nucl-th/9806058 18 Jun 1998

Bethe–Salpeter–Approach to Relativistic Two–Fermion–Systems with a Separable Nonstatic Interaction

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Abstract. To study the characteristic features of relativistic bound systems, the Bethe-Salpeter equation (BSE) for two equal mass spin 1/2 particles (like the deuteron) is solved in the cm-frame for a covariant separable interaction kernel. For that purpose the BSE is transformed to an eigenvalue problem which is diagonalized numerically. The Bethe–Salpeter amplitudes (BSAs) are obtained straightforwardly from the resulting eigenvectors. Only positive parity solutions of the eigenvalue problem are considered. To correlate the BSAs to standard quantum mechanical wavefunctions, the corresponding equal–time–wavefunctions (ETWs) are calculated. A decomposition of BSAs and ETWs in partial waves in angular momenta and parity is performed.

As a first application elastic electron–deuteron–scattering in the impulse approximation (IA) is considered. The charge, magnetic and quadrupole formfactors $F_C(k^2)$, $F_M(k^2)$, $F_Q(k^2)$ and tensor polarizations $\tilde{t}_{20}(k^2)$, $t_{20}(k^2, \theta_e = 70°)$ are obtained from three independent matrix elements of the deuteron current in the Breit–frame of elastic electron–deuteron–scattering. Additionally, the formfactors $A(k^2)$ and $B(k^2)$ of the *Rosenbluth formula* are calculated.

1 Introduction

The discussion of many body systems on a nuclear scale requires in most cases a quantum mechanical and relativistic treatment. One approach, most probably the correct description of such systems, is given by the BSE.
Unfortunately the practical solution of the BSE for most particle dynamics is highly non–trivial and extremely complex. There are different ways out of this dilemma. One of them

[1] FAU-TP3-96/2
[2] Work supported by Kernforschungszentrum Jülich under contract No. ER–41154523
[3] Thanks to Prof. G. Rupp (Lisbon) for many fruitful discussions and remarks
[4] See also: Few-Body Systems 15 (1993) N24

is to give up covariance and to use static interaction kernels in a Schrödinger or Dirac like description of the many particle problem, excluding retardation effects in interaction.
The way we follow in this note is to choose a covariant separable interaction kernel to satisfy covariance and to simplify the mathematical treatment of the BSE. As we are interested in bound systems we solve the homogenous BSE. Our test particle is the deuteron as a weakly bound two–fermion–system with two constituents, the proton and the neutron. The choice of our phenomenological interaction kernel results in a manageable, though still complex mathematical problem. Goal of this paper is the formulation and solution of the problem with a first simplistic application to elastic electron–deuteron–scattering. A more detailed discussion of the formalism and applications to selected systems will be presented elsewhere (e.g. the discussion of relativistic effects on bound state wavefuctions and formfactors or the investigation of antiparticle effects in few or many body systems at high energies).

2 Solution of the BSE in the rest frame of the deuteron

Starting point is the homogenous BSE describing the deuteron (mass M_d) as a bound system of a proton and a neutron (four—momenta p_1 and p_2, mass m) in terms of the Jacobi four–momenta $P := p_1 + p_2$ and $q := \frac{1}{2}\,(p_1 - p_2)$:

$$S_{F2}^{-1}(P,q)\,\Psi(P,q) = -\frac{i}{(2\pi)^4}\int d^4k\; K(P,q\,;k)\,\Psi(P,k) \tag{1}$$

with

$$S_{F2}(P,q) = \left[\,(\frac{1}{2}\not{P} + \not{q} - m + i\varepsilon)\otimes(\frac{1}{2}\not{P} - \not{q} - m + i\varepsilon)\right]^{-1} \tag{2}$$

(to begin with we drop self–energy corrections to the nucleon mass m).
The most general parity conserving interaction kernel in terms of the various invariants of Dirac matrices is given as:

$$\begin{aligned} K(P\,,q\,;k) \;=\;\; & g^2\cdot(\;\; g_s\;\; f_s(P\,,q\,;k) \quad 1_4\otimes 1_4 \\ & +\;\; g_v\;\; f_v(P\,,q\,;k) \quad \gamma_\mu\otimes\gamma^\mu \\ & +\;\; g_t\;\; f_t(P\,,q\,;k) \quad \sigma_{\mu\nu}\otimes\sigma^{\mu\nu} \\ & +\;\; g_p\;\; f_p(P\,,q\,;k) \quad \gamma_5\otimes\gamma^5 \\ & +\; g_{pv}\, f_{pv}(P\,,q\,;k) \quad \gamma_5\gamma_\mu\otimes\gamma^5\gamma^\mu\;) \end{aligned} \tag{3}$$

For the practical calculation we use as interaction, respecting covariance, retardation effects and integrability of the Bethe–Salpeter–Equation (BSE), the following separable kernel ($q^2 = (q^0)^2 - |\vec{q}|^2$):

$$K(q\,;k) \;=\; g^2\,(g_s\,v_s(q)\,v_s(k)\,\Gamma_s + g_v\,v_v(q)\,v_v(k)\,\Gamma_v + g_p\,v_p(q)\,v_p(k)\,\Gamma_p) \tag{4}$$

with

$$g_s = 1 \qquad \text{and} \qquad v_i(q) := \frac{\Lambda_i}{q^2 - \Lambda_i^2 + i\varepsilon} \qquad (i = s,v,p) \tag{5}$$

and with the spin–structures:

$$\begin{aligned} \Gamma_s &= 1_4 \otimes 1_4 \\ \Gamma_v &\stackrel{!}{=} -\gamma^0 \otimes \gamma^0 + g_0 \sum_{i=1}^{3} \gamma^i \otimes \gamma^i \\ \Gamma_p &= \gamma^5 \otimes \gamma^5 \end{aligned} \tag{6}$$

The interaction parameters Λ_s, Λ_v, Λ_p and the coupling constants g_v, g_p, g_0 are input parameters. The coupling constant g^2 will be calculated from the eigenvalue condition.

2.1 Formulation of the eigenvalue problem

Combination of the interaction kernel (4) and the homogenous BSE (1) yields:

$$S_{F2}^{-1}(P,q)\,\Psi(P,q) = -\frac{i}{(2\pi)^4}\int d^4k \left(g^2 \cdot \sum_j g_j\, v_j(q)\, v_j(k)\, \Gamma_j \right) \Psi(P,k) \qquad (j = s,v,p) \tag{7}$$

or after some transformations:

$$\Psi(P,q) = -\frac{ig^2}{(2\pi)^4} S_{F2}(P,q) \sum_j v_j(q)\, g_j\, \Gamma_j \int d^4k\, v_j(k)\, \Psi(P,k) \tag{8}$$

Multiplication by $v_i(q)$ and integration over q leads to:

$$\int d^4q\, v_i(q)\, \Psi(P,q) = -\frac{ig^2}{(2\pi)^4} \int d^4q\, S_{F2}(P,q) \sum_j v_i(q)\, v_j(q)\, g_j\, \Gamma_j \int d^4k\, v_j(k)\, \Psi(P,k) \tag{9}$$

$$(i,j = s,v,p)$$

For convenience we introduce the following quantities:

$$X_i := \int d^4q\, v_i(q)\, \Psi(P,q) \tag{10}$$

$$A_{ij} := -\frac{i}{(2\pi)^4} \int d^4q\, S_{F2}(P,q)\, v_i(q)\, v_j(q) \tag{11}$$

(Note that X_i and A_{ij} are functions of the total four–momentum P !)
Using the definitions above the eigenvalue problem is obtained from (9) as:

$$\frac{1}{g^2}\, X_i = \sum_j A_{ij}\, g_j\, \Gamma_j\, X_j \qquad (i,j = s,v,p) \tag{12}$$

or equivalently ($\lambda := g^{-2}$):

$$\begin{pmatrix} (g_s A_{ss}\Gamma_s) & (g_v A_{sv}\Gamma_v) & (g_p A_{sp}\Gamma_p) \\ (g_s A_{vs}\Gamma_s) & (g_v A_{vv}\Gamma_v) & (g_p A_{vp}\Gamma_p) \\ (g_s A_{ps}\Gamma_s) & (g_v A_{pv}\Gamma_v) & (g_p A_{pp}\Gamma_p) \end{pmatrix} \begin{pmatrix} X_s \\ X_v \\ X_p \end{pmatrix} \quad = \quad \lambda \begin{pmatrix} X_s \\ X_v \\ X_p \end{pmatrix} \tag{13}$$

The (unnormalized) BSAs $\Psi(P,q)$ are calculated from the eigenvectors X_j and the corresponding eigenvalues g^2 from eq. (8) by:

$$\Psi(P,q) = -\frac{ig^2}{(2\pi)^4} S_{F2}(P,q) \sum_j v_j(q)\, g_j\, \Gamma_j\, X_j \tag{14}$$

Not all eigensolutions of the BSE are "physical" solutions; there are e.g. solutions with negative norm. Unphysical solutions of the BSE are sometimes called "Bethe–Salpeter ghosts" and are discussed e.g. in [Nak69].
For our interaction kernel the normalization condition for the BSA reads:

$$\pi i \int d^4q\, \tilde{\Psi}_{B'}(P,q) \frac{\partial}{\partial P_\mu} \left[S_{F2}(P,q)\right]^{-1}\Big|_{P^2=M_d^2} \Psi_B(P,q) \quad = \quad P^\mu\, \delta_{B'B} \tag{15}$$

Here $\tilde{\Psi}$ is the adjoint BSA, B and B' are additional quantum numbers of the two–body bound states.

2.2 Calculation of A_{ij} in the rest frame of the two–particle system

For the numerical treatment of the eigenvalue problem (13) the matrix A_{ij} from eq. (11) has to be calculated. By the use of the definition of $v_i(q)$ in (5) and the rationalized version of the free two–fermion–propagator $S_{F2}(P,q)$ from (2):

$$S_{F2}(P,q) \quad = \quad \frac{(\frac{1}{2}\not{P} + \not{q} + m) \otimes (\frac{1}{2}\not{P} - \not{q} + m)}{((\frac{P}{2}+q)^2 - m^2 + i\varepsilon)((\frac{P}{2}-q)^2 - m^2 + i\varepsilon)} \tag{16}$$

the matrix A_{ij} can be rewritten as:

$$A_{ij} = -\frac{i}{(2\pi)^4} \int d^4q\, \frac{Z(P,q,m)}{N^{(ij)}(P,q,m)} \cdot \Lambda_i \Lambda_j \tag{17}$$

with

$$Z(P,q,m) := (\frac{1}{2}\not{P} + \not{q} + m) \otimes (\frac{1}{2}\not{P} - \not{q} + m) \tag{18}$$

$$N^{(ij)}(P,q,m) := ((\frac{P}{2}+q)^2 - m^2 + i\varepsilon)((\frac{P}{2}-q)^2 - m^2 + i\varepsilon)(q^2 - \Lambda_i^2 + i\varepsilon)(q^2 - \Lambda_j^2 + i\varepsilon) \tag{19}$$

Multiplying out the numerator $Z(P,q,m)$ leads to:

$$
\begin{aligned}
A_{ij} \stackrel{!}{=} \frac{\Lambda_i \Lambda_j}{(2\pi)^4} \Big\{ & \left(\tfrac{1}{4} P_\mu P_\nu \, I^{(ij)} - I^{(ij)}_{\mu\nu}\right) \quad (\gamma^\mu \otimes \gamma^\nu) & + \\
+ & \left(\tfrac{m}{2} P_\mu \, I^{(ij)}\right) \quad ((\gamma^\mu \otimes 1_4) + (1_4 \otimes \gamma^\mu)) & + \\
+ & \left(m^2 \, I^{(ij)}\right) \quad (1_4 \otimes 1_4) & \Big\}
\end{aligned}
\tag{20}
$$

with the integrals:

$$
\begin{aligned}
I^{(ij)} &:= -i \int d^4q \, \frac{1}{N^{(ij)}(P,q,m)} \\
I^{(ij)}_\mu &:= -i \int d^4q \, \frac{q_\mu}{N^{(ij)}(P,q,m)} \quad \stackrel{!}{=} \quad 0 \\
I^{(ij)}_{\mu\nu} &:= -i \int d^4q \, \frac{q_\mu q_\nu}{N^{(ij)}(P,q,m)}
\end{aligned}
\tag{21}
$$

These integrations have been performed in the rest frame of the bound system; specifically for the deuteron $P^\mu = (M_d, \vec{0})$. The connection between M_d and E_B (E_B = binding energy of the deuteron) is:

$$
P^2 \quad = \quad (2\,m - E_B)^2 \quad = \quad M_d^2 \tag{22}
$$

2.3 Parity, angular momentum and the BSA

The 16×16–matrices $(g_j A_{ij} \Gamma_j)$ $(i,j = s,v,p)$ in (13) may be represented by 4×4–matrices with matrix elements being themselves 4×4–matrices, consisting of the 4×4–unity–matrix 1_4, the two particle spin operator σ and its square σ^2.
The two particle spin operator σ is defined as follows (σ^k with $k = 1,2,3$ are the Pauli matrices in the z–representation):

$$
\sigma \quad = \quad \sum_{k=1}^{3} \sigma^k \otimes \sigma^k \quad = \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Its eigenvalues λ^S_σ are:

$$
\begin{aligned}
\lambda^0_\sigma &= -3 \quad \text{(singlet)} \\
\lambda^1_\sigma &= \quad 1 \quad \text{(triplet)}
\end{aligned}
$$

The corresponding orthonormal eigenvectors $\chi^S_{m_S}$ are for the singlet ($S = 0$) and for the triplet ($S = 1$) spin states:

$$\chi_0^0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \qquad \chi_1^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \chi_0^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \qquad \chi_{-1}^1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The subspace can be separated from the eigenvalue problem (13) by the ansatz for the (16–component–)eigenvector X_j:

$$X_j = X_j^{(S)} = \begin{pmatrix} X_j^{++(S)} \\ X_j^{+-(S)} \\ X_j^{-+(S)} \\ X_j^{--(S)} \end{pmatrix} \chi_{m_S}^S \qquad (j = s, v, p) \tag{23}$$

The parity operator $\hat{\mathcal{P}}$ for the BSA is defined as:

$$\hat{\mathcal{P}} \quad := \quad (\gamma^0 \otimes \gamma^0) \;\; \hat{\mathcal{P}}^0 \tag{24}$$

with the so called "orbital parity operator" $\hat{\mathcal{P}}^0$:

$$\hat{\mathcal{P}}^0 \; \Psi(P^0, \vec{P}; q^0, \vec{q}) \quad = \quad \Psi(P^0, -\vec{P}; q^0, -\vec{q}) \tag{25}$$

Finally, in our representation:

$$\gamma^0 \otimes \gamma^0 \quad = \quad \begin{pmatrix} 1_4 & 0 & 0 & 0 \\ 0 & -1_4 & 0 & 0 \\ 0 & 0 & -1_4 & 0 \\ 0 & 0 & 0 & 1_4 \end{pmatrix} \tag{26}$$

It is easy to show the effect of the parity operator $\hat{\mathcal{P}}$ on the BSA of (14) for $P^\mu = (M_d, \vec{0})$:

$$\hat{\mathcal{P}} \; \Psi(P, q) \stackrel{!}{=} -\frac{ig^2}{(2\pi)^4} S_{F2}(P, q) \; \sum_j v_j(q) \, g_j \, \Gamma_j \; (\gamma^0 \otimes \gamma^0) \, X_j \qquad (j = s, v, p) \tag{27}$$

i.e. the parity of the BSA only depends on the operation of $\gamma^0 \otimes \gamma^0$ on X_j. As the parity operator $\hat{\mathcal{P}}$ commutes with the interaction kernel $K(q\,;k)$ in (4), there exist eigensolutions of the BSA with positive and negative parity which can be selected (as one can easily see from (27) and (26)) with the following X_j $(j = s, v, p)$:

$$X_{+\,j}^{(S\,m_S)} := \begin{pmatrix} X_j^{++(S)} \\ 0 \\ 0 \\ X_j^{--(S)} \end{pmatrix} \chi_{m_S}^S \qquad , \qquad X_{-\,j}^{(S\,m_S)} := \begin{pmatrix} 0 \\ X_j^{+-(S)} \\ X_j^{-+(S)} \\ 0 \end{pmatrix} \chi_{m_S}^S \qquad (j = s, v, p) \tag{28}$$

For the deuteron only the states of positive parity (e.g. ${}^3S_1$, ${}^3D_1$, ...) (spectroscopic notation: ${}^{2\,S+1}L_J$) are observed. For that reason we only will consider eigensolutions of positive parity in the next few pages. For completeness we mention that in case of negative parity states the eigenvalue problem (13) can be solved analytically.

For a decomposition of the BSA into angular momenta we introduce the two fermion spherical harmonics $Y^M_{LSJ}(\Omega)$ which couple the spin angular momentum of a spin 1 particle (represented by the four–spinors $\chi^S_{m_S}$) to the orbital angular momentum (represented by the ordinary sperical harmonics $Y_{Lm_L}(\Omega)$):

$$Y^M_{LSJ}(\Omega) = \sum_{m_L,m_S} < LSm_Lm_S|JM > Y_{Lm_L}(\Omega)\,\chi^S_{m_S} \tag{29}$$

with the orthonormality relation:

$$\int d\Omega\, Y^{M'\,\dagger}_{L'S'J'}(\Omega)\, Y^M_{LSJ}(\Omega) = \delta_{M'M}\,\delta_{L'L}\,\delta_{S'S}\,\delta_{J'J} \tag{30}$$

(The Clebsch–Gordan–coefficients follow the convention of Condon & Shortley [Con35]). The application of the projections:

$$\chi^J_M = \sqrt{4\pi}\,Y^M_{0JJ}(\Omega)\ \ (J=0,\ M=0 \text{ or } J=1,\ M=0,\pm1) \tag{31}$$

$$(\frac{\vec\sigma\cdot\vec q}{|\vec q|}\otimes 1_2)\,\chi^0_0 = -\sqrt{4\pi}\,Y^0_{110}(\Omega) \tag{32}$$

$$(\frac{\vec\sigma\cdot\vec q}{|\vec q|}\otimes 1_2)\,\chi^1_M = \sqrt{\frac{4\pi}{3}}\,(Y^M_{101}(\Omega) - \sqrt{2}\,Y^M_{111}(\Omega)) \qquad (M=0,\pm1) \tag{33}$$

$$(1_2\otimes\frac{\vec\sigma\cdot\vec q}{|\vec q|})\,\chi^0_0 = \sqrt{4\pi}\,Y^0_{110}(\Omega) \tag{34}$$

$$(1_2\otimes\frac{\vec\sigma\cdot\vec q}{|\vec q|})\,\chi^1_M = \sqrt{\frac{4\pi}{3}}\,(-Y^M_{101}(\Omega) - \sqrt{2}\,Y^M_{111}(\Omega)) \qquad (M=0,\pm1) \tag{35}$$

$$(\frac{\vec\sigma\cdot\vec q}{|\vec q|}\otimes\frac{\vec\sigma\cdot\vec q}{|\vec q|})\,\chi^0_0 = -\sqrt{4\pi}\,Y^0_{000}(\Omega) \tag{36}$$

$$(\frac{\vec\sigma\cdot\vec q}{|\vec q|}\otimes\frac{\vec\sigma\cdot\vec q}{|\vec q|})\,\chi^1_M = \frac{\sqrt{4\pi}}{3}\,(Y^M_{011}(\Omega) + \sqrt{8}\,Y^M_{211}(\Omega)) \qquad (M=0,\pm1) \tag{37}$$

to the positive parity BSA combining (14) and (28):

$$\Psi^{(JM)}(P,q) = -\frac{i(g^{(J)}(P))^2}{(2\pi)^4} S_{F2}(P,q)\sum_j v_j(q)\,g_j\,\Gamma_j\,X^{(JM)}_{+j}(P) \qquad (j=s,v,p) \tag{38}$$

leads to the following decomposition of the positive parity BSA in partial waves:

$$\Psi^{(00)}(M_d,\vec 0;q) = \begin{pmatrix} \Psi_{\,^1S^+_0}(M_d;q_0,|\vec q|)\,Y^0_{000}(\Omega) \\ \Psi_{\,^3P^+_0}(M_d;q_0,|\vec q|)\,Y^0_{110}(\Omega) \\ \Psi_{\,^3P^-_0}(M_d;q_0,|\vec q|)\,Y^0_{110}(\Omega) \\ \Psi_{\,^1S^-_0}(M_d;q_0,|\vec q|)\,Y^0_{000}(\Omega) \end{pmatrix} \tag{39}$$

$$\Psi^{(1\,M)}\left(M_d\,,\vec{0};q\right) \;=\; \begin{pmatrix} \Psi\,{}^3S_1^+\left(M_d\,;q_0,|\vec{q}|\right)\,Y_{011}^M(\Omega) \;+\; \Psi\,{}^3D_1^+\left(M_d\,;q_0,|\vec{q}|\right)\,Y_{211}^M(\Omega) \\ \Psi\,{}^3P_1^+\left(M_d\,;q_0,|\vec{q}|\right)\,Y_{111}^M(\Omega) \;+\; \Psi\,{}^1P_1^+\left(M_d\,;q_0,|\vec{q}|\right)\,Y_{101}^M(\Omega) \\ \Psi\,{}^3P_1^-\left(M_d\,;q_0,|\vec{q}|\right)\,Y_{111}^M(\Omega) \;+\; \Psi\,{}^1P_1^-\left(M_d\,;q_0,|\vec{q}|\right)\,Y_{101}^M(\Omega) \\ \Psi\,{}^3S_1^-\left(M_d\,;q_0,|\vec{q}|\right)\,Y_{011}^M(\Omega) \;+\; \Psi\,{}^3D_1^-\left(M_d\,;q_0,|\vec{q}|\right)\,Y_{211}^M(\Omega) \end{pmatrix} \tag{40}$$

in the rest frame of the deuteron.

3 The equal-time-wavefunction $\Phi\left(P,\vec{q}\right)$ (ETW)

To correlate the determined BSAs to conventional bound state wavefunctions, the corresponding ETWs $\Phi\left(P,\vec{q}\right)$ are computed by:

$$\Phi\left(P,\vec{q}\right) \;:=\; \int dq^0\;\Psi(P,q)\,e^{i\,q^0x^0}\Big|_{x^0\,=\,0} \;\stackrel{!}{=}\; \int dq^0\;\Psi(P,q) \tag{41}$$

With the definition of:

$$I_{_{(i)}}(P,\vec{q}) \;:=\; \int dq^0\;S_{F2}\left(P,q\right)v_i(q) \qquad (i=s,v,p) \tag{42}$$

the BSA (38) changes in the rest frame to:

$$\Phi^{(JM)}(M_d\,,\vec{0};\vec{q}) = -\frac{i(g^{(J)}(M_d\,,\vec{0}))^2}{(2\pi)^4}\sum_j I_{_{(j)}}(M_d\,,\vec{0};\vec{q})\,g_j\,\Gamma_j\,X_{+\,j}^{(JM)}(M_d\,,\vec{0}) \quad (j=s,v,p) \tag{43}$$

The decomposition in partial waves gives for the positive parity ETW in the rest frame:

$$\Phi^{(00)}\left(M_d\,,\vec{0};\vec{q}\right) \;=\; \begin{pmatrix} \Phi\,{}^1S_0^+\left(M_d\,;|\vec{q}|\right)\,Y_{000}^0(\Omega) \\ \Phi\,{}^3P_0^+\left(M_d\,;|\vec{q}|\right)\,Y_{110}^0(\Omega) \\ \Phi\,{}^3P_0^-\left(M_d\,;|\vec{q}|\right)\,Y_{110}^0(\Omega) \\ \Phi\,{}^1S_0^-\left(M_d\,;|\vec{q}|\right)\,Y_{000}^0(\Omega) \end{pmatrix} \tag{44}$$

$$\Phi^{(1\,M)}(M_d\,,\vec{0};\vec{q}) \;=\; \left(\begin{array}{lcl} \Phi_{\;^3S_1^+}(M_d\,;|\vec{q}|)\;Y^M_{011}(\Omega) & + & \Phi_{\;^3D_1^+}(M_d\,;|\vec{q}|)\;Y^M_{211}(\Omega) \\ \Phi_{\;^3P_1^+}(M_d\,;|\vec{q}|)\;Y^M_{111}(\Omega) & + & \Phi_{\;^1P_1^+}(M_d\,;|\vec{q}|)\;Y^M_{101}(\Omega) \\ \Phi_{\;^3P_1^-}(M_d\,;|\vec{q}|)\;Y^M_{111}(\Omega) & + & \Phi_{\;^1P_1^-}(M_d\,;|\vec{q}|)\;Y^M_{101}(\Omega) \\ \Phi_{\;^3S_1^-}(M_d\,;|\vec{q}|)\;Y^M_{011}(\Omega) & + & \Phi_{\;^3D_1^-}(M_d\,;|\vec{q}|)\;Y^M_{211}(\Omega) \end{array}\right) \tag{45}$$

The free two–fermion–propagator can be expanded with the help of the two fermion energy projection operators $\Lambda^{\pm\oplus}(\vec{q}) := \Lambda^{\pm}(\vec{q}) \otimes \Lambda^{\oplus}(-\vec{q})$:

$$S_{F2}(M_d\,,\vec{0}\,;q) \quad =$$

$$= \quad \left(\sum_{\pm\oplus} \frac{\Lambda^{\pm\oplus}(\vec{q})}{((E_N+q^0)\pm(-\,\omega(\vec{q})+i\varepsilon))\,((E_N-q^0)\bigoplus(-\,\omega(\vec{q})+i\varepsilon))}\right)\;\left(\gamma^0\otimes\gamma^0\right) \tag{46}$$

$$(E_N := \frac{M_d}{2} \quad , \quad \omega(\vec{q}) := \sqrt{|\vec{q}|^2+m^2} \quad , \quad \omega_i(\vec{q}) := \sqrt{|\vec{q}|^2+\Lambda_i^2} \quad)$$

Upon performing the integration in (42), we find in the rest frame of the bound state:

$$\begin{aligned} I_{_{(i)}}(M_d\,,\vec{0}\,;\vec{q}) \;=\; & \frac{2\,\pi}{i} \;\cdot\; \Big(\quad w^{++}_{_{(i)}}(E_N;\vec{q})\;\Lambda^{++}(\vec{q}) \quad + \\ & + \quad w^{+-}_{_{(i)}}(E_N;\vec{q})\;\Lambda^{+-}(\vec{q}) \quad + \\ & + \quad w^{-+}_{_{(i)}}(E_N;\vec{q})\;\Lambda^{-+}(\vec{q}) \quad + \\ & + \quad w^{--}_{_{(i)}}(E_N;\vec{q})\;\Lambda^{--}(\vec{q})\Big)\;\left(\gamma^0\otimes\gamma^0\right) \end{aligned} \tag{47}$$

with

$$\begin{aligned} w^{++}_{_{(i)}}(E_N;\vec{q}) \;:=\; & \frac{\Lambda_i}{2\,(E_N-\omega(\vec{q}))\,((E_N-\omega(\vec{q}))^2-\omega_i^2(\vec{q}))} \quad + \\ & + \quad \frac{\Lambda_i}{((E_N-\omega(\vec{q}))^2-\omega_i^2(\vec{q}))}\;\cdot\;\frac{1}{2\,\omega_i(\vec{q})} \end{aligned}$$

$$w^{+-}_{_{(i)}}(E_N;\vec{q}) \;:=\; \frac{\Lambda_i}{(E_N^2-(\omega(\vec{q})+\omega_i(\vec{q}))^2)}\;\cdot\;\frac{1}{2\,\omega_i(\vec{q})}$$

$$w_{(i)}^{-+}(E_N;\vec{q}) := \frac{\Lambda_i}{(E_N^2-(\omega(\vec{q})+\omega_i(\vec{q}))^2)} \cdot \frac{1}{2\,\omega_i(\vec{q})}$$

$$w_{(i)}^{--}(E_N;\vec{q}) := \frac{-\Lambda_i}{2\,(E_N+\omega(\vec{q}))\,((E_N+\omega(\vec{q}))^2-\omega_i^2(\vec{q}))} +$$

$$+ \frac{\Lambda_i}{((E_N+\omega(\vec{q}))^2-\omega_i^2(\vec{q}))} \cdot \frac{1}{2\,\omega_i(\vec{q})} \tag{48}$$

Explicit expressions for the partial waves of equation (44) and (45) are given in appendix A. Partial waves of the ETW in the rest frame (in arbitary units) are shown in Fig. 1 (scalar interaction) and Fig. 2 (full interaction) for one selected eigensolution of the BSE. In the presented examples one can see the strong dependence of the ${}^3D_1^+$ wave and the antiparticle content of the BSA represented e.g. by the ${}^3S_1^-$ wave on the choice of the coupling constants in our model.

4 Elastic e-d-scattering in the IA and deuteron form factors

As a first application, we consider elastic electron–deuteron–scattering in impulse approximation (IA), following a similar route as in M.J. Zuilhof and J.A. Tjon 1980 [Zui80]. The connection between the BSA and the deuteron current matrix elements in the IA is given as [Mic92]:

$$< P_f, M_f|j_d^\mu|P_i, M_i> \quad =$$

$$\stackrel{!}{=} \quad 2\pi i\,\frac{e}{M_d}\int d^4q\;\tilde{\Psi}_{M_f}(P_f,\,q+\frac{1}{2}k)\;\left(\Gamma^\mu(k^2)\;\otimes\;(\frac{1}{2}\;\not{P}_i-\not{q}-m)\right)\;\Psi_{M_i}(P_i,q) \tag{49}$$

Thereby, M_i and M_f is the polarization of the deuteron before and after scattering, respectively, k is the four–momentum transfer by the photon. The vertex function $\Gamma^\mu(k^2)$ is evaluated with the isoscalar (dipole) formfactors $F_1^S(k^2)$ and $F_2^S(k^2)$ of the nucleon:

$$\Gamma^\mu(k^2) \quad = \quad \gamma^\mu\;F_1^S(k^2)\;+\;\frac{i}{2m}\,\sigma^{\mu\nu}\,k_\nu\;F_2^S(k^2) \tag{50}$$

For the nucleonic formfactors $F_1^S(k^2)$ and $F_2^S(k^2)$ we use fits of Iachello et al. [Iac73]. From the current matrix elements obtained we can calculate the observables of the deuteron (e.g. formfactors). First we introduce the following covariant and contravariant spherical unit vectors $\vec{\varepsilon}_M$ and $\vec{\varepsilon}^{\,M}$ $(M=+1,0,-1)$ expressed in terms of the cartesian ones $(\vec{e}_x,\vec{e}_y,\vec{e}_z)$:

$$\vec{\varepsilon}_{\pm1} := \mp\frac{1}{\sqrt{2}}(\vec{e}_x \pm i\vec{e}_y) =: (\vec{\varepsilon}^{\pm1})^* \quad , \quad \vec{\varepsilon}_0 := \vec{e}_z =: (\vec{\varepsilon}^{0})^*$$

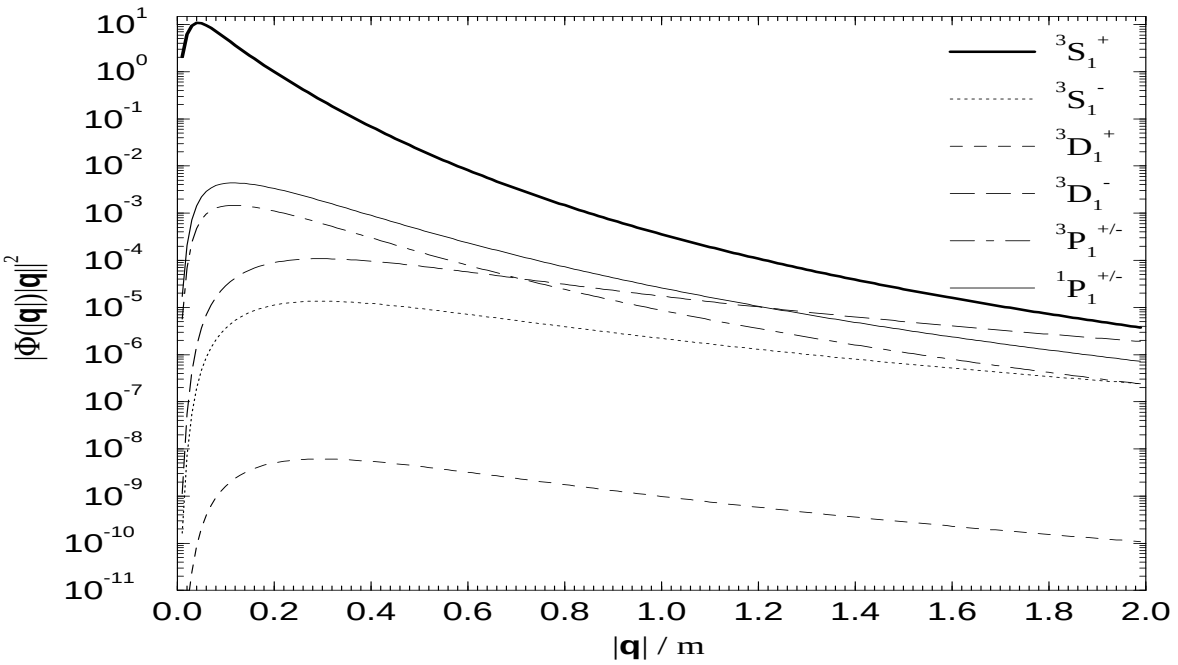

Figure 1: Partial wave decomposition of the $J^\pi = 1^+$ Bethe–Salpeter amplitude for $\Lambda_s = \Lambda_v = \Lambda_p = 0.34$, $g_v = g_0 = g_p = 0$, $E_B = 0.002$ (the cut–off masses Λ_i and the binding energy E_B are given in units of the nucleon mass)

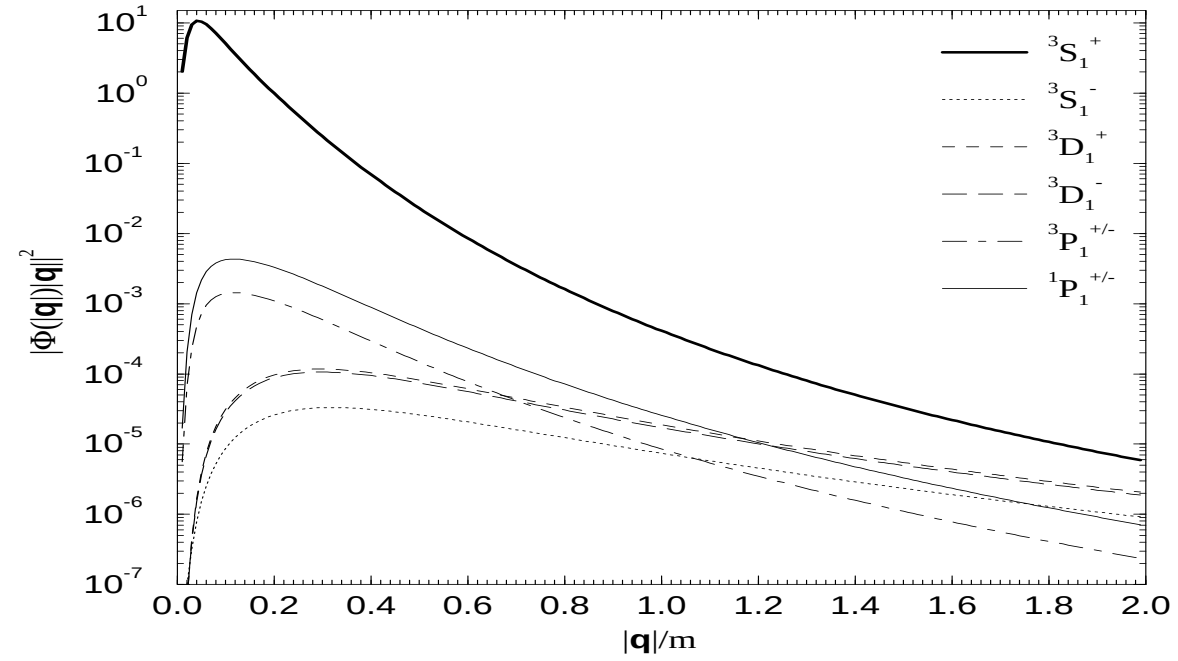

Figure 2: As Fig. 1, however for the parameters: $\Lambda_s = \Lambda_v = \Lambda_p = 0.34$, $g_v = 1.2$, $g_0 = 1.$, $g_p = -0.99$, $E_B = 0.002$

with the properties $(M, M' = +1, 0, -1)$:

$$\vec{\varepsilon}_M = (-1)^M \vec{\varepsilon}^{\,-M} \quad \text{and} \quad \vec{\varepsilon}^{\,M} \cdot \vec{\varepsilon}_{M'} = \delta^M_{M'}$$

Contravariant and covariant components of spherical vectors are defined by $\vec{A} = A^{\,M} \vec{\varepsilon}_M = A_M \vec{\varepsilon}^{\,M}$. Now we can define the polarization vectors for a massive spin 1 particle like the deuteron:

$$\varepsilon^{\,\mu\,M}(P) = (\varepsilon^{\,0\,M}(P)\,,\, \vec{\varepsilon}^{\,M}(P)) := \left(\frac{\vec{P} \cdot \vec{\varepsilon}^{\,M}}{M_d}\,,\, \vec{\varepsilon}^{\,M} + \frac{\vec{P} \cdot \vec{\varepsilon}^{\,M}}{M_d\,(P^0 + M_d)}\,\vec{P} \right)$$

where μ is a Lorentz index with $\mu = 0, 1, 2, 3$. The properties of the polarization vectors are well known:

$$\begin{aligned}
\varepsilon^{\,M}(P) \cdot \varepsilon_{M'}(P) \quad &= \quad \varepsilon^{\,\mu\,M}(P)\,\varepsilon_{\mu\,M'}(P) \quad &= \quad -\,\delta^M_{M'} \\
\sum_M \left(\varepsilon_{\mu\,M}(P)\right)^*\,\varepsilon_{\nu\,M}(P) \quad &= \quad \varepsilon_\mu^{\,M}(P)\,\varepsilon_{\nu\,M}(P) \quad &= \quad -\,g_{\mu\nu} + \frac{P_\mu P_\nu}{M_d^2} \\
& \qquad\qquad P^{\,\mu}\varepsilon_{\mu\,M}(P) \quad &= \quad 0
\end{aligned}$$

In terms of the momentum transfer $k = P_f - P_i$ and the polarization vectors the current matrix elements of a massive spin 1 particle can be expressed in the following covariant way [Gla57] [Zui80] [Rup90] [Hum90]:

$$< P_f, M_f | j^d_\mu | P_i, M_i > \quad = \quad -\,\frac{e}{2\,M_d}\,\varepsilon^*_{\rho\,M_f}(P_f)\;J^{\rho\sigma}_\mu\;\varepsilon_{\sigma M_i}(P_i) \tag{51}$$

with the current tensor:

$$J^{\rho\sigma}_\mu \quad = \quad (P_{f,\mu} + P_{i,\mu}) \left[g^{\rho\sigma}\,F_1(k^2) \;-\; \frac{k^\rho k^\sigma}{2\,M_d^2}\,F_2\,(k^2) \right] + i\,I^{\rho\sigma}_{\mu\nu} k^\nu\,G_1(k^2) \tag{52}$$

Here $F_1(k^2)$, $F_2(k^2)$, $G_1(k^2)$ are formfactors and $I^{\rho\sigma}_{\mu\nu} = i\,(g^\rho_\mu g^\sigma_\nu - g^\rho_\nu g^\sigma_\mu)$ are the generators of the Lorentz group. Going to the Breit–frame ($k^0 = 0$) and choosing $\vec{k}$ in the z–direction, i.e. $k^{\,\mu} = (0, 0, 0, k_z)$, evluation of equation (51) with respect to elastic scattering leads to the following relation between the formfactors and the current matrix elements ($\eta := -k^2/(2\,M_d)^2$):

$$\begin{aligned}
< P_f, M_f | j^0_d | P_i, M_i > \quad &= \quad e\,\sqrt{1+\eta}\,\Big\{ F_1\,\delta^{\,M_f}_{M_i} \;+ \\
&+ \quad 2\,\eta\;\left[F_1 + (1+\eta)\,F_2 - G_1\right]\,\delta^{\,M_f}_0\,\delta^0_{M_i} \Big\}
\end{aligned} \tag{53}$$

$$< P_f, M_f | j^1_d | P_i, M_i > \quad = \quad e\,\frac{k_z}{2\,M_d}\sqrt{\frac{1+\eta}{2}}\,G_1\,(\delta^{\,M_f}_{M_i+1} - \delta^{\,M_f}_{M_i-1}) \tag{54}$$

$$< P_f, M_f | j^2_d | P_i, M_i > \quad = \quad -\,ie\,\frac{k_z}{2\,M_d}\sqrt{\frac{1+\eta}{2}}\,G_1\,(\delta^{\,M_f}_{M_i+1} + \delta^{\,M_f}_{M_i-1}) \tag{55}$$

$$< P_f, M_f | j^3_d | P_i, M_i > \quad = \quad 0 \tag{56}$$

Equation (56) is the continuity equation for the deuteron current in our frame of reference. The charge, magnetic and quadrupole formfactors $F_C(k^2)$, $F_M(k^2)$, $F_Q(k^2)$ are related to the formfactors $F_1(k^2)$, $F_2(k^2)$, $G_1(k^2)$ by (see e.g. [Gou63]):

$$F_C(k^2) = F_1(k^2) + \frac{2}{3}\eta\left[F_1(k^2) + (1+\eta)\,F_2(k^2) - G_1(k^2)\right] \tag{57}$$

$$F_M(k^2) = G_1(k^2) \tag{58}$$

$$F_Q(k^2) = F_1(k^2) + (1+\eta)\,F_2(k^2) - G_1(k^2) \tag{59}$$

At $k^2 = 0$ they have the following values [Gar94]:

$$e\,F_C(0) = 1\,e \quad , \quad \frac{e}{2\,M_d}F_M(0) = \mu_d \quad , \quad \frac{e}{M_d^2}F_Q(0) = Q_d \tag{60}$$

$1\,e$, μ_d and Q_d are the deuteron's charge, magnetic and quadrupole moment (we note that in [Gla57] the quadrupole moment is defined by $Q_d = \frac{e}{M_d^2}F_2(0)$). Hence in the Breit-frame $F_C(k^2)$, $F_M(k^2)$, $F_Q(k^2)$ are obtained from three independent matrix elements of the deuteron current, e.g.:

$$e\sqrt{1+\eta}\,F_C = \frac{1}{3} < P_f, 0|j_d^0|P_i, 0 > + \frac{2}{3} < P_f, 1|j_d^0|P_i, 1 > \tag{61}$$

$$e\,\frac{k_z}{2\,M_d}\sqrt{\frac{1+\eta}{2}}\,F_M = < P_f, 1|j_d^1|P_i, 0 > \tag{62}$$

$$2\,\eta\,e\sqrt{1+\eta}\,F_Q = < P_f, 0|j_d^0|P_i, 0 > - < P_f, 1|j_d^0|P_i, 1 > \tag{63}$$

The formfactors $A(k^2)$, $B(k^2)$ of the *Rosenbluth formula* are calculated easily by (see e.g. [Gar94]):

$$A(k^2) = F_C^2 + \frac{8}{9}\eta^2\,F_Q^2 + \frac{2}{3}\eta\,F_M^2 \quad , \quad B(k^2) = \frac{4}{3}\eta\,(\eta+1)\,F_M^2 \tag{64}$$

with the appropriately boosted BSAs and deuteron currents from the deuteron rest frame to the Breit–frame. Knowing $F_C(k^2)$, $F_M(k^2)$, $F_Q(k^2)$ it is straightforward to calculate e.g. the simplified tensor polarization $\tilde{t}_{20}$ discussed in [Gar94]:

$$\tilde{t}_{20} = -\sqrt{2}\,\frac{x(x+2)}{1+2x^2} \quad \text{with} \quad x = \frac{2\eta F_Q}{3F_C} \tag{65}$$

and the complete (observable) tensor polarization t_{20} defined by:

$$t_{20} = -\sqrt{2}\,\frac{x(x+2)+y/2}{1+2\,(x^2+y)} \quad \text{with} \quad y = \frac{2\eta}{3}\,(\,\frac{1}{2} + (1+\eta)\,\tan^2\frac{\theta_e}{2})\,\left(\frac{F_M}{F_C}\right)^2 \tag{66}$$

As a characteristic result, formfactors and tensor polarizations of one selected eigensolution of the BSE are shown in Figs. 3 to 9 for the set of parameters summarized in Table 1.

——	$\Lambda_s = \Lambda_v = \Lambda_p = 0.24,$
	$E_B = 2.371 \cdot 10^{-3} \quad , \quad g_v = 1.285714 \quad , \quad g_0 = 1. \quad , \quad g_p = 0.897$
	$\Rightarrow \quad \mu_d = 6.685\,\mu_K \quad , \quad Q_d = 1.287\,\mathrm{fm}^2$
$\cdots$	$\Lambda_s = \Lambda_v = \Lambda_p = 0.24,$
	$E_B = 2.371 \cdot 10^{-3} \quad , \quad g_v = 1.097139 \quad , \quad g_0 = 1. \quad , \quad g_p = 1.522057$
	$\Rightarrow \quad \mu_d = 0.8570\,\mu_K \quad , \quad Q_d = 0.2860\,\mathrm{fm}^2$
- - -	$\Lambda_s = 0.30 \quad , \quad \Lambda_v = \Lambda_p = 0.24,$
	$E_B = 2.371 \cdot 10^{-3} \quad , \quad g_v = 2.309181 \quad , \quad g_0 = 1. \quad , \quad g_p = 2.608756$
	$\Rightarrow \quad \mu_d = 0.8571\,\mu_K \quad , \quad Q_d = 0.2860\,\mathrm{fm}^2$

Table 1: Parameters used in Figs. 3 to 9 (Interaction parameters and binding energy are given in units of the nucleon mass, coupling constants are dimensionless, $\mu_K = \frac{e}{2m}$ is the nuclear magneton)

Λ_s	Λ_v	Λ_p	g_v	g_p	μ_d/μ_K	Q_d [fm^2]
0.24	0.24	0.22	1.285714	0.89	6.413	1.261
0.24	0.24	0.24	1.285714	0.897	6.685	1.287
0.24	0.24	0.24	1.285714	2.85	6.674	1.340
0.32	0.32	0.32	1.171429	2.5	7.230	1.102
0.32	0.32	0.32	1.285714	2.1	6.977	0.7958
0.32	0.32	0.32	3.0	2.55	6.980	0.6127
0.32	0.32	0.32	9.142858	3.540875	6.973	0.7586

Table 2: Typical parameter dependence of the magnetic and quadrupole moment for $g_0 = 1$ and $E_B = 2.371 \cdot 10^{-3}$

To get a slight feeling for the dependence of observables like μ_d and Q_d on the parameters of our model we fixed the first node of the charge formfactor F_C to about $k_z^2 = 20\,\mathrm{fm}^{-2}$ (which is suggested by experiment) and varied coupling constants and interaction parameters. Typical results are listed in Table 2.
Clearly, at present our parameter studies are by far not exhaustive due to practical restrictions. For unequal interaction parameters the analytical and numerical integrations for the boost of BSAs from rest frame of the deuteron to the Breit frame are very involved and time expensive, additionally the positive parity eigensolutions of the BSE in our model are no more degenerate. Finally fixing the node of the charge formfactor to $k_z^2 = 20\,\mathrm{fm}^{-2}$ is only achieved by a very fine tuning of the coupling constants. All this makes a systematic discussion of the coupling space of our model extremly difficult. Nevertheless, by our experience with the model we draw some more general statements to our results in the conclusions.
We close with a final remark. It has been shown by B. Michel [Mic92] that for $\Lambda_s = \Lambda_v = \Lambda_p$ and $g_0 = 1$ positive parity eigensolutions of the model can be controlled by one simple parameter y_-, which is defined by:

$$y_- = \frac{[g_p + (4J-3)\,g_v] + (1-g_v)\,\dfrac{X^{--(J)}}{X^{++(J)}}}{(1-g_v) + [g_p + (4J-3)\,g_v]\,\dfrac{X^{--(J)}}{X^{++(J)}}} \tag{67}$$

and which is the ratio between the $++$ and the $--$ component of the BSA (because of the degeneracy of the interaction parameters the index j of $X_j^{++(J)}$ and $X_j^{--(J)}$ is dropped). We defer a more detailed discussion of this interesting feature to a forthcoming publication.

5 Conclusion

In this paper we developped the formalism for the covariant description of bound fermion–antifermion systems in the framework of the BSE. To faciliate the very complex solution of the problem, the kernel of BSE was represented by a covariant one–rank separable interaction piece for each of our 3 spin–invariants, which reduces the solution of the BSE to an algebraic problem. By integration over the relative energy variable, the full nonstatic BSAs were related to standard static 3–dimensional wave functions in momentum space.

As a first step we applied our formalism to the deuteron and investigated frame independent, i.e. covariant "deuteron" wavefuctions together with the corresponding IA formfactors and tensor polarizations. The normalization of the BSA was obtained by normalizing the charge formfactor to 1 for zero momentum transfer. We find that our simple interaction kernel is obviously not able to describe the deuteron accurately. Explicitly there are two scenarios: if, on the one hand, we reproduce magnetic and quadrupole moments, we fail to reproduce the k^2 dependence of the formfactors. On the other hand, upon fixing the k^2 dependence qualitatively from the first node of the charge formfactor F_C at about $k_z^2 = 20\,\mathrm{fm}^{-2}$, the moments turn out to be too large. Nevertheless, it is interesting to see that we are able to control the ${}^3D_1^+$ wave and the antiparticle content of our BSA over a wide range (i.e. it is no problem to obtain ${}^3D_1^+$ wave admixtures of 5 %, even without an explicit tensor force in our interaction kernel).
From its ansatz, our approach is just a first, crude step towards a more realistic covariant description of relativistic bound systems. The crucial point is certainly a more adequate formulation of the interaction kernel within a systematic separable expansion. Such an extension, which is presently under way, then opens up a variety of interesting questions within the model, to name only a systematic investigation of mesonic systems in standard coordinates and on the light cone.

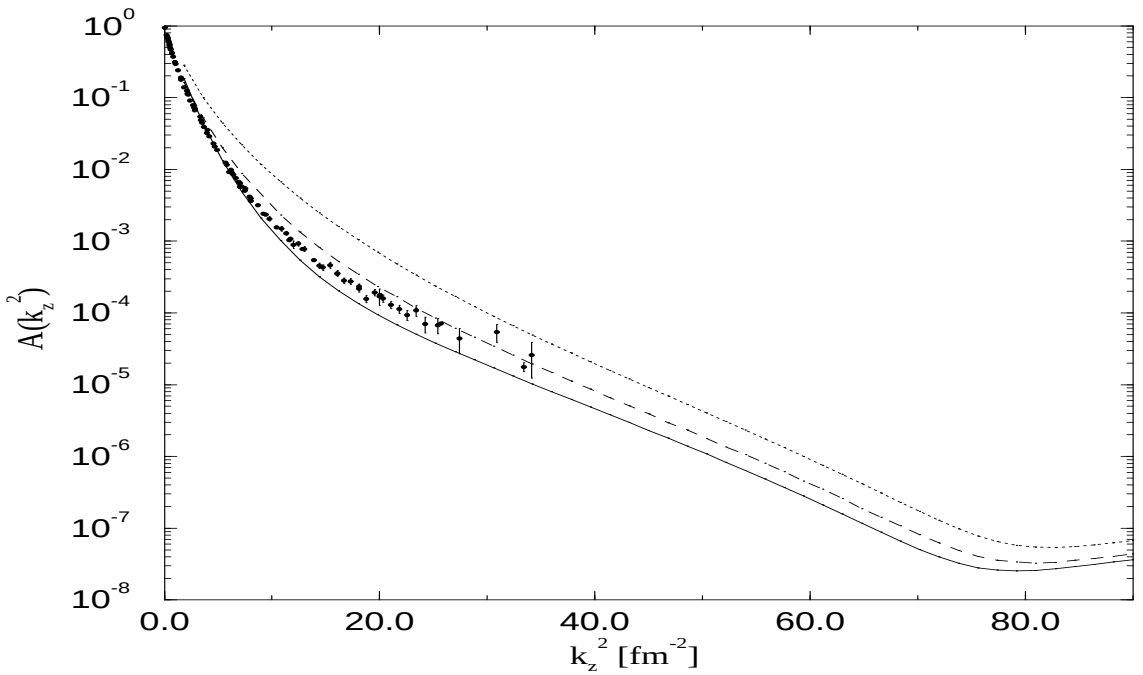

Figure 3: Dependence of the formfactor $A(k_z^2)$ from equ. (64) on the momentum transfer $k^2 = -k_z^2$ (The sets of model parameters compared are summarized in Tab. 1; for experimental data see appendix B)

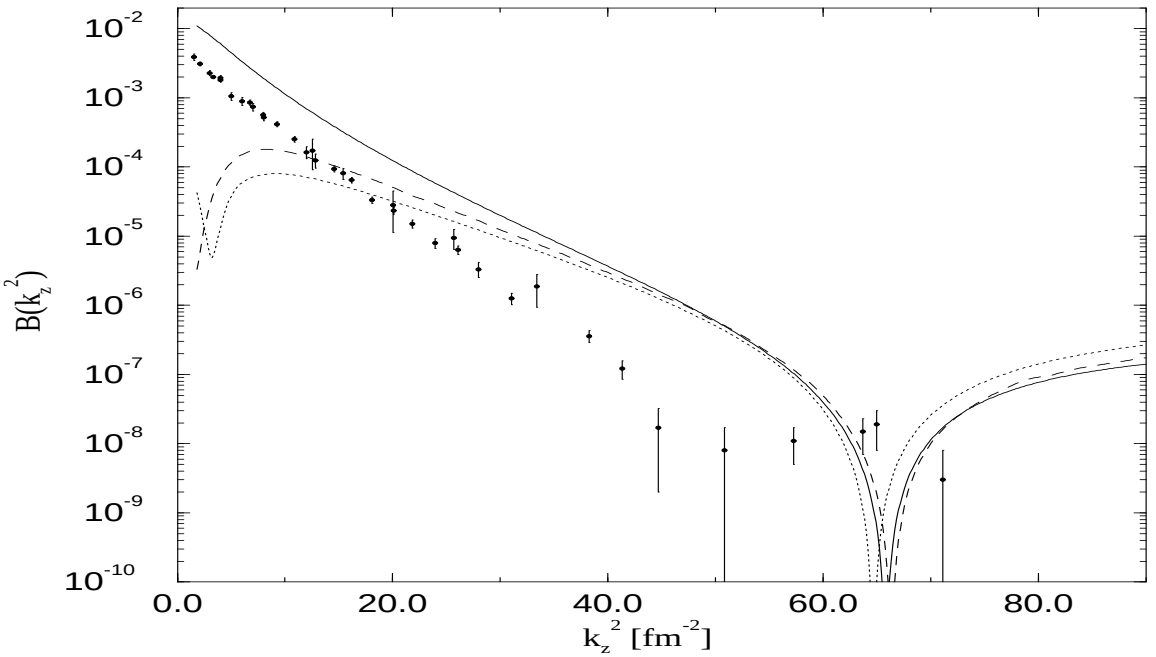

Figure 4: As Fig. 3, however for the formfactor $B(k_z^2)$ from equ. (64)

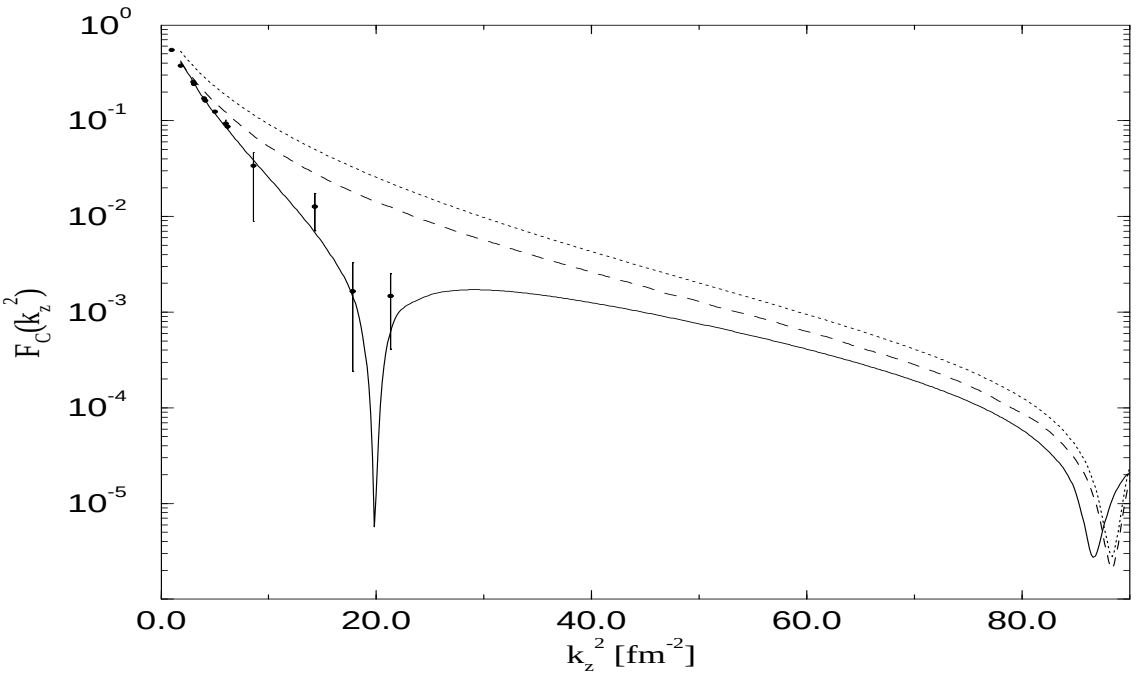

Figure 5: As Fig. 3, however for the formfactor $F_C(k_z^2)$ from equ. (57)

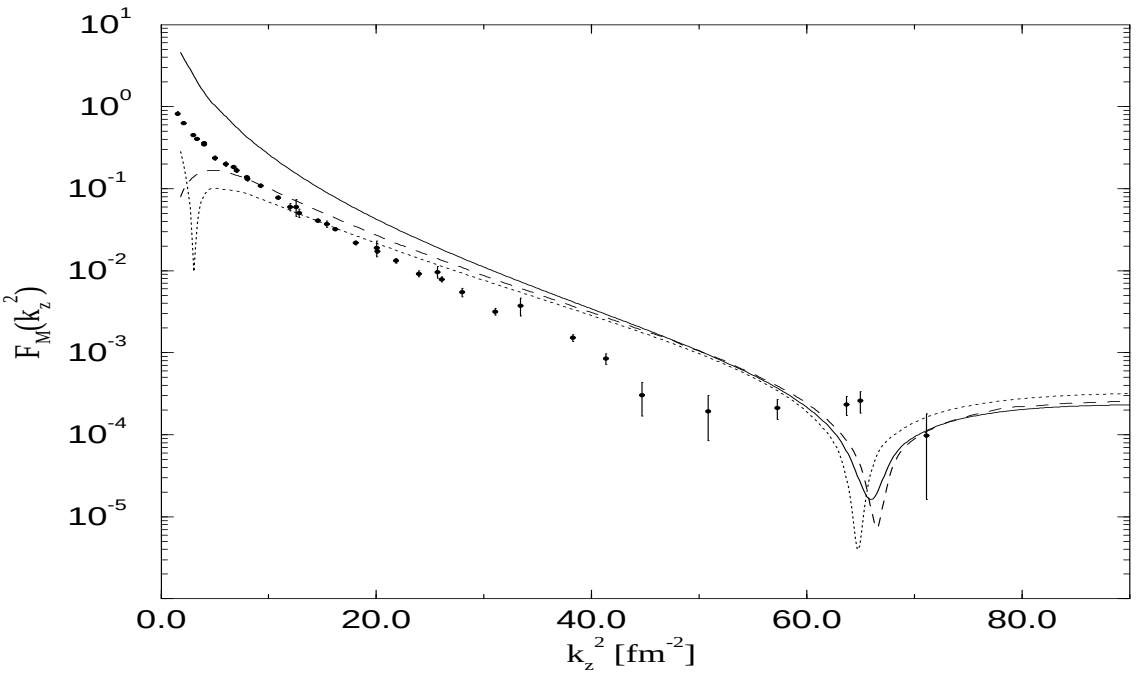

Figure 6: As Fig. 3, however for the formfactor $F_M(k_z^2)$ from equ. (58)

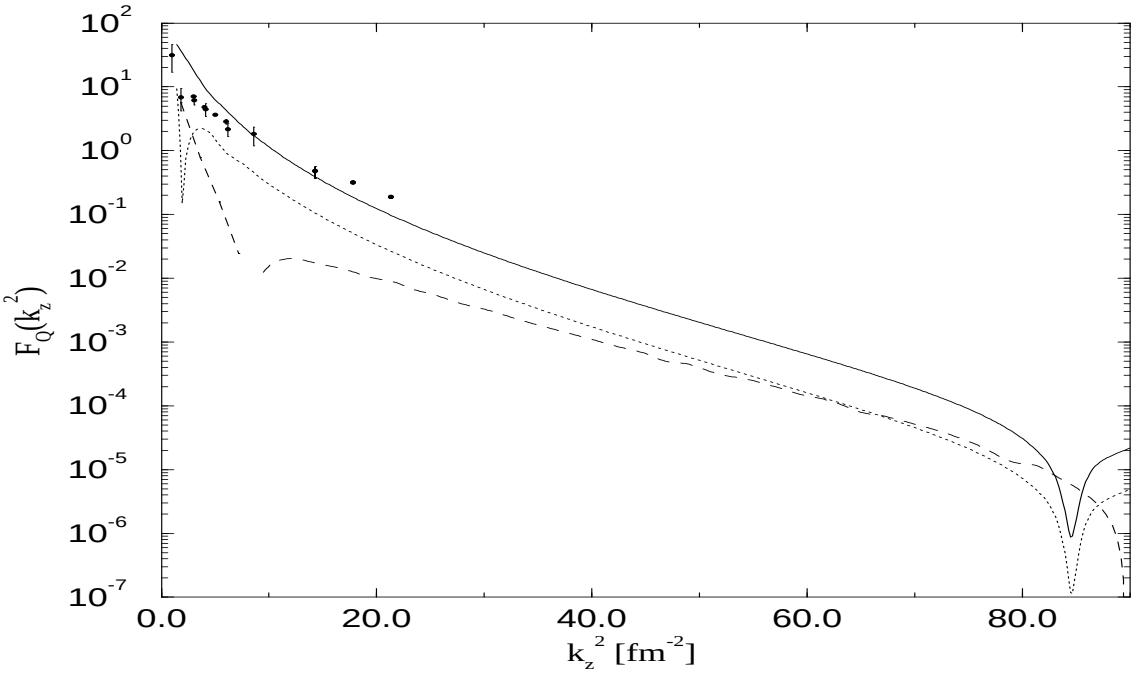

Figure 7: As Fig. 3, however for the formfactor $F_Q(k_z^2)$ from equ. (59)

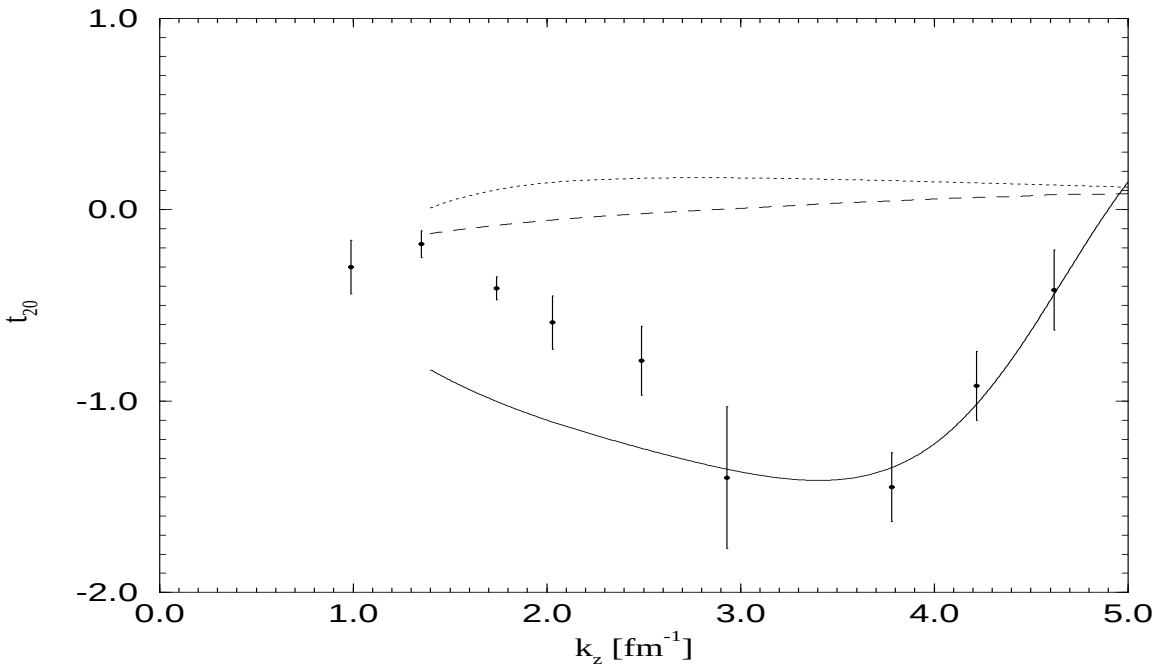

Figure 8: As Fig. 3, however for the tensor polarization $\tilde{t}_{20}(k_z)$ from equ. (65)

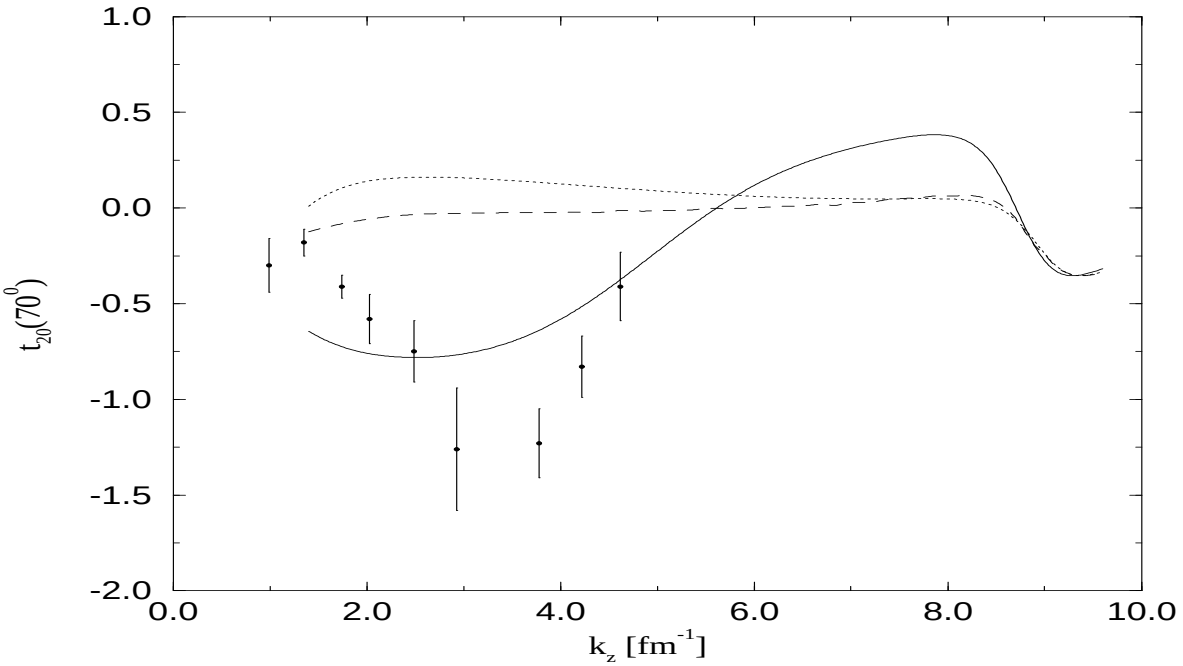

Figure 9: As Fig. 3, however for the tensor polarization $t_{20}(k_z, \theta_e = 70°)$ from equ. (66)

A The partial waves of the ETW in the rest frame of the deuteron

Using the definitions

$$\tilde{\omega}^{\pm}(\vec{q}) \quad := \quad \omega(\vec{q}) \pm m \qquad , \qquad \tilde{\omega}^{\oplus}(\vec{q}) \quad := \quad \omega(\vec{q}) \oplus m \tag{68}$$

we give here the expicit expressions for the partial waves obtained by evaluation of equation (43).
We get for $J = 0$:

$$\begin{aligned}
\Phi_{\,^1S_0^+}(M_d\,;|\vec{q}|) \;=\; & -\sum_{\pm\oplus}\frac{\sqrt{\pi}}{2}\cdot\frac{(g^{(0)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})} \quad \cdot \\
& \cdot\Bigg\{ \quad w_{(s)}^{\pm\oplus}(E_N;\vec{q})\,(\tilde{\omega}^{\pm}(\vec{q})\,\tilde{\omega}^{\oplus}(\vec{q})\,X_s^{++(0)} \pm \oplus \;\; X_s^{--(0)}\cdot|\vec{q}|^2) \quad + \\
& + \quad w_{(v)}^{\pm\oplus}(E_N;\vec{q})\,g_v\,\Big(-(\tilde{\omega}^{\pm}(\vec{q})\,\tilde{\omega}^{\oplus}(\vec{q}) \pm \oplus\; 3\,g_0\,|\vec{q}|^2)\,X_v^{++(0)} \quad - \\
& - \quad (3\,g_0\,\tilde{\omega}^{\pm}(\vec{q})\,\tilde{\omega}^{\oplus}(\vec{q}) \pm \oplus\; |\vec{q}|^2)\,X_v^{--(0)}\Big) \quad + \\
& + \quad w_{(p)}^{\pm\oplus}(E_N;\vec{q})\,g_p\,(\tilde{\omega}^{\pm}(\vec{q})\,\tilde{\omega}^{\oplus}(\vec{q})\,X_p^{--(0)} \pm \oplus \;\; X_p^{++(0)}\cdot|\vec{q}|^2)\Bigg\}
\end{aligned}$$

$$\begin{aligned}
\Phi_{\,^3P_0^+}(M_d\,;|\vec{q}|) \;=\; & \sum_{\pm\oplus}\frac{\sqrt{\pi}}{2}\cdot\frac{(g^{(0)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})}\cdot|\vec{q}|\;\cdot \\
& \cdot\Bigg\{ \quad w_{(s)}^{\pm\oplus}(E_N;\vec{q})\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_s^{--(0)} \oplus \tilde{\omega}^{\pm}(\vec{q})\;X_s^{++(0)}) \quad + \\
& + \quad w_{(v)}^{\pm\oplus}(E_N;\vec{q})\,g_v\,\Big(-(\pm\,3\,g_0\,\tilde{\omega}^{\oplus}(\vec{q}) \oplus \tilde{\omega}^{\pm}(\vec{q}))\,X_v^{++(0)} \quad - \\
& - \quad (\pm\,\tilde{\omega}^{\oplus}(\vec{q}) \oplus 3\,g_0\,\tilde{\omega}^{\pm}(\vec{q}))\,X_v^{--(0)}\Big) \quad + \\
& + \quad w_{(p)}^{\pm\oplus}(E_N;\vec{q})\,g_p\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_p^{++(0)} \oplus \tilde{\omega}^{\pm}(\vec{q})\;X_p^{--(0)})\Bigg\}
\end{aligned}$$

$$\begin{aligned}
\Phi_{\,^3P_0^-}(M_d\,;|\vec{q}|) \;=\; & \sum_{\pm\oplus}\frac{\sqrt{\pi}}{2}\cdot\frac{(g^{(0)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})}\cdot|\vec{q}|\;\cdot \\
& \cdot\Bigg\{ \quad w_{(s)}^{\pm\oplus}(E_N;\vec{q})\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_s^{++(0)} \oplus \tilde{\omega}^{\mp}(\vec{q})\;X_s^{--(0)}) \quad + \\
& + \quad w_{(v)}^{\pm\oplus}(E_N;\vec{q})\,g_v\,\Big(-(\pm\,3\,g_0\,\tilde{\omega}^{\oplus}(\vec{q}) \oplus \tilde{\omega}^{\mp}(\vec{q}))\,X_v^{--(0)} \quad -
\end{aligned}$$

$$
\begin{aligned}
&- \quad (\pm\,\tilde{\omega}^{\oplus}\,(\vec{q}) \oplus\!\!\!\!\!\!\ominus\, 3\,g_0\,\tilde{\omega}^{\mp}\,(\vec{q}))\,X_v^{++(0)}\Big) \quad + \\
&+ \quad w_{(p)}^{\pm\oplus}\,(E_N;\vec{q})\,g_p\,(\pm\,\tilde{\omega}^{\oplus}\,(\vec{q})\,X_p^{--(0)} \oplus\!\!\!\!\!\!\ominus\, \tilde{\omega}^{\mp}\,(\vec{q})\;X_p^{++(0)})\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\Phi\,{}^1S_0^-\,(M_d\,;|\vec{q}|) \;=\; & -\sum_{\pm\oplus}\frac{\sqrt{\pi}}{2}\;\cdot\;\frac{(g^{(0)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})} \quad\cdot \\
\cdot\Big\{ & \quad w_{(s)}^{\pm\oplus}\,(E_N;\vec{q})\,(\tilde{\omega}^{\mp}\,(\vec{q})\,\tilde{\omega}^{\oplus}\,(\vec{q})\,X_s^{--(0)} \pm \oplus\!\!\!\!\!\!\ominus\;\; X_s^{++(0)}\cdot|\vec{q}|^2) \quad + \\
& + \quad w_{(v)}^{\pm\oplus}\,(E_N;\vec{q})\,g_v\,\Big(-\,(\tilde{\omega}^{\mp}\,(\vec{q})\,\tilde{\omega}^{\oplus}\,(\vec{q}) \pm \oplus\!\!\!\!\!\!\ominus\; 3\,g_0\,|\vec{q}|^2)\,X_v^{--(0)} \quad - \\
& - \quad (3\,g_0\,\tilde{\omega}^{\mp}\,(\vec{q})\,\tilde{\omega}^{\oplus}\,(\vec{q}) \pm \oplus\!\!\!\!\!\!\ominus\; |\vec{q}|^2)\,X_v^{++(0)}\Big) \quad + \\
& + \quad w_{(p)}^{\pm\oplus}\,(E_N;\vec{q})\,g_p\,(\tilde{\omega}^{\mp}\,(\vec{q})\,\tilde{\omega}^{\oplus}\,(\vec{q})\,X_p^{++(0)} \pm \oplus\!\!\!\!\!\!\ominus\;\; X_p^{--(0)}\cdot|\vec{q}|^2)\Big\}
\end{aligned}
$$

and for $J=1$:

$$
\begin{aligned}
\Phi\,{}^3S_1^+\,(M_d\,;|\vec{q}|) \;=\; & -\sum_{\pm\oplus}\frac{\sqrt{\pi}}{2}\;\cdot\;\frac{(g^{(1)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})} \quad\cdot \\
\cdot\Big\{ & \quad w_{(s)}^{\pm\oplus}\,(E_N;\vec{q})\,(\tilde{\omega}^{\pm}\,(\vec{q})\,\tilde{\omega}^{\oplus}\,(\vec{q})\,X_s^{++(1)} \pm \oplus\!\!\!\!\!\!\ominus\; (-\frac{1}{3})\,X_s^{--(1)}\cdot|\vec{q}|^2) \quad + \\
& + \quad w_{(v)}^{\pm\oplus}\,(E_N;\vec{q})\,g_v\,\Big((-\,\tilde{\omega}^{\pm}\,(\vec{q})\,\tilde{\omega}^{\oplus}\,(\vec{q}) \pm \oplus\!\!\!\!\!\!\ominus\; (-\frac{1}{3})\,g_0\,|\vec{q}|^2)\,X_v^{++(1)} \quad + \\
& + \quad (g_0\,\tilde{\omega}^{\pm}\,(\vec{q})\,\tilde{\omega}^{\oplus}\,(\vec{q}) \pm \oplus\!\!\!\!\!\!\ominus\; \frac{1}{3}\,|\vec{q}|^2)\,X_v^{--(1)}\Big) \quad + \\
& + \quad w_{(p)}^{\pm\oplus}\,(E_N;\vec{q})\,g_p\,(\tilde{\omega}^{\pm}\,(\vec{q})\,\tilde{\omega}^{\oplus}\,(\vec{q})\,X_p^{--(1)} \pm \oplus\!\!\!\!\!\!\ominus\; (-\frac{1}{3})\,X_p^{++(1)}\cdot|\vec{q}|^2)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\Phi\,{}^3D_1^+\,(M_d\,;|\vec{q}|) \;=\; & \sum_{\pm\oplus}\pm\oplus\!\!\!\!\!\!\ominus\,\frac{\sqrt{2\,\pi}}{3}\;\cdot\;\frac{(g^{(1)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})}\;\cdot\;|\vec{q}|^2\;\cdot\;\Big\{w_{(s)}^{\pm\oplus}\,(E_N;\vec{q})\,X_s^{--(1)} \quad + \\
& + \quad w_{(v)}^{\pm\oplus}\,(E_N;\vec{q})\,g_v\,(-\,X_v^{--(1)} + \;g_0\,X_v^{++(1)}) \quad + \\
& + \quad w_{(p)}^{\pm\oplus}\,(E_N;\vec{q})\,g_p\,X_p^{++(1)}\Big\}
\end{aligned}
$$

$$
\Phi\,{}^3P_1^+\,(M_d\,;|\vec{q}|) \;=\; \sum_{\pm\oplus}\sqrt{\frac{\pi}{6}}\;\cdot\;\frac{(g^{(1)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})}\;\cdot\;|\vec{q}|\;\cdot
$$

$$\cdot \Bigg\{ \quad w_{(s)}^{\pm\oplus}(E_N;\vec{q})\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_s^{--(1)} - \bigoplus \tilde{\omega}^{\pm}(\vec{q})\;X_s^{++(1)}) \quad +$$

$$+ \quad w_{(v)}^{\pm\oplus}(E_N;\vec{q})\,g_v\,\Big((\pm\,g_0\,\tilde{\omega}^{\oplus}(\vec{q}) \bigoplus \tilde{\omega}^{\pm}(\vec{q}))\,X_v^{++(1)} \quad -$$

$$- \quad (\pm\,\tilde{\omega}^{\oplus}(\vec{q}) \bigoplus g_0\,\tilde{\omega}^{\pm}(\vec{q}))\,X_v^{--(1)}\Big) \quad +$$

$$+ \quad w_{(p)}^{\pm\oplus}(E_N;\vec{q})\,g_p\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_p^{++(1)} - \bigoplus \tilde{\omega}^{\pm}(\vec{q})\;X_p^{--(1)})\Bigg\}$$

$$\Phi\,{}^1P_1^+(M_d\,;|\vec{q}|) \;=\; -\sum_{\pm\oplus}\frac{\sqrt{\pi}}{2\sqrt{3}}\cdot\frac{(g^{(1)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})}\cdot|\vec{q}|\cdot$$

$$\cdot \Bigg\{ \quad w_{(s)}^{\pm\oplus}(E_N;\vec{q})\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_s^{--(1)} \bigoplus \tilde{\omega}^{\pm}(\vec{q})\;X_s^{++(1)}) \quad +$$

$$+ \quad w_{(v)}^{\pm\oplus}(E_N;\vec{q})\,g_v\,\Big((\pm\,g_0\,\tilde{\omega}^{\oplus}(\vec{q}) - \bigoplus \tilde{\omega}^{\pm}(\vec{q}))\,X_v^{++(1)} \quad -$$

$$- \quad (\pm\,\tilde{\omega}^{\oplus}(\vec{q}) - \bigoplus\; g_0\,\tilde{\omega}^{\pm}(\vec{q}))\,X_v^{--(1)}\Big) \quad +$$

$$+ \quad w_{(p)}^{\pm\oplus}(E_N;\vec{q})\,g_p\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_p^{++(1)} \bigoplus \tilde{\omega}^{\pm}(\vec{q})\;X_p^{--(1)})\Bigg\}$$

$$\Phi\,{}^3P_1^-(M_d\,;|\vec{q}|) \;=\; \sum_{\pm\oplus}\sqrt{\frac{\pi}{6}}\cdot\frac{(g^{(1)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})}\cdot|\vec{q}|\cdot$$

$$\cdot \Bigg\{ \quad w_{(s)}^{\pm\oplus}(E_N;\vec{q})\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_s^{++(1)} - \bigoplus \tilde{\omega}^{\mp}(\vec{q})\;X_s^{--(1)}) \quad +$$

$$+ \quad w_{(v)}^{\pm\oplus}(E_N;\vec{q})\,g_v\,\Big((\pm\,g_0\,\tilde{\omega}^{\oplus}(\vec{q}) \bigoplus \tilde{\omega}^{\mp}(\vec{q}))\,X_v^{--(1)} \quad -$$

$$- \quad (\pm\,\tilde{\omega}^{\oplus}(\vec{q}) \bigoplus g_0\,\tilde{\omega}^{\mp}(\vec{q}))\,X_v^{++(1)}\Big) \quad +$$

$$+ \quad w_{(p)}^{\pm\oplus}(E_N;\vec{q})\,g_p\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_p^{--(1)} - \bigoplus \tilde{\omega}^{\mp}(\vec{q})\;X_p^{++(1)})\Bigg\}$$

$$\Phi\,{}^1P_1^-(M_d\,;|\vec{q}|) \;=\; -\sum_{\pm\oplus}\frac{\sqrt{\pi}}{2\sqrt{3}}\cdot\frac{(g^{(1)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^{\,2}(\vec{q})}\cdot|\vec{q}|\cdot$$

$$\cdot \Bigg\{ \quad w_{(s)}^{\pm\oplus}(E_N;\vec{q})\,(\pm\,\tilde{\omega}^{\oplus}(\vec{q})\,X_s^{++(1)} \bigoplus \tilde{\omega}^{\mp}(\vec{q})\;X_s^{--(1)}) \quad +$$

$$
\begin{aligned}
&+ \quad w^{\pm\oplus}_{(v)}(E_N;\vec{q})\, g_v \Big((\pm g_0\, \tilde{\omega}^{\oplus}(\vec{q}) - \bigoplus \tilde{\omega}^{\mp}(\vec{q}))\, X_v^{--(1)} \quad - \\
&- \quad (\pm \tilde{\omega}^{\oplus}(\vec{q}) - \bigoplus g_0\, \tilde{\omega}^{\mp}(\vec{q}))\, X_v^{++(1)} \Big) \quad + \\
&+ \quad w^{\pm\oplus}_{(p)}(E_N;\vec{q})\, g_p\, (\pm \tilde{\omega}^{\oplus}(\vec{q})\, X_p^{--(1)} \bigoplus \tilde{\omega}^{\mp}(\vec{q})\, X_p^{++(1)}) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
\Phi\,{}^3S_1^-(M_d\,;|\vec{q}|) \quad &= \quad -\sum_{\pm\oplus} \frac{\sqrt{\pi}}{2} \cdot \frac{(g^{(1)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^2(\vec{q})} \quad \cdot \\
&\cdot \Big\{ \quad w^{\pm\oplus}_{(s)}(E_N;\vec{q})\, (\tilde{\omega}^{\mp}(\vec{q})\, \tilde{\omega}^{\oplus}(\vec{q})\, X_s^{--(1)} \pm \bigoplus (-\frac{1}{3})\, X_s^{++(1)} \cdot |\vec{q}|^2) \quad + \\
&+ \quad w^{\pm\oplus}_{(v)}(E_N;\vec{q})\, g_v \Big((-\tilde{\omega}^{\mp}(\vec{q})\, \tilde{\omega}^{\oplus}(\vec{q}) \pm \bigoplus (-\frac{1}{3})\, g_0\, |\vec{q}|^2)\, X_v^{--(1)} \quad + \\
&+ \quad (g_0\, \tilde{\omega}^{\mp}(\vec{q})\, \tilde{\omega}^{\oplus}(\vec{q}) \pm \bigoplus \frac{1}{3}\, |\vec{q}|^2)\, X_v^{++(1)} \Big) \quad + \\
&+ \quad w^{\pm\oplus}_{(p)}(E_N;\vec{q})\, g_p\, (\tilde{\omega}^{\mp}(\vec{q})\, \tilde{\omega}^{\oplus}(\vec{q})\, X_p^{++(1)} \pm \bigoplus (-\frac{1}{3})\, X_p^{--(1)} \cdot |\vec{q}|^2) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
\Phi\,{}^3D_1^-(M_d\,;|\vec{q}|) \quad &= \quad \sum_{\pm\oplus} \pm \bigoplus \frac{\sqrt{2\,\pi}}{3} \cdot \frac{(g^{(1)}(M_d\,,\vec{0}))^2}{(2\pi)^3\,\omega^2(\vec{q})} \cdot |\vec{q}|^2 \cdot \Big\{ w^{\pm\oplus}_{(s)}(E_N;\vec{q})\, X_s^{++(1)} \quad + \\
&+ \quad w^{\pm\oplus}_{(v)}(E_N;\vec{q})\, g_v\, (-X_v^{++(1)} + g_0\, X_v^{--(1)}) \quad + \\
&+ \quad w^{\pm\oplus}_{(p)}(E_N;\vec{q})\, g_p\, X_p^{--(1)} \Big\}
\end{aligned}
$$

B List of experimental data

The following tables consist of all experimental data we have used in our plots. They are either original or derived data from the references quoted at each line in the tables.

k_z^2 [fm^{-2}]	$B(k_z^2)$	$F_M(k_z^2)$	Ref.
.1550E+01	.3900E-02 $^{+\ .4000E-03}_{-\ .4000E-03}$	.8235E+00 $^{+\ .4223E-01}_{-\ .4223E-01}$	Sim81
.2100E+01	.3100E-02 $^{+\ .2000E-03}_{-\ .2000E-03}$	.6303E+00 $^{+\ .2033E-01}_{-\ .2033E-01}$	Sim81
.3000E+01	.2273E-02 $^{+\ .1915E-03}_{-\ .1915E-03}$	.4510E+00 $^{+\ .1900E-01}_{-\ .1900E-01}$	Ben66
.3300E+01	.2000E-02 $^{+\ .1000E-03}_{-\ .1000E-03}$	.4032E+00 $^{+\ .1008E-01}_{-\ .1008E-01}$	Sim81
.4000E+01	.1936E-02 $^{+\ .1614E-03}_{-\ .1614E-03}$	.3600E+00 $^{+\ .1500E-01}_{-\ .1500E-01}$	Ben66
.4000E+01	.1800E-02 $^{+\ .1000E-03}_{-\ .1000E-03}$	.3471E+00 $^{+\ .9641E-02}_{-\ .9641E-02}$	Sim81

k_z^2 [fm^{-2}]	$B(k_z^2)$	$F_M(k_z^2)$	Ref.
.5000E+01	.1052E-02 $^{+.1332E-03}_{-.1332E-03}$	.2370E+00 $^{+.1500E-01}_{-.1500E-01}$	Ben66
.6000E+01	.8920E-03 $^{+.1151E-03}_{-.1151E-03}$	.1990E+00 $^{+.1283E-01}_{-.1283E-01}$	Buc65
.6720E+01	.8510E-03 $^{+.7914E-04}_{-.7914E-04}$	.1834E+00 $^{+.8530E-02}_{-.8530E-02}$	Auf85
.7000E+01	.7390E-03 $^{+.9090E-04}_{-.9090E-04}$	.1674E+00 $^{+.1030E-01}_{-.1030E-01}$	Buc65
.7940E+01	.5660E-03 $^{+.4811E-04}_{-.4811E-04}$	.1374E+00 $^{+.5840E-02}_{-.5840E-02}$	Auf85
.8000E+01	.5230E-03 $^{+.6119E-04}_{-.6119E-04}$	.1316E+00 $^{+.7697E-02}_{-.7697E-02}$	Buc65
.9250E+01	.4170E-03 $^{+.3378E-04}_{-.3378E-04}$	.1091E+00 $^{+.4417E-02}_{-.4417E-02}$	Auf85
.1086E+02	.2520E-03 $^{+.2268E-04}_{-.2268E-04}$	.7808E-01 $^{+.3514E-02}_{-.3514E-02}$	Auf85
.1200E+02	.1640E-03 $^{+.3198E-04}_{-.3198E-04}$	.5983E-01 $^{+.5834E-02}_{-.5834E-02}$	Buc65
.1255E+02	.1720E-03 $^{+.8084E-04}_{-.8084E-04}$	.5987E-01 $^{+.1407E-01}_{-.1407E-01}$	Auf85
.1284E+02	.1250E-03 $^{+.2900E-04}_{-.2900E-04}$	.5054E-01 $^{+.5993E-02}_{-.5993E-02}$	Cra85
.1459E+02	.9380E-04 $^{+.9099E-05}_{-.9099E-05}$	.4090E-01 $^{+.1983E-02}_{-.1983E-02}$	Auf85
.1541E+02	.8090E-04 $^{+.1470E-04}_{-.1470E-04}$	.3696E-01 $^{+.3396E-02}_{-.3396E-02}$	Cra85
.1618E+02	.6460E-04 $^{+.5814E-05}_{-.5814E-05}$	.3216E-01 $^{+.1447E-02}_{-.1447E-02}$	Auf85
.1810E+02	.3340E-04 $^{+.3641E-05}_{-.3641E-05}$	.2181E-01 $^{+.1189E-02}_{-.1189E-02}$	Auf85
.2003E+02	.2810E-04 $^{+.1670E-04}_{-.1670E-04}$	.1896E-01 $^{+.4155E-02}_{-.4155E-02}$	Cra85
.2009E+02	.2350E-04 $^{+.2491E-05}_{-.2491E-05}$	.1732E-01 $^{+.9179E-03}_{-.9179E-03}$	Auf85
.2184E+02	.1510E-04 $^{+.1978E-05}_{-.1978E-05}$	.1328E-01 $^{+.8701E-03}_{-.8701E-03}$	Auf85
.2394E+02	.7940E-05 $^{+.1286E-05}_{-.1286E-05}$	.9175E-02 $^{+.7432E-03}_{-.7432E-03}$	Auf85
.2568E+02	.9480E-05 $^{+.3000E-05}_{-.3000E-05}$	.9649E-02 $^{+.1538E-02}_{-.1538E-02}$	Cra85
.2609E+02	.6330E-05 $^{+.8989E-06}_{-.8989E-06}$	.7826E-02 $^{+.5556E-03}_{-.5556E-03}$	Auf85
.2797E+02	.3330E-05 $^{+.8092E-06}_{-.8092E-06}$	.5469E-02 $^{+.6644E-03}_{-.6644E-03}$	Auf85
.3108E+02	.1260E-05 $^{+.2400E-06}_{-.2400E-06}$	.3179E-02 $^{+.3027E-03}_{-.3027E-03}$	Arn87
.3339E+02	.1870E-05 $^{+.9300E-06}_{-.9300E-06}$	.3716E-02 $^{+.9189E-03}_{-.9189E-03}$	Cra85
.3827E+02	.3600E-06 $^{+.7000E-07}_{-.7000E-07}$	.1517E-02 $^{+.1475E-03}_{-.1475E-03}$	Arn87
.4135E+02	.1220E-06 $^{+.3600E-07}_{-.3600E-07}$	.8465E-03 $^{+.1249E-03}_{-.1249E-03}$	Arn87
.4469E+02	.1700E-07 $^{+.1500E-07}_{-.1500E-07}$	.3027E-03 $^{+.1335E-03}_{-.1335E-03}$	Arn87
.5085E+02	.8000E-08 $^{+.9000E-08}_{-.9000E-08}$	.1932E-03 $^{+.1087E-03}_{-.1087E-03}$	Arn87
.5727E+02	.1100E-07 $^{+.6000E-08}_{-.6000E-08}$	.2118E-03 $^{+.5777E-04}_{-.5777E-04}$	Arn87
.6369E+02	.1500E-07 $^{+.8000E-08}_{-.8000E-08}$	.2328E-03 $^{+.6207E-04}_{-.6207E-04}$	Arn87
.6498E+02	.1900E-07 $^{+.1100E-07}_{-.1100E-07}$	.2590E-03 $^{+.7497E-04}_{-.7497E-04}$	Arn87
.7114E+02	.3000E-08 $^{+.5000E-08}_{-.5000E-08}$	.9765E-04 $^{+.8137E-04}_{-.8137E-04}$	Arn87

k_z^2 [fm^{-2}]	$F_C(k_z^2)$	$F_Q(k_z^2)$	Ref.
.9761E+00	.5500E+00 $^{+\ .7000E-02}_{-\ .7000E-02}$	.3140E+02 $^{+\ .1440E+02}_{-\ .1440E+02}$	Dmi85
.1831E+01	.3770E+00 $^{+\ .3000E-02}_{-\ .3000E-02}$	.6850E+01 $^{+\ .2680E+01}_{-\ .2680E+01}$	Dmi85
.3000E+01	.2550E+00 $^{+\ .7000E-02}_{-\ .7000E-02}$	.7019E+01 $^{+\ .2322E+00}_{-\ .2322E+00}$	Ben66
.3028E+01	.2420E+00 $^{+\ .2000E-02}_{-\ .2000E-02}$	.6130E+01 $^{+\ .9000E+00}_{-\ .9000E+00}$	Sch84
.4000E+01	.1710E+00 $^{+\ .5000E-02}_{-\ .5000E-02}$	.4851E+01 $^{+\ .1548E+00}_{-\ .1548E+00}$	Ben66
.4121E+01	.1630E+00 $^{+\ .3000E-02}_{-\ .4000E-02}$	.4430E+01 $^{+\ .1020E+01}_{-\ .1020E+01}$	Sch84
.5000E+01	.1250E+00 $^{+\ .3500E-02}_{-\ .3500E-02}$	.3664E+01 $^{+\ .1032E+00}_{-\ .1032E+00}$	Ben66
.6000E+01	.9300E-01 $^{+\ .8000E-02}_{-\ .8000E-02}$	.2838E+01 $^{+\ .2322E+00}_{-\ .2322E+00}$	Ben66
.6200E+01	.8670E-01 $^{+\ .2900E-02}_{-\ .3900E-02}$	.2160E+01 $^{+\ .5000E+00}_{-\ .5000E+00}$	Gil90
.8585E+01	.3400E-01 $^{+\ .1270E-01}_{-\ .2510E-01}$	.1840E+01 $^{+\ .5000E+00}_{-\ .6500E+00}$	Gil90
.1429E+02	.1270E-01 $^{+\ .4700E-02}_{-\ .5600E-02}$	.4820E+00 $^{+\ .7700E-01}_{-\ .1160E+00}$	The91
.1429E+02	.1270E-01 $^{+\ .4700E-02}_{-\ .5600E-02}$	.4820E+00 $^{+\ .7700E-01}_{-\ .1160E+00}$	Gar94
.1781E+02	.1660E-02 $^{+\ .1610E-02}_{-\ .1420E-02}$	.3150E+00 $^{+\ .1000E-01}_{-\ .1100E-01}$	The91
.1781E+02	.1660E-02 $^{+\ .1610E-02}_{-\ .1420E-02}$	.3150E+00 $^{+\ .1000E-01}_{-\ .1100E-01}$	Gar94
.2134E+02	-.1470E-02 $^{+\ .1060E-02}_{-\ .1040E-02}$	.1890E+00 $^{+\ .7000E-02}_{-\ .8000E-02}$	The91
.2134E+02	-.1470E-02 $^{+\ .1060E-02}_{-\ .1040E-02}$	.1890E+00 $^{+\ .7000E-02}_{-\ .8000E-02}$	Gar94

k_z [fm^{-1}]	$\tilde{t}_{20}$	$t_{20}(70°)$	Ref.
.9761E+00	-.3000E+00 $^{+\ .1400E+00}_{-\ .1400E+00}$	-.3000E+00 $^{+\ .1400E+00}_{-\ .1400E+00}$	Dmi85
.1831E+01	-.1800E+00 $^{+\ .7000E-01}_{-\ .7000E-01}$	-.1800E+00 $^{+\ .7000E-01}_{-\ .7000E-01}$	Dmi85
.3028E+01	-.4100E+00 $^{+\ .6000E-01}_{-\ .6000E-01}$	-.4100E+00 $^{+\ .6000E-01}_{-\ .6000E-01}$	Sch84
.4121E+01	-.5900E+00 $^{+\ .1400E+00}_{-\ .1400E+00}$	-.5800E+00 $^{+\ .1300E+00}_{-\ .1300E+00}$	Sch84
.6200E+01	-.7900E+00 $^{+\ .1800E+00}_{-\ .1800E+00}$	-.7500E+00 $^{+\ .1600E+00}_{-\ .1600E+00}$	Gil90
.8585E+01	-.1400E+01 $^{+\ .3700E+00}_{-\ .3700E+00}$	-.1260E+01 $^{+\ .3200E+00}_{-\ .3200E+00}$	Gil90
.1429E+02	-.1450E+01 $^{+\ .1800E+00}_{-\ .1800E+00}$	-.1230E+01 $^{+\ .1800E+00}_{-\ .1800E+00}$	Gar94
.1781E+02	-.9200E+00 $^{+\ .1800E+00}_{-\ .1800E+00}$	-.8300E+00 $^{+\ .1600E+00}_{-\ .1600E+00}$	Gar94
.2134E+02	-.4200E+00 $^{+\ .2100E+00}_{-\ .2100E+00}$	-.4100E+00 $^{+\ .1800E+00}_{-\ .1800E+00}$	Gar94

k_z^2 [fm^{-2}]	$A(k_z^2)$	Ref.	k_z^2 [fm^{-2}]	$A(k_z^2)$	Ref.
.4400E-01	.9358E+00 $^{+.1500E-02}_{-.1500E-02}$	Sim81	.3300E+01	.5390E-01 $^{+.4000E-03}_{-.4000E-03}$	Sim81
.2110E+00	.7450E+00 $^{+.2800E-02}_{-.2800E-02}$	Sim81	.3360E+01	.4844E-01 $^{+.8700E-03}_{-.8700E-03}$	Pla90
.2500E+00	.7101E+00 $^{+.1500E-02}_{-.1500E-02}$	Sim81	.3480E+01	.4478E-01 $^{+.6700E-03}_{-.6700E-03}$	Pla90
.3000E+00	.6659E+00 $^{+.1600E-02}_{-.1600E-02}$	Sim81	.3680E+01	.3928E-01 $^{+.6700E-03}_{-.6700E-03}$	Pla90
.3500E+00	.6220E+00 $^{+.1600E-02}_{-.1600E-02}$	Sim81	.3960E+01	.3233E-01 $^{+.6100E-03}_{-.6100E-03}$	Pla90
.4000E+00	.5911E+00 $^{+.1500E-02}_{-.1500E-02}$	Sim81	.4000E+01	.3550E-01 $^{+.4000E-03}_{-.4000E-03}$	Sim81
.4500E+00	.5548E+00 $^{+.1400E-02}_{-.1400E-02}$	Sim81	.4170E+01	.2874E-01 $^{+.4300E-03}_{-.4300E-03}$	Pla90
.4600E+00	.5455E+00 $^{+.9300E-02}_{-.9300E-02}$	Pla90	.4540E+01	.2282E-01 $^{+.5000E-03}_{-.5000E-03}$	Pla90
.5000E+00	.5247E+00 $^{+.2000E-02}_{-.2000E-02}$	Sim81	.4680E+01	.2072E-01 $^{+.3100E-03}_{-.3100E-03}$	Pla90
.5500E+00	.4972E+00 $^{+.2200E-02}_{-.2200E-02}$	Sim81	.4900E+01	.1856E-01 $^{+.2800E-03}_{-.2800E-03}$	Pla90
.6000E+00	.4662E+00 $^{+.2800E-02}_{-.2800E-02}$	Sim81	.5650E+01	.1247E-01 $^{+.1900E-03}_{-.1900E-03}$	Pla90
.6000E+00	.4698E+00 $^{+.7100E-02}_{-.7100E-02}$	Pla90	.5760E+01	.1164E-01 $^{+.1800E-03}_{-.1800E-03}$	Pla90
.7000E+00	.4198E+00 $^{+.1800E-02}_{-.1800E-02}$	Sim81	.6000E+01	.9180E-02 $^{+.5324E-03}_{-.5324E-03}$	Buc65
.7000E+00	.4148E+00 $^{+.7500E-02}_{-.7500E-02}$	Pla90	.6160E+01	.9860E-02 $^{+.5000E-03}_{-.5000E-03}$	Gal71
.8100E+00	.3743E+00 $^{+.6400E-02}_{-.6400E-02}$	Pla90	.6310E+01	.8904E-02 $^{+.1690E-03}_{-.1690E-03}$	Pla90
.9700E+00	.3107E+00 $^{+.5600E-02}_{-.5600E-02}$	Pla90	.6400E+01	.8421E-02 $^{+.1600E-03}_{-.1600E-03}$	Pla90
.1000E+01	.3103E+00 $^{+.1300E-02}_{-.1300E-02}$	Sim81	.6600E+01	.7570E-02 $^{+.4500E-03}_{-.4500E-03}$	Gal71
.1040E+01	.2954E+00 $^{+.4400E-02}_{-.4400E-02}$	Pla90	.6880E+01	.6730E-02 $^{+.1010E-03}_{-.1010E-03}$	Pla90
.1250E+01	.2387E+00 $^{+.4500E-02}_{-.4500E-02}$	Pla90	.7000E+01	.5710E-02 $^{+.3198E-03}_{-.3198E-03}$	Buc65
.1550E+01	.1911E+00 $^{+.1100E-02}_{-.1100E-02}$	Sim81	.7040E+01	.6400E-02 $^{+.4100E-03}_{-.4100E-03}$	Gal71
.1550E+01	.1778E+00 $^{+.3400E-02}_{-.3400E-02}$	Pla90	.7160E+01	.5743E-02 $^{+.9300E-04}_{-.9300E-04}$	Pla90
.1550E+01	.1858E+00 $^{+.2800E-02}_{-.2800E-02}$	Pla90	.7460E+01	.5028E-02 $^{+.7500E-04}_{-.7500E-04}$	Pla90
.1840E+01	.1400E+00 $^{+.2400E-02}_{-.2400E-02}$	Pla90	.7500E+01	.5390E-02 $^{+.3800E-03}_{-.3800E-03}$	Gal71
.2100E+01	.1241E+00 $^{+.7000E-03}_{-.7000E-03}$	Sim81	.7920E+01	.4169E-02 $^{+.6700E-04}_{-.6700E-04}$	Pla90
.2130E+01	.1127E+00 $^{+.2200E-02}_{-.2200E-02}$	Pla90	.8000E+01	.3630E-02 $^{+.1815E-03}_{-.1815E-03}$	Buc65
.2130E+01	.1140E+00 $^{+.1700E-02}_{-.1700E-02}$	Pla90	.8040E+01	.3958E-02 $^{+.5900E-04}_{-.5900E-04}$	Pla90
.2210E+01	.1109E+00 $^{+.1700E-02}_{-.1700E-02}$	Pla90	.8670E+01	.3151E-02 $^{+.8200E-04}_{-.8200E-04}$	Pla90
.2390E+01	.9145E-01 $^{+.2010E-02}_{-.2010E-02}$	Pla90	.9220E+01	.2413E-02 $^{+.5100E-04}_{-.5100E-04}$	Pla90
.2630E+01	.7825E-01 $^{+.2040E-02}_{-.2040E-02}$	Pla90	.9400E+01	.2343E-02 $^{+.7000E-04}_{-.7000E-04}$	Pla90
.2740E+01	.7249E-01 $^{+.1200E-03}_{-.1200E-03}$	Pla90	.9750E+01	.2050E-02 $^{+.1500E-03}_{-.1500E-03}$	Gal71
.2820E+01	.7014E-01 $^{+.1050E-02}_{-.1050E-02}$	Pla90	.1041E+02	.1562E-02 $^{+.3600E-04}_{-.3600E-04}$	Pla90
.2850E+01	.6633E-01 $^{+.1920E-02}_{-.1920E-02}$	Pla90	.1090E+02	.1490E-02 $^{+.1200E-03}_{-.1200E-03}$	Gal71

k_z^2 [fm^{-2}]	$A(k_z^2)$	Ref.
.1130E+02	.1287E-02 $^{+.7800E-04}_{-.7800E-04}$	Gal71
.1160E+02	.1038E-02 $^{+.2900E-04}_{-.2900E-04}$	Pla90
.1170E+02	.1077E-02 $^{+.6400E-04}_{-.6400E-04}$	Gal71
.1200E+02	.8950E-03 $^{+.1065E-03}_{-.1065E-03}$	Buc65
.1242E+02	.9260E-03 $^{+.6800E-04}_{-.6800E-04}$	Gal71
.1284E+02	.7790E-03 $^{+.3400E-04}_{-.3400E-04}$	Cra85
.1300E+02	.7790E-03 $^{+.6300E-04}_{-.6300E-04}$	Gal71
.1391E+02	.5403E-03 $^{+.1840E-04}_{-.1840E-04}$	Pla90
.1442E+02	.4560E-03 $^{+.4150E-04}_{-.4150E-04}$	Eli69
.1472E+02	.4288E-03 $^{+.3690E-04}_{-.3690E-04}$	Eli69
.1541E+02	.4630E-03 $^{+.4500E-04}_{-.4500E-04}$	Cra85
.1607E+02	.3575E-03 $^{+.3400E-04}_{-.3400E-04}$	Eli69
.1608E+02	.3499E-03 $^{+.1570E-04}_{-.1570E-04}$	Pla90
.1666E+02	.2808E-03 $^{+.2530E-04}_{-.2530E-04}$	Eli69
.1729E+02	.2760E-03 $^{+.2580E-04}_{-.2580E-04}$	Eli69
.1806E+02	.2335E-03 $^{+.1990E-04}_{-.1990E-04}$	Pla90
.1807E+02	.2166E-03 $^{+.2110E-04}_{-.2110E-04}$	Eli69
.1878E+02	.1575E-03 $^{+.1790E-04}_{-.1790E-04}$	Eli69
.1952E+02	.1926E-03 $^{+.2060E-04}_{-.2060E-04}$	Eli69
.2000E+02	.1710E-03 $^{+.4395E-04}_{-.4395E-04}$	Buc65
.2003E+02	.1770E-03 $^{+.7000E-05}_{-.7000E-05}$	Cra85
.2033E+02	.1588E-03 $^{+.1870E-04}_{-.1870E-04}$	Eli69
.2105E+02	.1302E-03 $^{+.1480E-04}_{-.1480E-04}$	Eli69
.2182E+02	.1129E-03 $^{+.1470E-04}_{-.1470E-04}$	Eli69
.2258E+02	.9330E-04 $^{+.1500E-04}_{-.1500E-04}$	Eli69
.2338E+02	.1081E-03 $^{+.1870E-04}_{-.1870E-04}$	Eli69
.2425E+02	.7010E-04 $^{+.1760E-04}_{-.1760E-04}$	Eli69
.2538E+02	.6750E-04 $^{+.1620E-04}_{-.1620E-04}$	Eli69
.2568E+02	.7100E-04 $^{+.2100E-05}_{-.2100E-05}$	Cra85
.2743E+02	.4380E-04 $^{+.1670E-04}_{-.1670E-04}$	Eli69
.3090E+02	.5400E-04 $^{+.1490E-04}_{-.1490E-04}$	Eli69
.3339E+02	.1770E-04 $^{+.2600E-05}_{-.2600E-05}$	Cra85
.3410E+02	.2570E-04 $^{+.1340E-04}_{-.1340E-04}$	Eli69

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