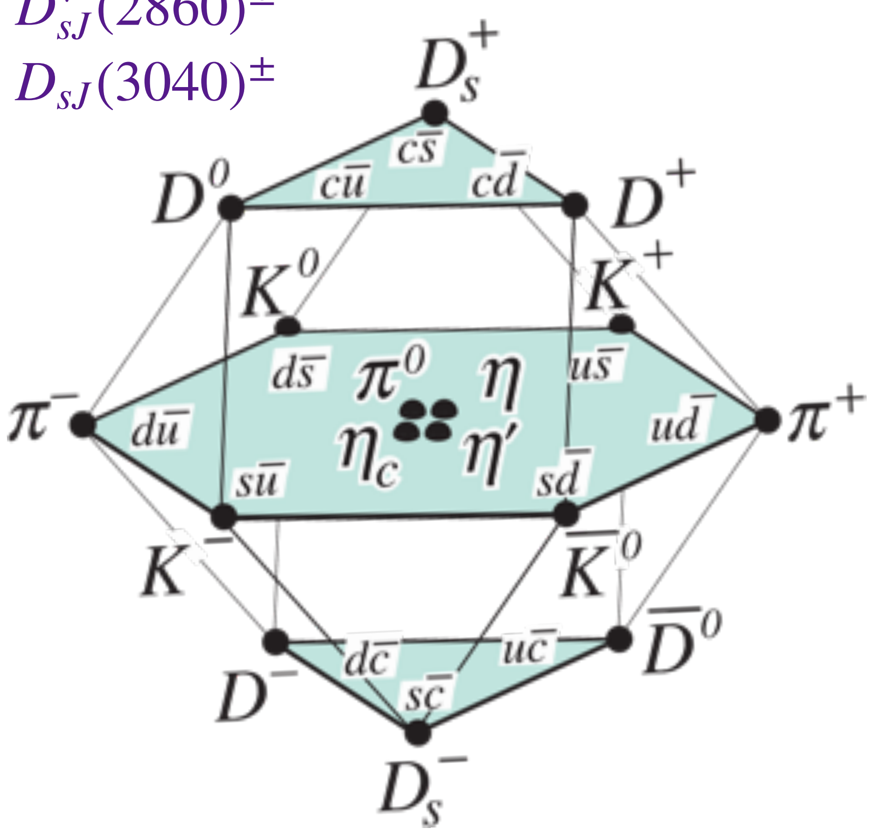

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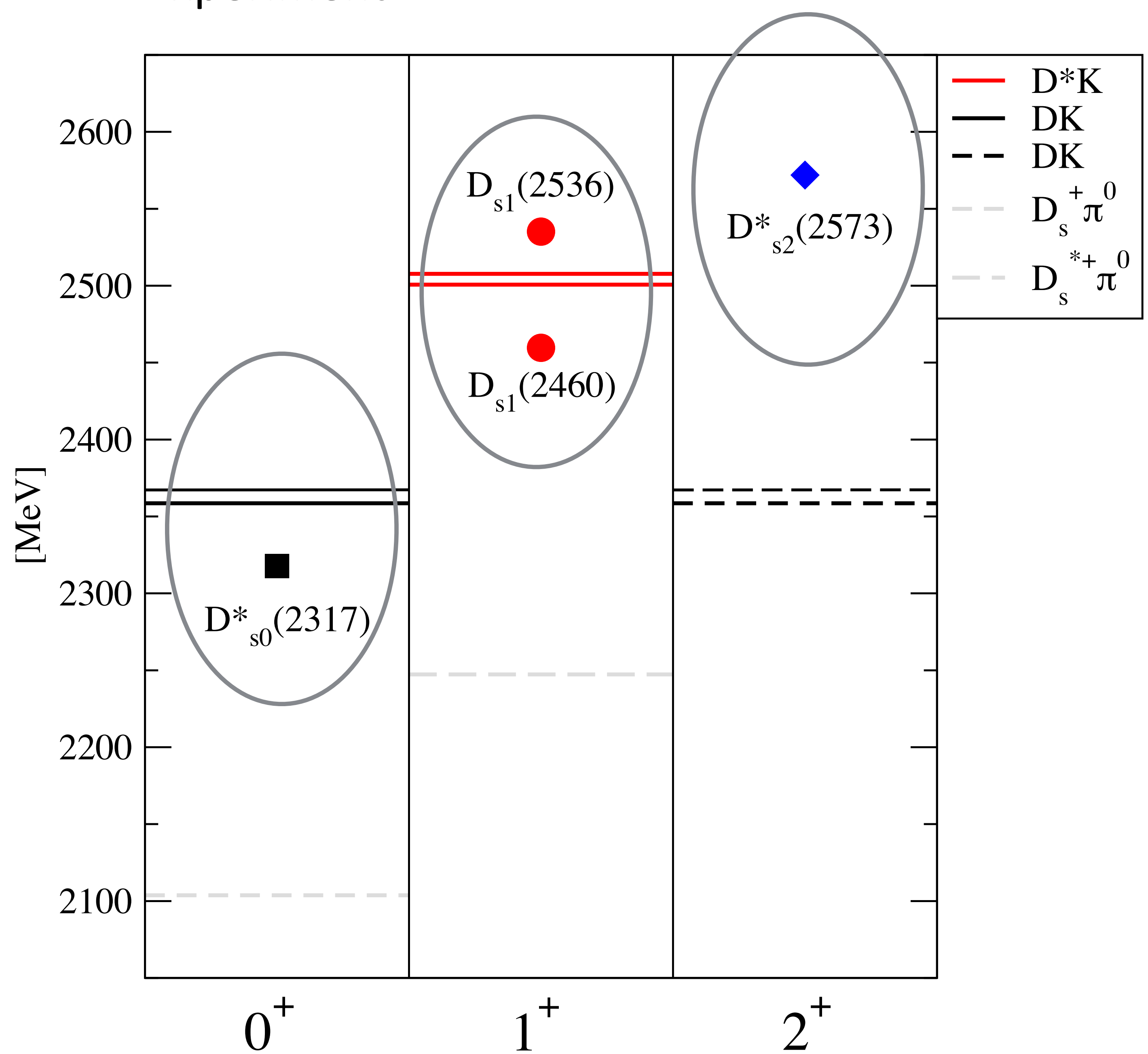


Charmed mesons: D_s

Quark model:

$D^*_{s0}(2317)$ and $D_{s1}(2460)$ are *above* thresholds DK and D^*K

cf. Godfrey/Isgur PRD 32, 189 (1985)

But: threshold effects may be important

van Beveren/Rupp PRL 91(2003) 012003
Godfrey, PRD 72, 054029 (2005)

ChU coupled channels:

Martinez Torres et al., PR D 85 (2012) 014027

Lattice QCD:

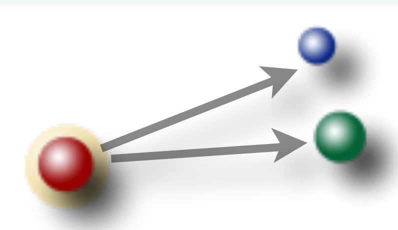
Single hadron (c<u>s</u>) studies give too high values

large pion mass: D^*_{s0} below threshold

small pion mass: D^*_{s0} above threshold

Namekawa et al., Phys. Rev. D 84, 074505 (2011)
Mohler/Woloshyn, Phys. Rev. D 84, 054505 (2011)
Bali et al., J. Phys. Conf. Ser. 426, 012017 (2013)
Bali et al., PoS LATTICE2011, 135 (2011),
Moir et al, JHEP 05, 021 (2013)
Kalinowski et al., A. Phys. Pol. B PS. 6, 991 (2013)
Wagner et al. 1310.5513.

Include meson meson interpolators!



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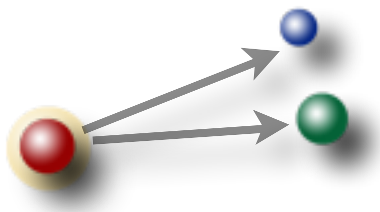


2 Configuration ensembles

cbl et al., PRD 90(2014) 034510
arXiv:1403.8103

- Ensemble 1:
  - Hasenfratz et al., PRD 78, 014515 & 054511 (2008)
  - n_f=2 Wilson improved, 4 nHYP
  - 16^3x32, L_x=2 fm, 279 configs.
  - m_π=266 MeV, m_K=552 MeV

- Ensemble 2:
  - PACS-CS, Aoki et al, PRD 79, 034503 (2009)
  - n_f=2+1 Wilson improved, 3D HYP
  - 32^3x64, L_x=2.9 fm, 196 configs.
  - m_π=156 MeV, m_K=504 MeV

Propagators

- Distillation
  - HSC, Peardon et al., PRD80, 054506 (2009)
  - n_v=96
  - perambulators

$$\tau_{ij}^{\overline{\alpha}\overline{\beta}}(t',t) = v_i^*(t')\, G^{\overline{\alpha}\overline{\beta}}(t';\, t)\, v_j(t)$$

- Stochastic Distillation
  - Morningstar et al., PRD 83, 114505 (2011)
  - n_v=192, n_b=12,n_{ti}=8
  - stochastic sources

$$S_b^{\overline{\alpha}[r]}(\vec{x},c;t) = \sum_i v_i(\vec{x},c;t)\; \eta_{ib}^{\overline{\alpha}[r]}$$

  - (half) stochastic perambulators

$$T_{ib}^{[r]}(t,t') = v_i^*(t)\, G(t;t')\, S_b^{[r]}(t')$$

Quark mass parameters and dispersion relation

u d	m_π=266(6)MeV	m_π=156(7)
s	valence m_Φ=1016(12) MeV m_K=552(7) MeV	partially quenched m_Φ=1018(14) MeV m_K=504(7) MeV $m_{\eta s}$=693(10)MeV
c	valence	valence

$m_{\eta s}$=688(2) MeV from Dowdall et al., PRD 88, 074504(2013)

Heavy quark dispersion relation: Fermilab method

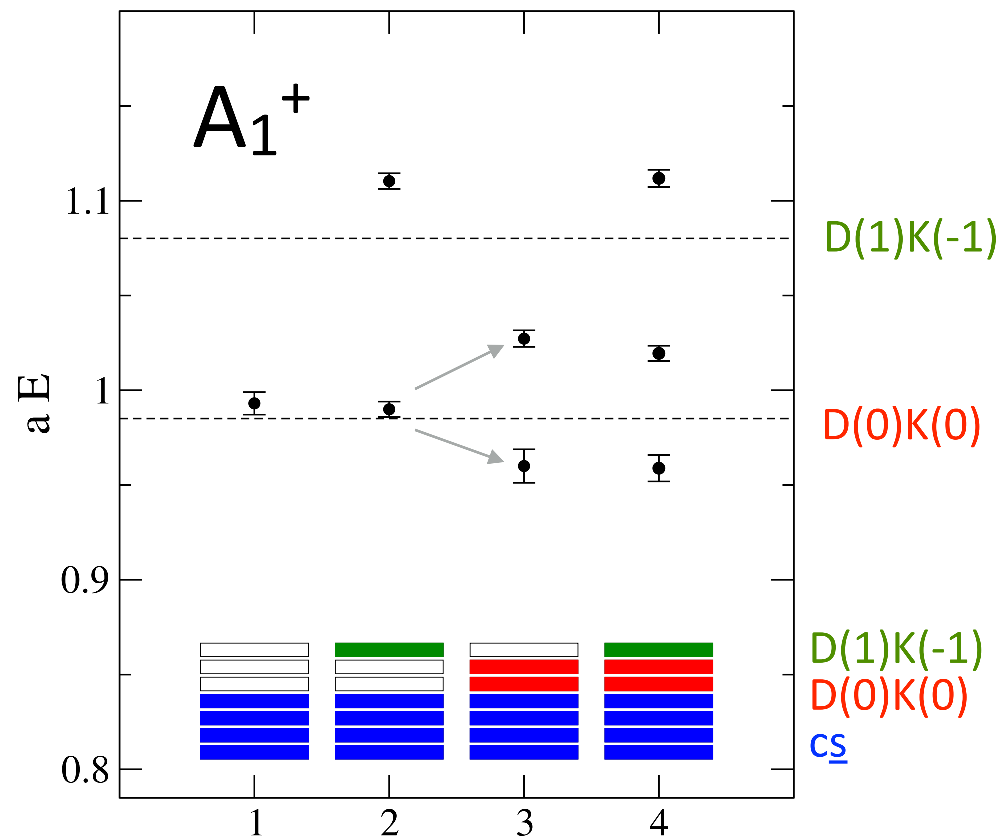
Tune spin-average mass $\bar{m}$ (M_2 in the d.r.) for D, D_s and charmonium, respectively and determine $m - \bar{m}$

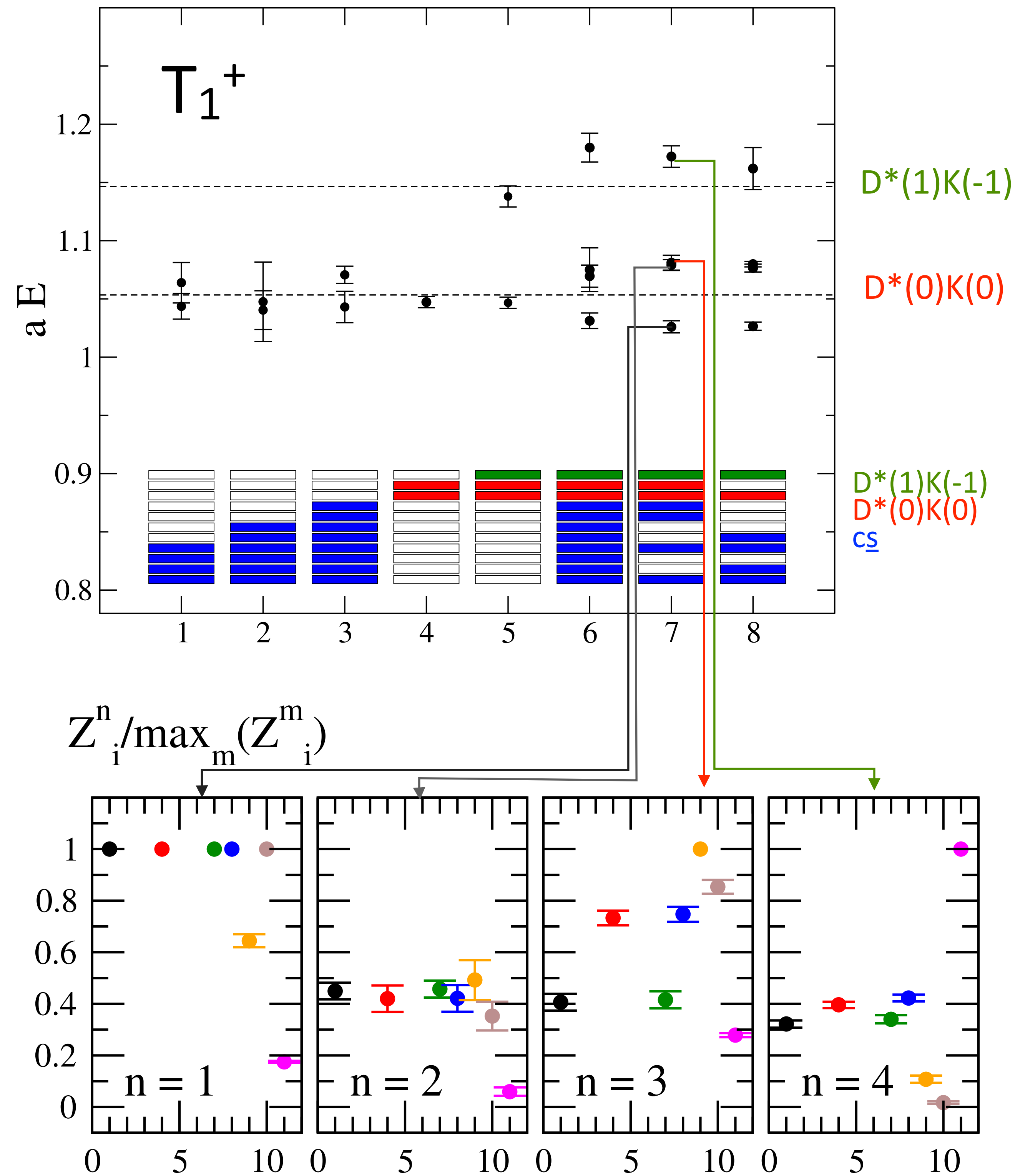
D.rel.: $$E(p) = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{(\mathbf{p}^2)^2}{8M_4^3}$$

E.g. $\bar{m} = \frac{1}{4}(m_{D_s} + 3m_{D_s^*})$

El Khadra et al., PRD 55, 3933 (1997),...,
C. Bernard et al., PRD 83, 034503 (2011)

Composition of eigenstates (ensemble 2: PACS-CS)



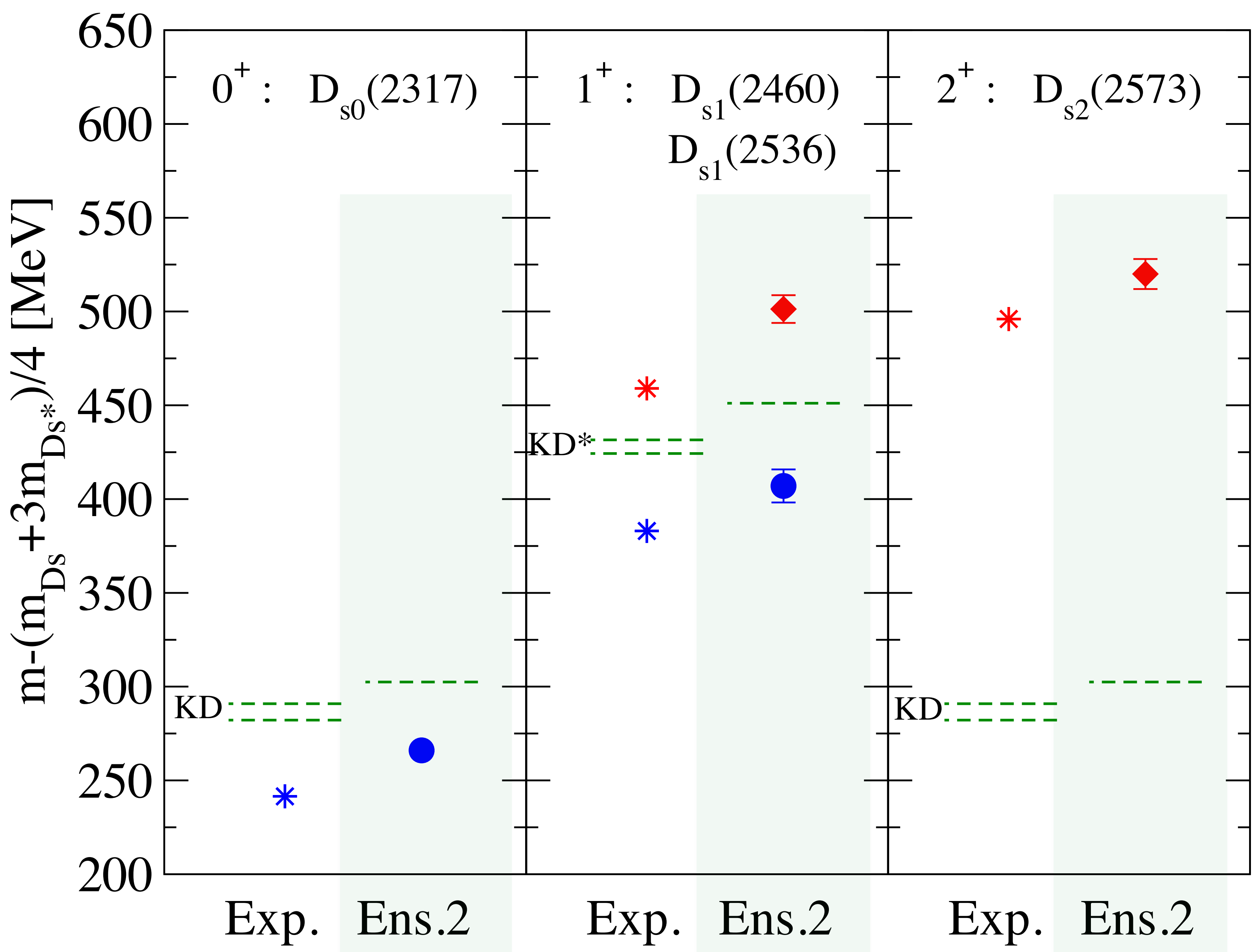


Overlap factor

$$Z_i^n = \langle \mathcal{O}_i | n \rangle$$

Results summary

cbl et al., PRD 90(2014) 034510
arXiv:1403.8103



Charmonium et al.: J/ψ π and more

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"Exotic" Channels, e.g. $J^{PC}=1^{+-}$

Charged charmonium Z_c^+ (3900, 4020,..?)

18 interpolators of meson-meson type covering all imaginable states up to 4.1 GeV (small volume bonus)

4 tetraquark operators

$$\mathcal{O}_1^{4q} = N_L^3\ \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5 \bar{d}_c\ c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i \bar{d}_c\ c_{b'}\gamma_5 C u_{c'})\,,$$

$$\mathcal{O}_2^{4q} = N_L^3\ \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C \bar{d}_c\ c_{b'}\gamma_i\gamma_5 C u_{c'} - \bar{c}_b C\gamma_i\gamma_5 \bar{d}_c\ c_{b'} C u_{c'})\,,$$

4.14 GeV $J/\psi(11)\pi(-11)$

$\psi_3(0)\pi(0)$

$D(1)D^*(-1)$

$\eta_c(1)\rho(-1)$

$\psi_{3700}(0)\pi(0)$

$D^*(0)D^*(0)$

$\psi_{2S}(0)\pi(0)$

$D(0)D^*(0)$

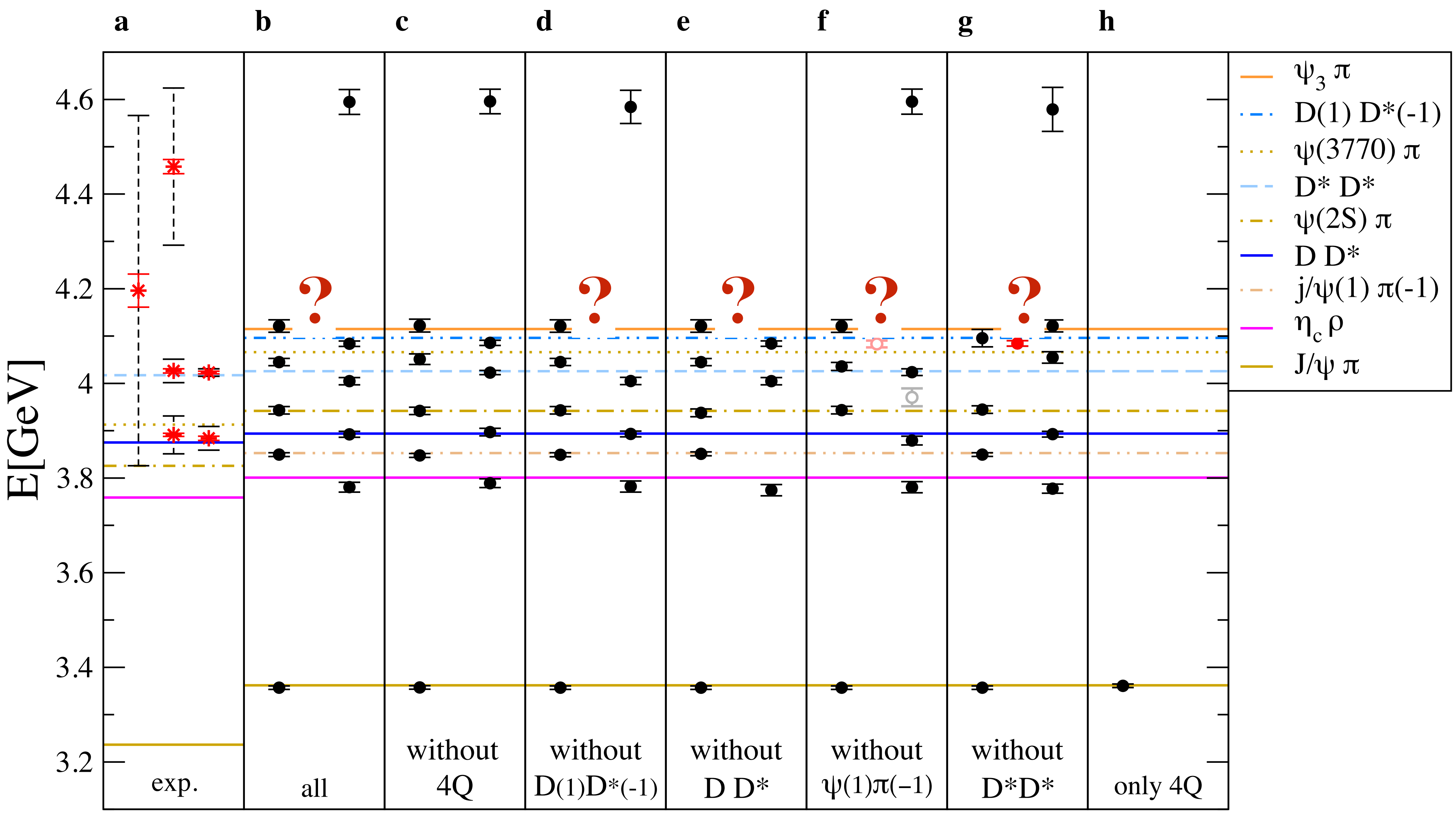
$J/\psi(1)\pi(-1)$

$\eta_c(0)\rho(0)$

3.36 GeV $J/\psi(0)\pi(0)$

Latt

Level hunting



Prelovsek et al., under construction

Summary and outlook

- $\pi\pi$ p-wave: ρ resonance
- πK I=1/2, 1/2 s- and p-wave: κ and K^*
- a_1 and b_1
- πN $1/2^-$ s-wave
- D_s
- $\psi\pi^+$

Challenges:

Coupled channels, inelastic regime → cf. Döring et al., EPJ A47(2011)139; Hansen & Sharpe, PRD86 (2012) 016007; Briceno, PRD89 (2014) 074507

Physically large lattices (many more levels)

…and, finally

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