A unitarized model for tetraquarks with a color flip-flop potential

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Motivation

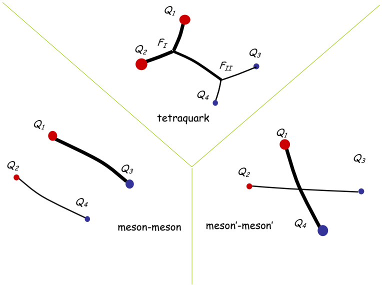
- The existence of tetraquarks is still debated
- Various experimental tetraquark candidates
- Theoretically is a four-body problem, with a four body force
- The system of two mesons is one of the simplest system with more than one color singlet

Method

- Begin with a microscopic quark model for $Q_1Q_2\bar{Q}_3\bar{Q}_4$
- Compute meson states
- Integrate internal degrees of freedom
  - Compute meson-meson potentials
- Find bound states
- Compute the T matrix and find resonances

Potential I

- Static potential (ground state) from the lattice:
  $V_{FF} = \min(V_I, V_{II}, V_T)$



- Correct infrared behavior but insufficient to describe the $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$ system

Potential II

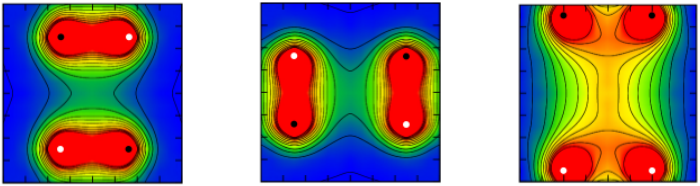
- In more detail: V_I and V_{II} are the two-meson potentials

$$V_I = V_M(r_{13}) + V_M(r_{24}) \qquad \text{and} \qquad V_{II} = V_M(r_{14}) + V_M(r_{23})$$

  with $V_M = K - \frac{\gamma}{r} + \sigma\, r$

- V_T is the tetraquark potential:

$$V_T = 2K - \gamma \sum_{i<j} \frac{C_{ij}}{r_{ij}} + \sigma\, L_{min}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$$



Color states

- We have two linearly independent color singlets
- Two meson states (non-orthognal basis):

$$|\mathcal{C}_I\rangle = \frac{1}{3}|Q_i Q_j \overline{Q}_i \overline{Q}_j\rangle \qquad \text{and} \qquad |\mathcal{C}_{II}\rangle = \frac{1}{3}|Q_i Q_j \overline{Q}_j \overline{Q}_i\rangle$$

- Antisymmetric and symmetric color states:

$$|\mathcal{A}\rangle = \frac{\sqrt{3}}{2}\big(|\mathcal{C}_I\rangle - |\mathcal{C}_{II}\rangle\big) \qquad \text{and} \qquad |\mathcal{S}\rangle = \sqrt{\frac{3}{8}}\big(|\mathcal{C}_I\rangle + |\mathcal{C}_{II}\rangle\big)$$

- Potential has to be a 2×2 matrix : Need to obtain both eigenvalues and the eigenvectors

Potential IV

- Ground state eigenvector shall be $|\mathcal{C}_I\rangle$, $|\mathcal{C}_{II}\rangle$ or $|\mathcal{A}\rangle$
- Excited state should orthogonal to ground state
- Assume simple hypothesis for the excited state

$v_0 = V_{ff}$	$\|u_0\rangle$	v_1	$\|u_1\rangle$
V_I	$\|\mathcal{C}_I\rangle$	$\min(V_{II}, V_T)$	$\|\bar{\mathcal{C}}_I\rangle$
V_{II}	$\|\mathcal{C}_{II}\rangle$	$\min(V_I, V_T)$	$\|\bar{\mathcal{C}}_{II}\rangle$
V_T	$\|\mathcal{A}\rangle$	$\min(V_I, V_{II})$	$\|\mathcal{S}\rangle$

with $\langle \mathcal{C}_A | \bar{\mathcal{C}}_A \rangle = 0$

From quarks to mesons I

- Since we will be studying meson-meson interaction, our natural choice for the color structure basis is the $|\mathcal{C}_I\rangle$ and $|\mathcal{C}_{II}\rangle$
- This basis is non-orthogonal $g_{AB} = \langle\mathcal{C}_A|\mathcal{C}_B\rangle \neq \delta_{AB}$

$$g = \begin{pmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{pmatrix}$$

- Equation becomes

$$g_{AB}\hat{T}_q\Psi^B + \hat{V}_{AB}\Psi^B = E g_{AB}\Psi^B$$

From quarks to mesons II

- Mesons kinetic energy operator is different from quarks kinetic energy operator

$$T_I = T_q + V_I \neq T_{II} = T_q + V_{II} \neq T_q$$

- Trying

$$T_M = \begin{pmatrix} T_Q + V_I & \\ & T_Q + V_{II} \end{pmatrix}$$

potential becomes non-hermitian

$$V_S' = V_q - g \begin{pmatrix} V_I & \\ & V_{II} \end{pmatrix} = \begin{pmatrix} V_{11} - V_I & V_{12} - \frac{1}{3} V_{II} \\ V_{21} - \frac{1}{3} V_I & V_{22} - V_{II} \end{pmatrix}$$

From quarks to mesons III

- Hermitian decomposition

$$\hat{T}_S = \begin{pmatrix} \hat{T}_I & \frac{\hat{T}_I + \hat{T}_{II}}{6} \\ \frac{\hat{T}_I + \hat{T}_{II}}{6} & \hat{T}_{II} \end{pmatrix}$$

and

$$\hat{V}_S = \begin{pmatrix} V_{11} - V_I & V_{12} - \frac{V_I + V_{II}}{6} \\ V_{12} - \frac{V_I + V_{II}}{6} & V_{22} - V_{II} \end{pmatrix}$$

- Schrödinger Equation given by

$$T_{AB}\Psi^B + V_{AB}\Psi^B = E g_{AB}\Psi^B$$

Meson-Meson interaction

- Expand Ψ^A in the multiple meson-meson states

$$\Psi^I = \sum_n \phi_{13}^n(\boldsymbol{\rho}_{13})\phi_{24}^n(\boldsymbol{\rho}_{24})\psi^I(\mathbf{r}_{13,24})$$
$$\Psi^{II} = \sum_n \phi_{14}^n(\boldsymbol{\rho}_{14})\phi_{23}^n(\boldsymbol{\rho}_{23})\psi^{II}(\mathbf{r}_{14,23})$$

- Schrödinger Equation becomes

$$\hat{T}_{\alpha\beta}\psi^\beta + \hat{V}_{\alpha_\beta}\psi^\beta = E g_{\alpha\beta}\psi^\beta$$

with

$$\hat{V}_{AiAj}\psi^{Aj} = V_{ij}(\mathbf{r})\psi^{Aj}(\mathbf{r})$$
$$\hat{V}_{AiBj}\psi^{Bj} = \int d^3\mathbf{r}'_B\, v_{ij}(\mathbf{r}_A, \mathbf{r}'_B)\,\psi^{Bj}(\mathbf{r}'_B) \qquad \text{with } A \neq B$$

- $\hat{T}$ and $\hat{V}$ have a similar structure

Asymptotic behavior

- These states have the asymptotic behavior:

$$u^{\alpha}(r) \to A_{i\alpha}\sqrt{\frac{\mu_\alpha}{k_\alpha}}\sin(k_\alpha r - \frac{l_\alpha \pi}{2} + \varphi_{i\alpha}) + f_{i\alpha}e^{i(k_\alpha r - \frac{l_\alpha \pi}{2})}$$

which leads to the T matrix

$$T_{ij} = \sum_\alpha \sqrt{\frac{k_\alpha}{\mu_\alpha}}\left[A^*_{i\alpha}e^{-i\varphi_{i\alpha}}f_{j\alpha} + A^*_{j\alpha}e^{-i\varphi_{j\alpha}}f_{i\alpha}\right]$$

obeying the optical theorem:

$$\mathrm{Im}[T] = T^\dagger T$$

Method

- Generate N_{open} eigenfunctions of the $\hat{T}$ operator

$$\hat{T}\Psi_0 = Eg\Psi_0$$

- Orthogonalize the basis using Gram-Schmidt procedure

$$\langle \Psi_{0i} | \Psi_{0j} \rangle = \sum_{\alpha} A^*_{i\alpha} A_{j\alpha} \cos(\varphi_{i\alpha} - \varphi_{j\alpha})$$

- Calculate Scattering wave-functions by solving (with $\Psi_i = \Psi_{0i} + \chi_i$)

$$(\hat{T} + \hat{V})\chi_i = Eg\chi_i - V\Psi_{0i}$$

- Calculate T matrix

Meson-meson scattering

- For now we do the following approximations:
  - Consider only heavy quarks
  - Neglect spin effects
  - Neglect dynamical quark effects
  - Non-relativistic kinematics

Exotic channels

- For the exotic $qq\bar{Q}\bar{Q}$ channels, the wave-function

$$\Psi = \Phi(\boldsymbol{\rho}_{13}, \boldsymbol{\rho}_{24})\psi(\mathbf{r}_{13,24})\mathcal{C}_I + \xi\Phi(\boldsymbol{\rho}_{14}, \boldsymbol{\rho}_{23})\psi(\mathbf{r}_{14,23})\mathcal{C}_{II}$$

  has the properties

$$\begin{aligned} P_{12}^{RC}\Psi &= (-1)^{L_r} s \\ P_{34}^{RC}\Psi &= \xi(-1)^{L_r} s \end{aligned}$$

  with $\xi = \pm 1$, $s = \pm 1$ and $\Phi(\mathbf{y}, \mathbf{x}) = s\Phi(\mathbf{x}, \mathbf{y})$
- Including spin we must have

$$P_{12}\Psi = (-1)^{1+S_{12}} P_{12}^{RC}\Psi = (-1)^{1+S_{34}} P_{34}^{RC}\Psi = -\Psi$$

- Imposing $s(-1)^{L_r} = 1$, $\xi = 1$ and $L = 0$, gives us $J = 0$

Finding bound states

- We need a very large box in order to be able to accurately find bound states, if we use Dirichlet boundary conditions
- Instead, we consider the boundary conditions to be dependent on the energy

$$\big[H + B(E)\big]u = Egu$$

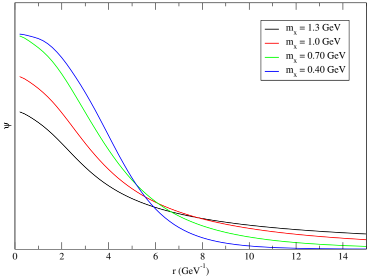
- Bound states are given by the zeros in $\det\big[H + B(E) - E\big]$, found by using Newton's method

$$E^{(n+1)} = E^{(n)} - \frac{1}{\mathrm{Tr}[(H + B(E) - Eg)^{-1}(B'(E) - g)]}$$

Results I

- Bound states for system $xx\bar{b}\bar{b}$ with $L = 0$ and $P = +$:

m_x(GeV)	B(MeV)	$R = \frac{1}{q_0}$(fm)
1.30	$\simeq 0$	18.8
1.00	-0.95	2.60
0.70	-7.91	0.92
0.40	-48.54	0.38



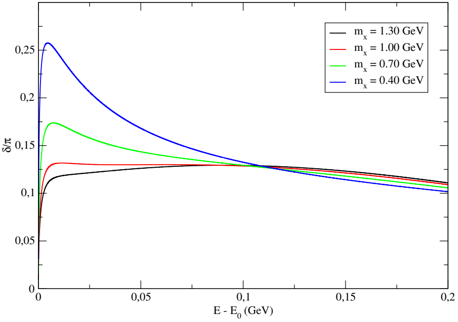
Results II

- Resonances for the 0^+ $qq\bar{Q}\bar{Q}$ system:

m_x(GeV)	E(GeV)	N_{open}
1.30	12.998 - 0.0179i	2
1.00	12.505 - 0.0192i	2
0.70	12.050 - 0.0215i	2
0.40	11.666 - 0.0171i	2

Results III

- $xx\bar{c}\bar{c}$ system
- No bound states or resonances found



Crypto-exotic Tetraquark

- For the crypto-exotic $xb\bar{x}\bar{b}$ sytem:
  - No bound states where found for $m_x \in \{0.4, 0.7, 1.0, 1.3\}$ GeV
  - The following resonances were found (preliminary results):

m_x(GeV)	E(GeV)	N_{open}
0.70	11.545 - 0.237i	1
	12.019 - 0.033i	2
0.40	11.431 - 0.024i	1
	11.687 - 0.114i	2

Discussion/Conclusion

- An unitarized method to compute the meson-meson scattering was developed
- We find bound states and resonances for the 0^+ $xx\bar{b}\bar{b}$ system
- Resonances can be found in the crypto-exotic $xb\bar{x}\bar{b}$ system
- Refinements should be easy to include
  - Spin-spin interactions, etc
  - Other potential models