Tetraquarks
why it is so difficult to model them

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Complementing Eef’s work: systems without annihilation/disconnected diagrams, with a four-body tetraquark flux tube.
Outline

1. Introduction
   - Tetraquark models since the onset of QCD
   - Experimental tetraquarks candidates
   - Tetraquarks computed in lattice QCD

2. Tetraquarks potentials in Lattice QCD
   - Static tetraquarks potentials and flux tubes
   - Dynamical-antistatic dynamical-antistatic potentials in lattice QCD

3. Problems and solutions in modeling tetraquarks
   - Lattice QCD inspired potentials
   - Flip-flop potentials
   - Non-orthogonality, and the colour excited states
   - Unitarity, phase shifts and resonances

4. Conclusion
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4 Conclusion
In the 1950’s, 1960’s and 1970’s the best particle physicists, including many Nobel Prize winners, worked out the hadron puzzle piece by piece.

they finally discovered the **quarks** and produced a new theory, QuantumChromoDynamics (QCD).

Already at the onset of QCD, the bag model predicted many tetraquarks.

Recent studies of tetraquark binding in the quark model

- Lattice QCD changed the paradigm for quark confinement.
- Nevertheless, a flip-flop potential with meson meson and tetraquark linear confinement, also produces tetraquark binding.

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Observation of a hybrid, or tetraquark, by COMPASS at CERN

**COMPASS at CERN observe an exotic hadron resonance with M= 1660 (74) MeV , Γ= 269 (85) MeV, J^{PC} I = 1^{--}1**

the charged quarkonia \( Z^+ \) and \( Z^- \)

- Several charged quarkonia \( Z_c^q \) and \( Z_b^q \) are observed in different collaborations, after the initial observation by BELLE.
- LHCb at CERN confirms the the crypto-exotic (exotic if we neglect \( c\bar{c} \) annihilation) hadron resonance \( Z_c(4430)^- \) with a resonance mass of 4475 MeV and width of 172 MeV.
- In a talk at Scadron 70, organized by George, we predicted the correct partial decay width to \( \pi J/\psi \) with simple Resonant Group Method calculations.

Observation of a resonance-like structure in the \( \pi^- \psi' \) mass distribution in exclusive \( B \rightarrow K \pi^- \psi' \) decays, Belle collaboration, S. Choi et al., Phys. Rev. Lett. 100 (2008) 142001.


M. Cardoso and P. Bicudo, Microscopic calculation of the decay of Jaffe-Wilczek tetraquarks, and the \( Z(4433)^- \), [arXiv:0805.2260].
Tetraquarks, or other exotics, are difficult to observe experimentally. Some observations ended up being unconfirmed.

Clearly they are excited states, with complicated flavour or other quantum numbers, and theorist cannot exactly predict where to look for them.

Whether their complexity makes them really hard to observe or they simply are rare, is open to debate.


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Discretizing QCD

- To discretize QCD in a finite space-time lattice, we utilize finite differences.
- Different points in the lattice must be connected by gauge links to preserve gauge invariance. Thus the quarks field are placed on the lattice vertices and the gluon fields on the links.
- Each link is a $SU(3)$ matrix.
- While the quark action is obvious, the gluon action is obtained with the plaquette

$$
\sum_c F^c_{\mu \nu} F^c_{\mu \nu} = \frac{2 \beta}{a^4} P_{\mu \nu} + O(a),
$$

a closed loop in the lattice,

$$
P_{\mu \nu}(\mathbf{r}) = 1 - \frac{1}{3} \text{Re} \text{Tr} \left[ U_\mu(\mathbf{r}) U_\nu(\mathbf{r} + \mu) U^\dagger_\mu(\mathbf{r} + \nu) U^\dagger_\nu(\mathbf{r}) \right].
$$
Ljubljana-Graz tetraquarks

14 meson-meson components and 4 diquark-diquark components

Calcutational Method

The energies $E_n$ and the overlaps $Z_j^n$ of the physical eigenstates $n$ are extracted from the correlation matrix

$$C_{jk}(t) = \langle \Omega | O_j(t_{src} + t) O_k^\dagger(t_{src}) | \Omega \rangle = \sum_n Z_j^n Z_k^n e^{-E_n t}, \quad Z_j^n = \langle \Omega | O_j | n \rangle, \quad j, k = 1, \ldots, 18.$$  \hspace{1cm} (3)

The physical system for given quantum numbers is created from the vacuum $|\Omega\rangle$ using creation operators $O_j$ at time $t_{src}$ and the system propagates for time $t$ before being annihilated at $t_{ana}$ by $O_j$. The creation/annihilation operators are called interpolators. The right-hand side is obtained by means of spectral decomposition after $I = \sum_n |n\rangle \langle n|$ is inserted.

We employ fourteen interpolators $O_{M_1 M_2}$ that couple well to the two-particle states and add diquark-antidiquark interpolators $O^{d\bar{d}}$ which may be important for the exotic state $Z_c^*$. A full description of our interpolators is given in the Supplementary Material.

Lattice QCD shows evidence the $Z_c^-$ has a large tetraquark component.

Evidence for a charged charmonium-like $Z+c$ from QCD, Sasa Prelovsek, C.B. Lang, Luka Leskovec, Daniel Mohler, [arXiv:1405.7623]
Ljubljana-Graz tetraquarks

Figure 3: The spectrum in the channel $I^G(J^{PC}) = 1^+(1^{+-})$. a,b same as in Fig. 1 where the lattice spectrum is based on the full 18 x 18 correlation matrix; c shows the lattice spectrum based on the 14 x 14 correlation matrix without diquark-antidiquark interpolating fields $O^{bq}_{14}$; spectra d-g are based on truncated correlation matrices as described in the figure; spectrum h is based on $O^{bq}_{14}$ only. The horizontal lines represent energies of the non-interacting two-particle states (2). Statistical errors on the lattice spectrum are shown.
Excited tetraquarks are resonances, decaying into many channels, 30 for the last experimental $Z_{c}^−$ candidates.

For an absolute evidence, the different partial decay widths should be computed as well in lattice QCD.

With present computers and the phase shift Lüscher method, only resonances with just one open decay channel have been studied in lattice QCD, with sufficient detail.
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Conclusion
A four-body potential is computed in lattice QCD, consistent with the 4-body confining potentials used in confining quark models.

Detailed analysis of the tetraquark potential and flip-flop in SU(3) lattice QCD, Fumiko Okiharu, Hideo Suganuma, Toru T. Takahashi,

Flux Tubes

At $T = 0$, we find a 4-body tetraquark flux tube.

By the way, we also find a 5-body pentaquark flux tube.

Colour Fields of the Static Pentaquark System Computed in SU(3) Lattice QCD, Nuno Cardoso, Pedro Bicudo, Phys.Rev. D87 (2013)
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Conclusion
Mechanism: the light quarks screen the antistatic-antistatic interaction;
however screening only acts at a distance typical of the light quark wavefunction.

Lattice QCD signal for a bottom-bottom tetraquark, Pedro Bicudo, Marc Wagner, Phys.Rev. D87 (2013) 11, 114511
In some channels we find attraction, in others we find attraction.

We utilize this potential computed in lattice QCD to solve a Schrödinger equation.

We find binding in the $B - B$ system, but not in $D - B$ or $D - D$ systems.
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Only the spin-scalar and static potentials have been computed for exotic systems

- The spin-tensor (hyperfine, spin-orbit, tensor) potentials have been computed in lattice QCD only for static mesonic systems.
- Already in baryons we are not sure what potential to utilize.
- In exotic systems hybrids, glueballs, tetraquarks, pentaquarks ...) we have no idea on the spin-tensor potentials.
- Approximation: start by ignoring spin-tensor potentials.

Even the spin-scalar potential is not exactly determined yet

- Static quark potentials may be different from light/dynamical potentials.
- There are Coulomb and Logarithmic corrections to the confining linear, spin-scalar potential.
- The constant shift is undetermined.
- The constituent quark mass is gauge dependent.

**Approximation:** start with linear and Coulomb potentials

- Data fit: $A + B \cdot \ln(R)$
  
  \begin{align*}
  A &= 0.1477 \pm 0.0035 \\
  B &= 0.0762 \pm 0.0090 \\
  \chi^2/\text{dof} &= 0.383
  \end{align*}

Inside the SU(3) quark-antiquark QCD flux tube: screening versus quantum widening, N. Cardoso, M. Cardoso, P. Bicudo, arXiv:1302.3633

Fermat points - where the flux tubes match

- The string sections linking the multiquarks join at Fermat points.
- There is a geometrical method to construct the two Fermat points $F_I$ and $F_{II}$ of a tetraquark. Notice in general the quarks are not coplanar.
- **Solution:** We develop a numerical sequence converging to the Fermat points.

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Conclusion
Since there is no evidence for long distance polarization forces, or Van der Waals forces, in hadron-hadron interactions, the two-body confinement potentials cannot be right, they are wrong!

Flip-flop potentials have the correct screening

Solution: use a flip-flop potential, where confining flux tubes or strings take the geometry minimizing the energy of the system.

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Fermi Golden Rule cannot be applied to tetraquarks

When the vector basis of the initial state and decay state are orthogonal,

$$\langle i | f \rangle = 0$$  \hspace{1cm} (1)

for instance when the initial state is an excited atom, and the final state is an atom in a less excited state plus a photon, $$\langle e^- | e^- \gamma \rangle = 0$$; then it is standard to compute decay widths, either with the Fermi Golden Rule, or with Feynman Diagrams,

$$T_{i \rightarrow f} = \frac{2\pi}{\hbar} \langle i | H | f \rangle^2 \rho$$ \hspace{1cm} (2)
Meson - meson and tetraquark are not orthogonal

- However, when studying the tetraquark decay to meson-meson channels, all three colour states are non-orthogonal:
  \[
  \langle 1\ 1 | 1\ 1' \rangle = \frac{1}{3}
  \]
  \[
  \langle 1\ 1 | \bar{3}\ 3 \rangle = \sqrt{\frac{1}{3}}
  \]
  \[
  \langle \bar{3}\ 3 | 1\ 1' \rangle = \sqrt{\frac{1}{3}},
  \]

- contrary to the decay of 1 meson → 2 mesons, in orthogonal Fock spaces, here the Fock space of on tetraquark and two mesons are not orthogonal. This is a orthogonality problem, it may lead to a non-hermitian hamiltonian.

- the potential is open in some variables, compact/confined in others.

- **Solution**: use a complete basis.

Decays of tetraquark resonances in a two-variable approximation to the triple flip-flop potential Pedro Bicudo, Marco Cardoso, Phys.Rev. D83 (2011) 094010
Approximation: Use the second lower potential as the colour excitation, say lowest meson-meson, $1^- - 1$ or $1^- - 1'$, as the excitation of the tetraquark $\bar{3} - 3$, etc.

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Conclusion
Our solution shown in Marco Cardoso’s talk this morning consists in:

- utilizing a non-orthogonal basis of meson-meson wavefunctions,
- we have a flip-flop potentials with 3 sectors: meson-meson, meson-meson and tetraquark potentials,
- we also have a flip-flop in the colour excited potential,
- in the center of mass we have 9 coordinates, but we fold the 8 confined directions with integrals in the mesonic wavefunctions,
- we are left with coupled channel meson-meson Schrödinger equations,
- from the spherical outgoing wavefunctions, we extract phase shifts,
- from a simple phase shift analysis we determine the boundstates and resonances.

Summary

- Tetraquarks have been suggested by theorists since the onset of QCD.
- Tetraquarks have been difficult to observe experimentally, but recently $1^{-+}\pi_1$ (perhaps) and $Z^-$ (most likely) have been confirmed. Tetraquarks are a main highlight in particle physics.
- Lattice QCD is crucial for the theoretical confirmation of tetraquarks. The observed tetraquarks are not yet fully computed in Lattice QCD, but lattice QCD progresses constantly and will eventually compute them.
- Quark models are necessary to understand tetraquarks. One encounters several new, interesting, difficult problems in tetraquark resonances. Only a perfect model will be able to simulate tetraquarks quantitatively.

_to be continued_ ...